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TPCGEN 95-001
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Ideas on Particle Identification using Ionization Energy Loss

The following is a Monte Carlo study on the estimation of a charged particle's velocity by repeated measurement of its ionization energy loss upon traversing thin layers of gas. Both the truncated mean technique (currently used in ALEPH) and a maximum-likelihood technique are investigated. Significant improvement in particle identification is found when using maximum likelihood.

The probability density $f(\Delta;\beta)$ for the energy loss Δ of a charged particle when traversing a layer of matter was first derived by Landau [Lan44], and is given by

$$f(\Delta;\beta) = \frac{1}{\xi}\,\phi(\lambda) \; , \tag{1}$$

where ξ is a parameter related to the properties of the material and the velocity of the particle $\beta = v/c$, and $\phi(\lambda)$ is the probability density of the dimensionless variable λ. λ is related to the properties of the material, the velocity β, and the energy loss Δ. These quantities are given by

$$\xi = \frac{2\pi N_A e^4 \rho \sum Z}{m_e c^2 \sum A}\,\frac{d}{\beta^2} \; , \tag{2}$$

$$\lambda = \frac{1}{\xi}\left[\Delta - \xi\left(\ln\frac{\xi}{\epsilon'} + 1 - \gamma_E\right)\right] \; , \tag{3}$$

$$\epsilon' = \frac{I^2\, e^{\beta^2}}{2 m_e c^2 \beta^2 \gamma^2} \; , \tag{4}$$

where N_A is Avagadro's number, m_e and e are the mass and charge of the electron, $\sum Z$ and $\sum A$ are the sums of the atomic numbers and atomic weights of the molecular substance, ρ is its density, d is the thickness of the layer, $I = I_0 Z$ with $I_0 \approx 13.5$ eV is an ionization energy characteristic of the material, $\gamma = 1/\sqrt{1-\beta^2}$, and $\gamma_E = 0.5772\ldots$ is Euler's constant. The function $\phi(\lambda)$ is given by

$$\phi(\lambda) = \frac{1}{2\pi i}\int_{\sigma - i\infty}^{\sigma + i\infty} \exp(u \ln u + \lambda u) du \; , \tag{5}$$

where σ is infinitesimal and positive, or equivalently after a variable transformation by

$$\phi(\lambda) = \frac{1}{\pi}\int_0^\infty \exp[-u(\ln u + \lambda)]\, \sin \pi u \, du \; . \tag{6}$$

The integral must be evaluated numerically (see e.g. [Mac69], [CER95]). The energy loss distribution is shown in Fig. 1(a) for several values of the velocity $\beta = v/c$. Because of the long "Landau" tail, the mean and higher moments of the Landau distribution do not exist, i.e. the integral $\int_0^\infty \Delta^n f(\Delta) d\Delta$ diverges for $n \geq 1$. As can be seen from the figure, however, the most probable value Δ_{mp} (i.e. the maximum of the p.d.f.) is sensitive to the particle's velocity. This has been computed numerically in [Mac69] to be

$$\Delta_{mp} = \xi \left[\ln(\xi/\epsilon') + 0.198 \right] , \tag{7}$$

and is shown in Fig. 1(b). Equation (7) (the "Bethe-Bloch formula") is typically used as the basis for identification of charged particles via measurement of their ionization energy loss; see e.g. [All80].

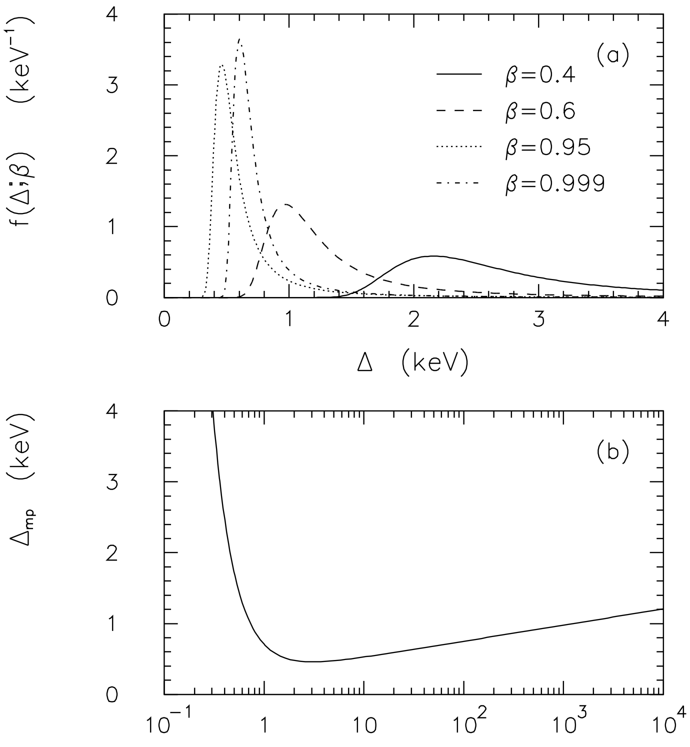

Figure 1: (a) The Landau probability density for the energy loss Δ of a charged particle traversing a 4 mm thick layer of argon gas for various values of the velocity β. (b) The peak position (mode) of the distributions in (a) as a function of $\beta\gamma$ as given by equation (7).

The ALEPH TPC provides up to 350 measurements of a track's ionization energy loss in gas layers of thickness $d/\sin\theta$ with $d = 4$ mm, where θ is the polar angle of a track with respect to the beam line. To roughly simulate these conditions, a Monte Carlo study has been carried out for argon gas with a layer thickness of $d = 0.4$ cm. The gas parameters were: $I_0 = 13.5$ eV, $Z = 18$, $A = 39.95$ and $\rho = 1.783 \cdot 10^{-3}$ g/cm^3.

Samples of 300 measurements of the energy loss Δ were generated according to a Landau distribution $f(\Delta;\beta)$ using the CERNLIB routine RANLAN (G110) with various values of the velocity parameter $\beta\gamma$. A maximum likelihood fit for $\beta\gamma$ was done using the routine DENLAN (also included in CERNLIB routine G110). An example is shown in Fig. 2 generated with $\beta\gamma = 4$.

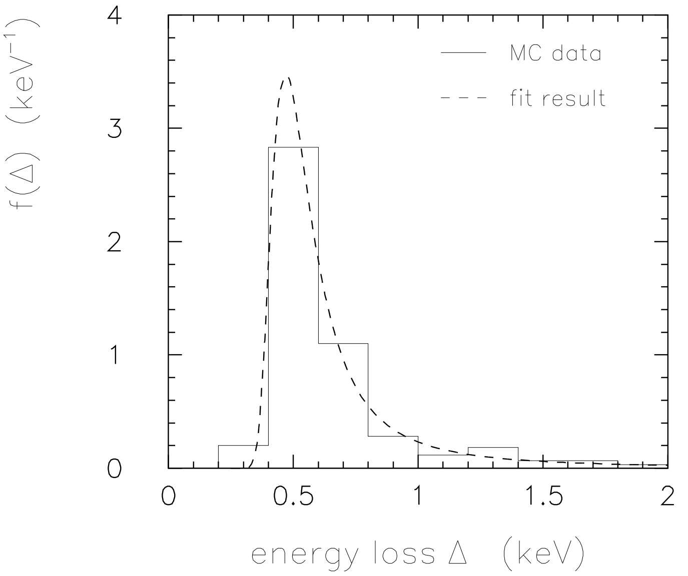

Figure 2: Maximum likelihood fit of a Landau distribution to 300 MC generated energy-loss values.

The relative standard deviations of the fitted $\beta\gamma$ values $\sigma_{\beta\gamma}/\beta\gamma$ are shown in Fig. 3 as a function of the input value of $\beta\gamma$. Around $\beta\gamma \approx 3$ one is close to the minimum value of the most probable energy loss. Since the position of Δ_{mp} gives the most information on $\beta\gamma$, and Δ_{mp} is to first order independent of $\beta\gamma$ around the minimum, the error in the estimated $\beta\gamma$ increases significantly in this region. In the relativistic rise region, the relative error in $\beta\gamma$ is around 5%. Assuming the error in the momentum measurement is negligible, this would correspond to a relative error in the estimated mass $m = \frac{p}{\beta\gamma}$ of 5%.

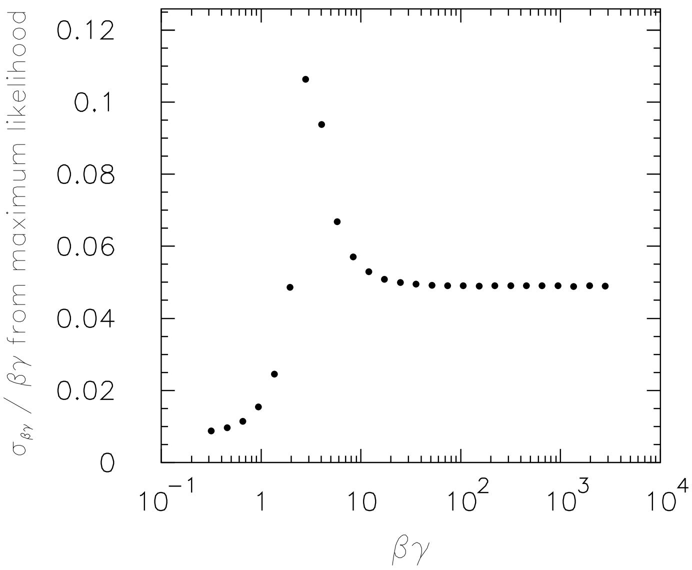

Figure 3: The relative standard deviation of the estimated $\beta\gamma$ from a maximum-likelihood fit $\sigma_{\beta\gamma}/\beta\gamma$ as a function of the generated $\beta\gamma$.

An interesting feature of the maximum likelihood approach is that one can distinguish between values of $\beta\gamma$ that correspond to the same value of the most probable energy loss

Δ_{mp}. Two likelihood functions are shown in Fig. 4 as a function of $\beta\gamma$, where 300 values of Δ were generated using $\beta\gamma = 4$ (a) and $\beta\gamma = 6$ (b). In both cases one sees two maxima, corresponding to the two values of $\beta\gamma$ with the same value of the most probable energy loss Δ_{mp}. The maximum-likelihood estimators for $\beta\gamma$ are found to be $\widehat{\beta\gamma} = 4.45 \pm 0.42$ and $\widehat{\beta\gamma} = 6.46 \pm 0.42$, in each case corresponding to the higher of the two maxima. If only the truncated mean of the observed energy-loss values were used, the solutions from the two maxima would be equally valid (see Fig. 1). By using the likelihood function, however, one sees that the correct solutions are clearly preferred over the alternative values.

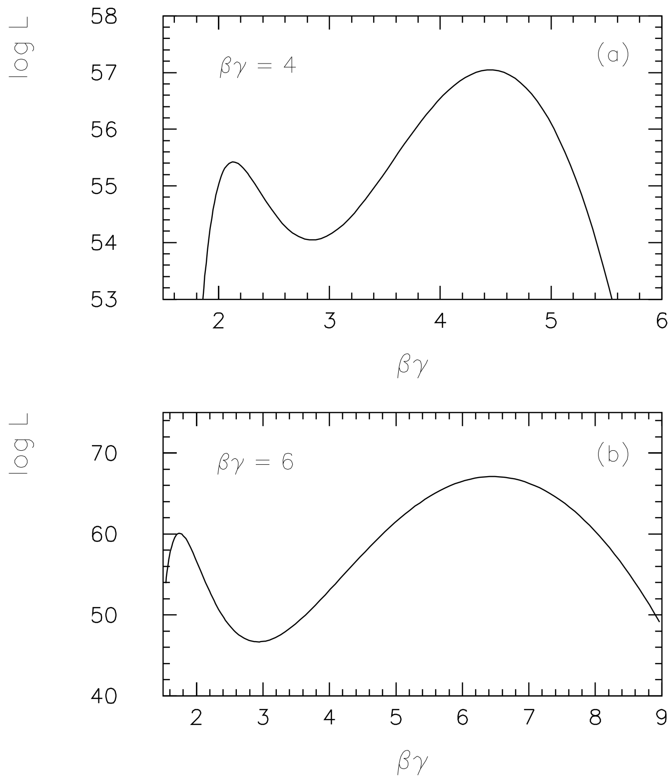

Figure 4: Log-likelihood as a function of $\beta\gamma$ for two samples of 300 energy loss values, with (a) $\beta\gamma = 4$ and (b) $\beta\gamma = 6$. The two maxima correspond to the two values of $\beta\gamma$ with the same most probable energy loss Δ_{mp}.

The results presented are significantly better than those currently obtained by the ALEPH TPC. If the MC model here is used to compute truncated mean values, however, the results are also significantly better than ALEPH's current measurements, indicating that there exist other effects degrading the particle i.d. information besides just the loss of information associated with the truncated mean.

This was investigated by computing values of the 60% truncated mean for the 300 generated energy-loss values described above. This is shown in Figure 5 along with the most probable energy loss from equation (7). The truncated mean is clearly a good estimator for Δ_{mp}. The standard deviation of the truncated mean was estimated by generating 100 samples of 300 energy-loss values at each value of $\beta\gamma$. Figure 6 shows the relative resolution for the truncated mean, and Fig. 7 shows the corresponding relative $\beta\gamma$ resolution. This shows a structure similar to that found using maximum likelihood; cf. Fig. 3. The values of $\sigma_{\beta\gamma}/\beta\gamma$ for $\beta\gamma > 10$ are, however, a factor of $2-4$ higher than those obtained using maximum likelihood.

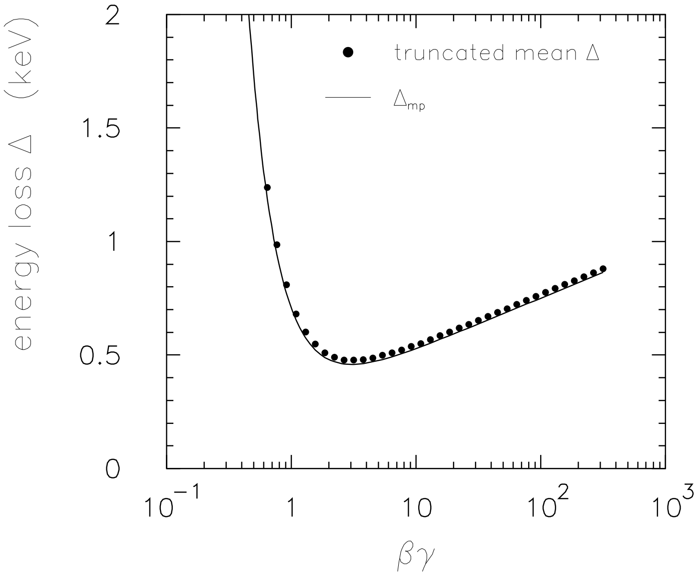

Figure 5: The truncated mean energy loss averaged over 100 samples and the most probable energy loss Δ_{mp} as a function of $\beta\gamma$.

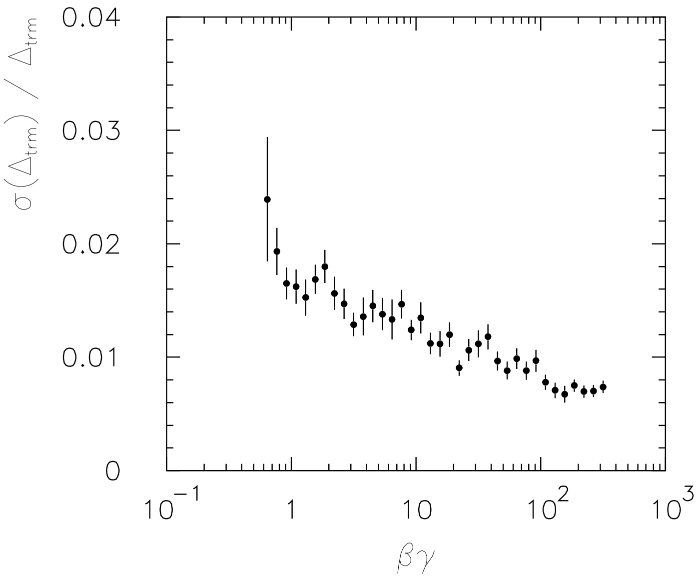

Figure 6: Relative resolution of the truncated mean as a function of $\beta\gamma$.

The relative resolution of the truncated mean is in the 1% range, a factor of $4-5$ better than what is currently achieved with the ALEPH TPC. The resolution in $\beta\gamma$ is about 20% for $\beta\gamma \approx 10$, and improves somewhat for increasing $\beta\gamma$. This improvement is not realistic, since the density effect is not included in the model. But a 20% $\beta\gamma$ resolution (i.e. a 5σ kaon-proton separation) is still significantly better than what is currently attained ($\approx 1\sigma$). The reason for this discrepancy is not entirely clear.

An interesting project would be to analyze a sample of the ALEPH raw data in order to see if a significant improvement in particle identification can be achieved with the

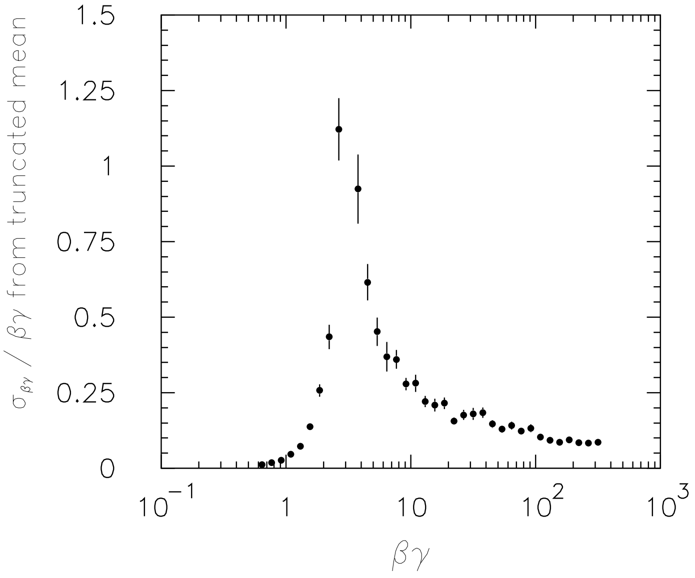

Figure 7: Relative resolution for the estimated $\beta\gamma$ based on the truncated mean energy loss as a function of $\beta\gamma$.

maximum-likelihood technique. In addition one could investigate why the MC model predicts five times better resolution for the truncated mean than that actually achieved. Despite this discrepancy with current measurements, the maximum-likelihood method gives a resolution for $\beta\gamma$ a factor of $2 - 4$ better than the truncated-mean method, i.e. $\sigma_{\beta\gamma}/\beta\gamma$ around 5% rather than $10-20\%$, so some improvement from maximum likelihood can be expected.

A number of refinements to Landau's energy loss model would be necessary. For example, the density effect would need to be included not only in the most probable energy loss, but for the entire energy-loss distribution. Calibration constants related to temperature and pressure variations would need to be applied to all of the energy-loss measurements, not just to the truncated mean. If a significant improvement could be attained, it might be worth reprocessing a subset of the ALEPH data, particularly for special events (rare decays, Higgs candidates, etc.) for which one would like to exploit as much information as possible.

References

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[Lan44] L. Landau, J. Phys. USSR, **8** (1944) 201.

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