Studying QCD in e^+e^- collisions

Part I: jets, event-shapes, α_s, etc.

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Outline

(i) QCD in $e^+e^- \rightarrow$ hadrons

- general theoretical and experimental picture
- Monte carlo models
- defining observables

(ii) QCD with jets and event-shapes

- parton spin
- $\alpha_{\rm s}$ from jets and event-shapes
- running of $\alpha_{\rm s}$
- flavour independence of $\alpha_{\rm s}$, measuring m_b

(iii) $\alpha_{\rm s}$ from total cross sections and BRs: R_l, R_τ

(iv) Conclusions (and what we missed)

Why study QCD? (experimentalist's view)

(i) To see if QCD is Nature's theory of strong interactions?

> Sure, but most already convinced.

(ii) To measure its free parameters?

> Yes: α_s, quark masses,
> needed to understand bigger picture.

(iii) To help search for physics beyond the SM?

> Yes: understand 'QCD background',
> deviation from firm QCD prediction $\rightarrow$ new physics

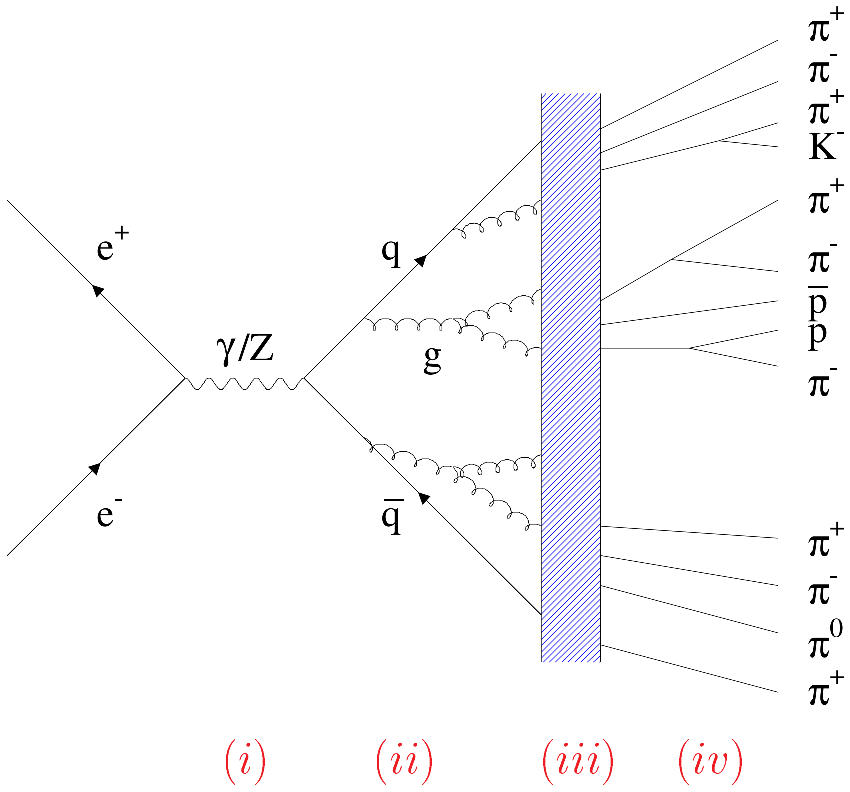
(iv) Test validity of QCD calculations?

> Yes, but not usually driving concern.

(v) Help understand non-perturbative QCD?

> Ball (mostly) in theorists' court.

$e^+e^- \to$ hadrons



(i) electroweak

(ii) perturbative QCD

(iii) hadronization (non-perturbative QCD)

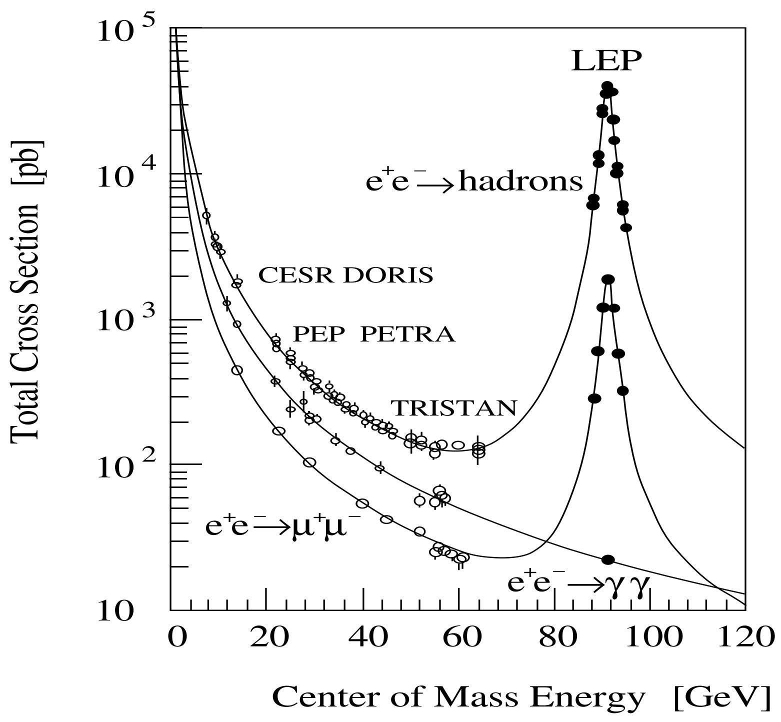
(iv) resonance decays

Usually define observable to be sensitive to only one of the above.

QCD with e^+e^- annihilation: the data

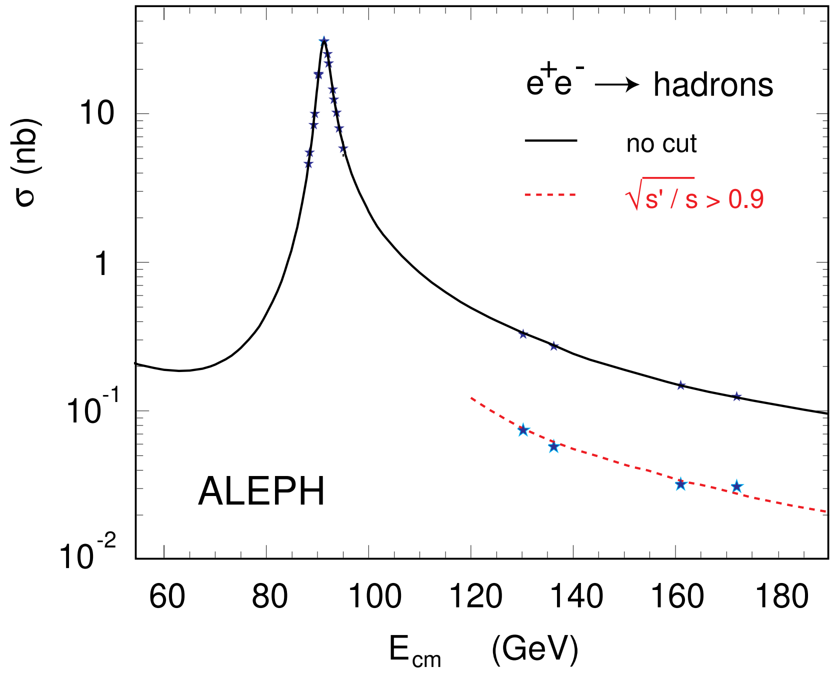
SPEAR (1972)	$E_{\rm cm} = 8$ GeV
PETRA (1978)	14 GeV $< E_{\rm cm} <$ 44 GeV
PEP (1980)	$E_{\rm cm} = 29$ GeV
TRISTAN (1987)	$E_{\rm cm} = 64$ GeV
SLC (1989)	$E_{\rm cm} = 91$ GeV
LEP I (1989)	$E_{\rm cm} = 91$ GeV

ca. 4×10^6 hadronic events each for ALEPH, DELPHI, L3, OPAL at $E_{\rm cm} \approx M_{\rm Z}$.



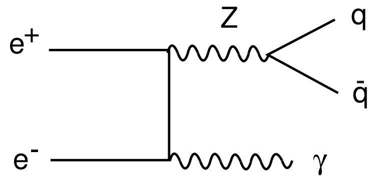
More e^+e^- annihilation data

LEP II (1996) $\quad$ 130 GeV $< E_{\rm cm} <$ 208 GeV



LEP II hadronic cross section $\sim 10^3$ smaller than at Z peak

Many events with initial state photon radiation: $m_{q\bar{q}} \approx M_{\rm Z}$



'Background' from $e^+e^- \to W^+W^- \to$ hadrons

Important for tests of QCD $E_{\rm cm}$ dependence

Monte Carlo models: perturbative phase

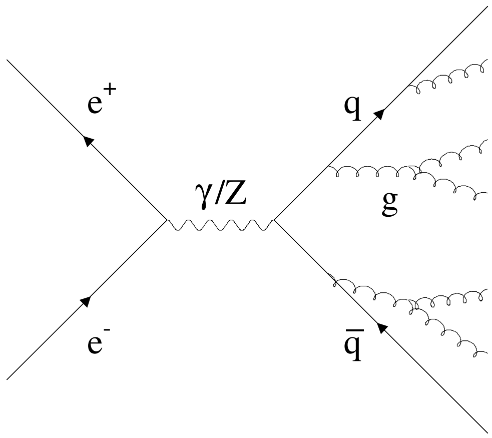
Use random numbers to select a partonic final state and generate all momentum vectors

Usually based on $\mathcal{O}(\alpha_s)$ QCD combined with 'parton shower': leading-log approx. (valid in limit of collinear gluon radiation)

$+$ angular ordering

$+$ (sometimes) next-to-leading logs

$+ \ldots$



$\rightarrow$ generates set of partons for each event

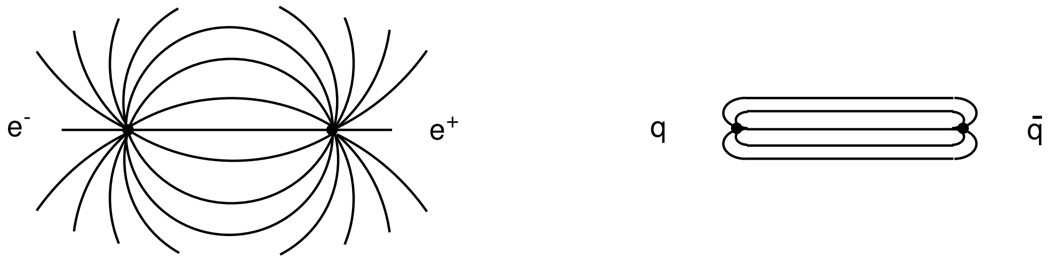
This *is* perturbative QCD but not at its most accurate.
(α_s in MC $\neq$ α_s in $\overline{\mathrm{MS}}$ scheme)

MC with $\mathcal{O}(\alpha_s^2)$ matrix element also available, but without parton shower (max of 4 partons in event)

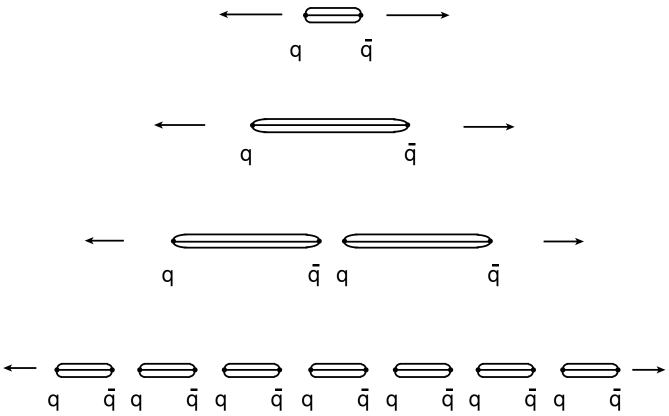
Monte Carlo models: string hadronization

QCD inspired models (e.g. string) convert partons into hadrons

In contrast to electric charges, ‘chromoelectric’ field between $q\overline{q}$ pair confined to narrow flux tube (string):



$q\overline{q}$ production in flux tube $\rightarrow$ string breaks $\rightarrow$ mesons



MC generates flavours of $q\overline{q}$ pairs ($u : d : s \approx 1 : 1 : 0.3$)

space–time location of breaks $\rightarrow$ momenta of hadrons

Gluons $\rightarrow$ momentum carrying kinks in string

$\rightarrow$ ‘Lund family’ of models: JETSET, ARIADNE, PYTHIA …

Monte Carlo models: cluster hadronization

Cluster model (program HERWIG):

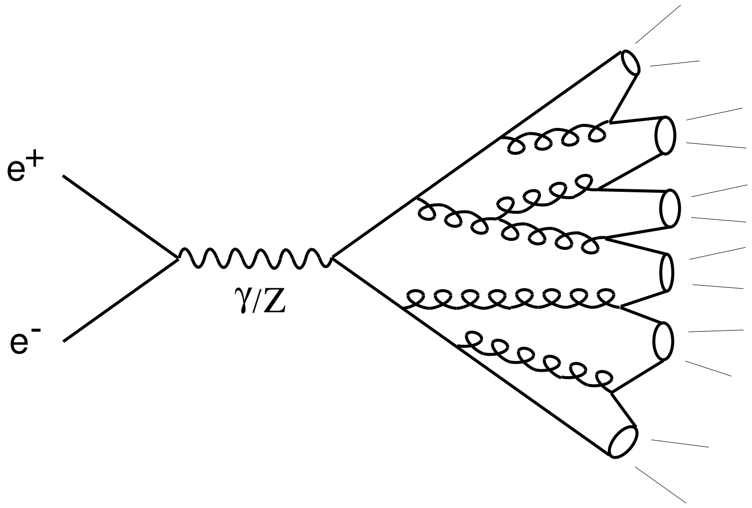
parton shower ends with virtual mass of all partons $= Q_0$;

gluons split into $q\overline{q}$ pairs;

neighbouring q and $\overline{q}$ form colour neutral clusters;

clusters (usually) decay isotropically into two hadrons;

exceptions allowed for very light and very heavy clusters



Parameters:

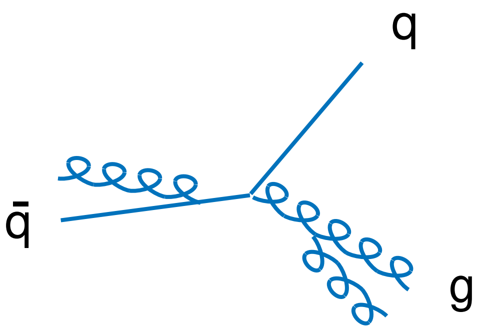
Λ_{QCD}, Q_0, quark masses,

parameters for treatment of very light/heavy clusters

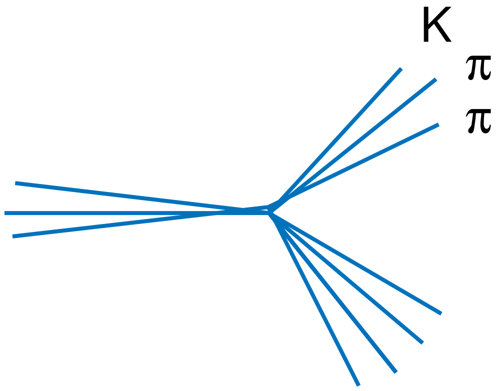
and other tweaks mainly related to flavour production.

Comparing theory and experiment

Need to compare QCD prediction . . .



with measurement . . .



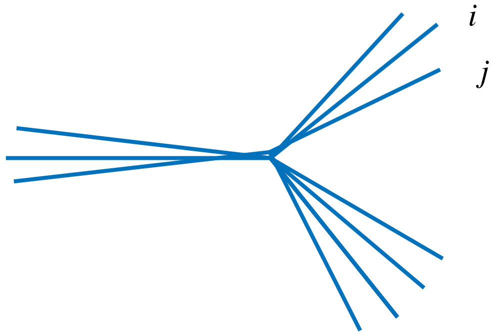
using appropriately defined jet rates, event-shape variables

infrared, collinear safe;

not overly sensitive to hadronization effects

Defining jets

Clustering algorithms: for every pair, compute 'distance' y_{ij}



← partons or hadrons

$$\text{e.g.}\quad y_{ij} = \begin{cases} \dfrac{2E_iE_j(1-\cos\theta_{ij})}{s} & \text{(JADE)} \\[2ex] \dfrac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{s} & \text{(Durham)} \end{cases}$$

(i) Find pair with smallest y_{ij}

(ii) if less than a given y_{cut}, replace i, j with pseudoparticle:

$$p^\mu = p_i^\mu + p_j^\mu \qquad \text{('E' scheme)}$$

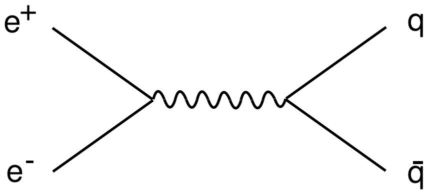
(iii) iterate until all $y_{ij} > y_{\mathrm{cut}}$

remaining pseudoparticles → jets

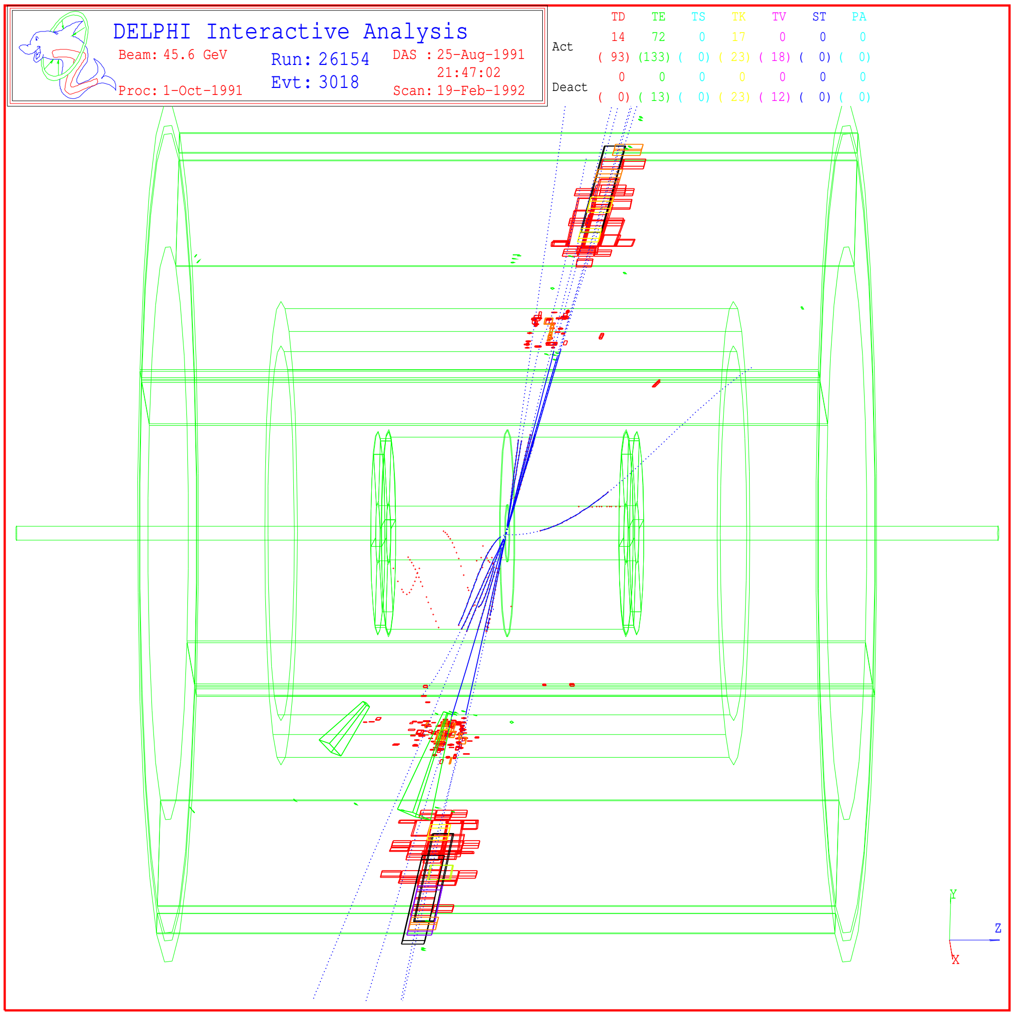
Other jet definitions also used, e.g., cone algorithm.

Two-jet events

$e^+e^- \to q\overline{q}$ leads to two back-to-back jets of hadrons

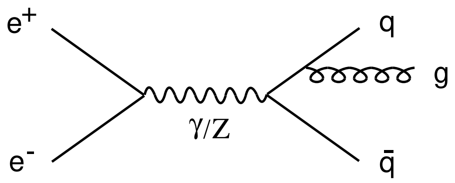


$\to$ angular distribution of jets depends on quark spin

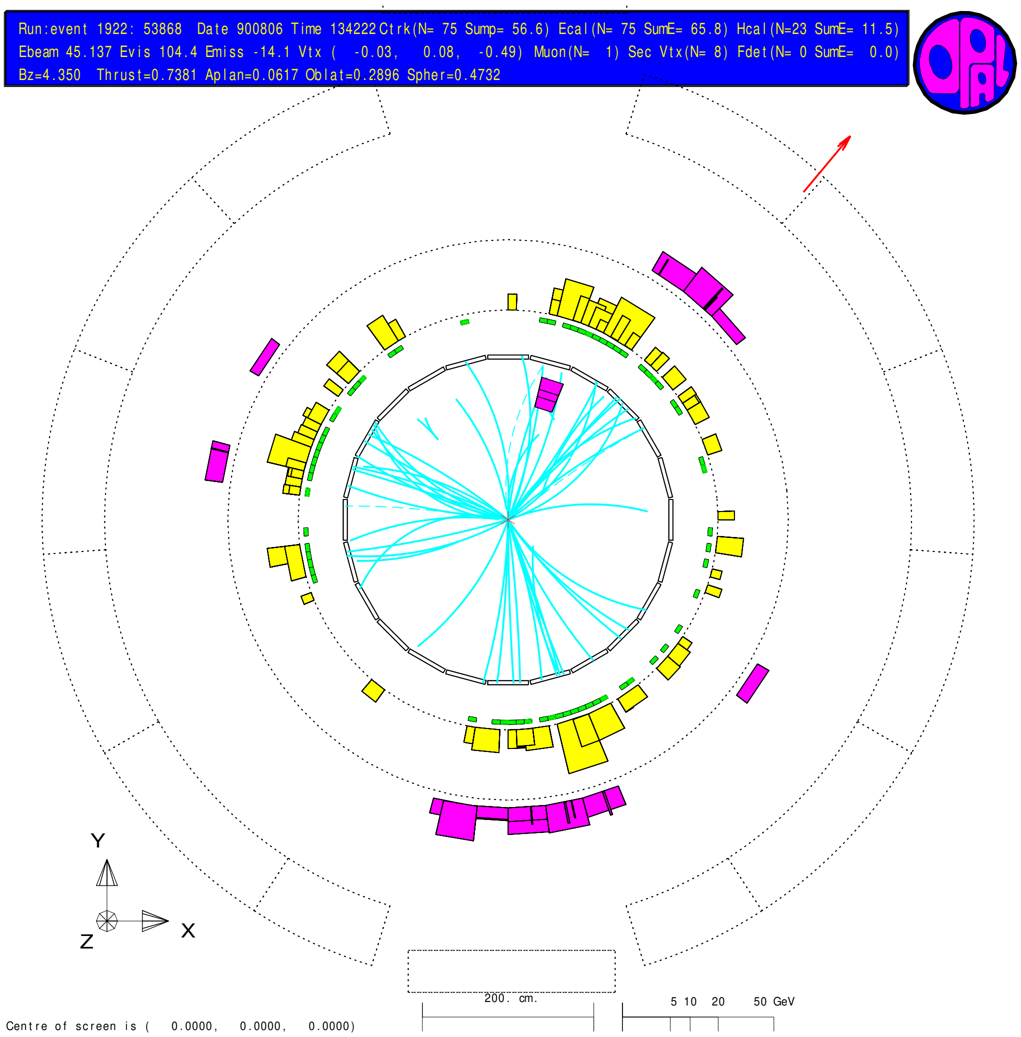


Three-jet events

Bremsstrahlung-like gluon radiation (cf. $e^+e^- \to \mu^+\mu^-\gamma$)

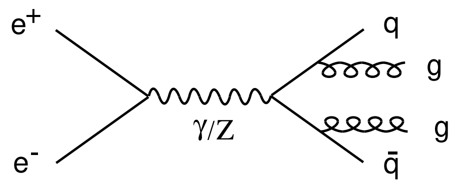


Additional jets $\to$ rate sensitive to strong coupling α_s

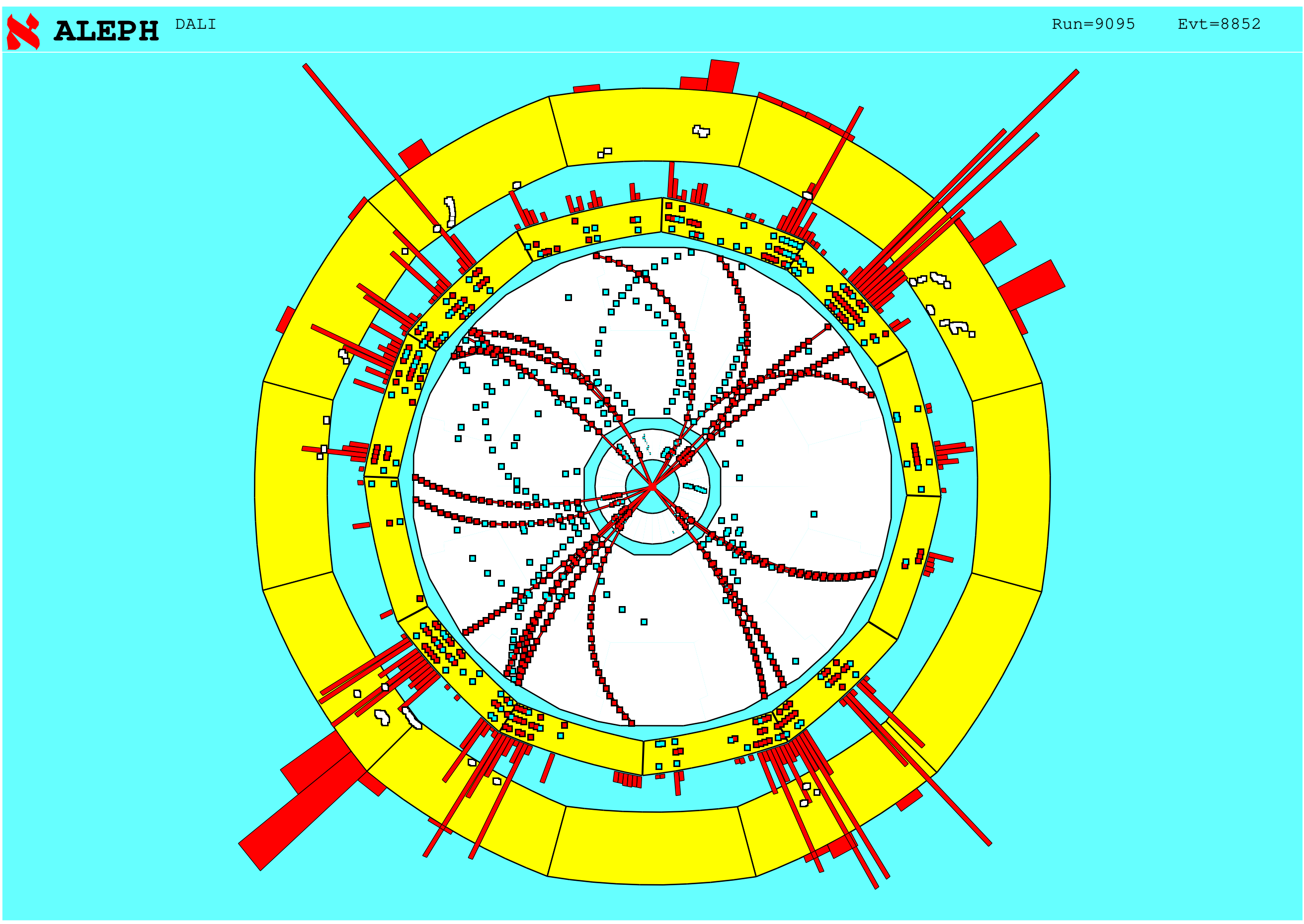


Multijet events

More gluon radiation leads to multijet events

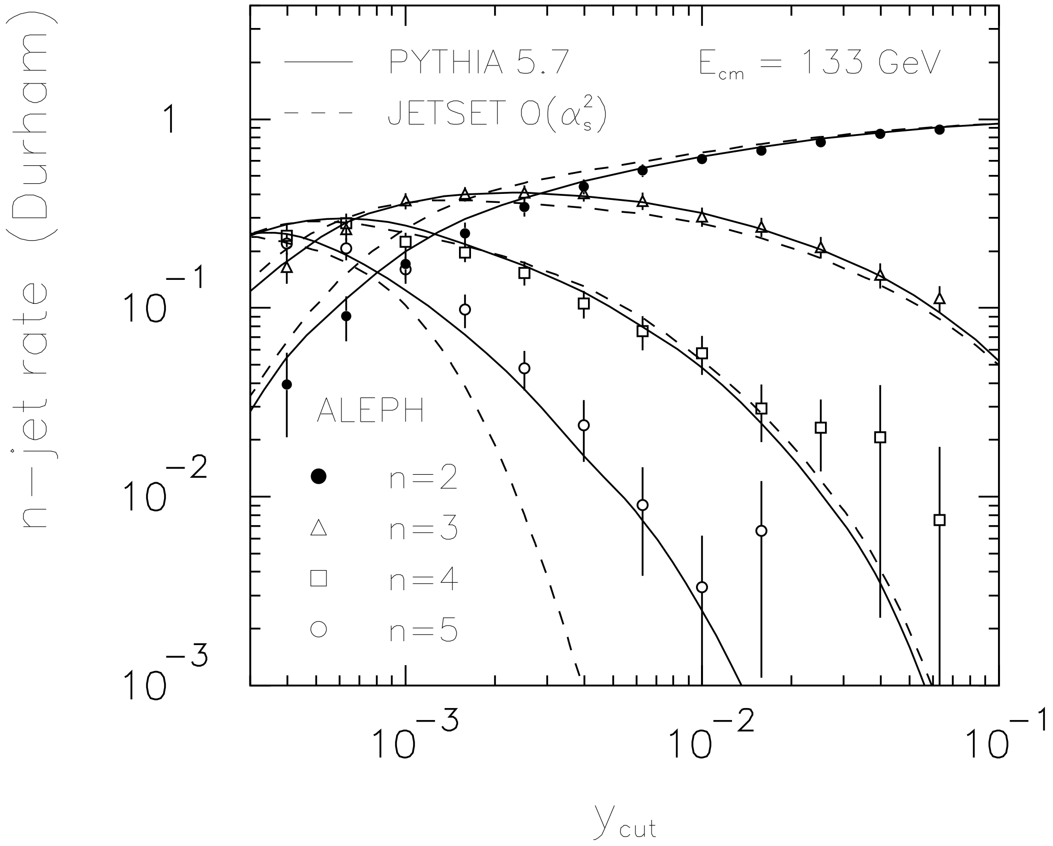


→ sensitive to triple-gluon vertex, non-abelian character of QCD



Jet production rates

Relative rate of finding n jets for $n = 2, 3, 4, 5$

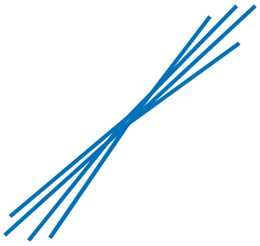


Parton shower based model (PYTHIA) gives good description of multijet rates

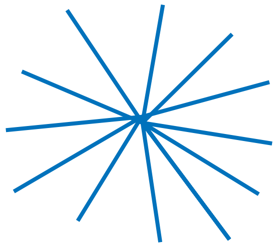
$\mathcal{O}(\alpha_s^2)$ based model has at most 4 partons in final state, falls short for rates of $n \geq 5$ jets

Event-shape variables

Thrust: $$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\rm T}|}{\sum_i |\vec{p}_i|}$$



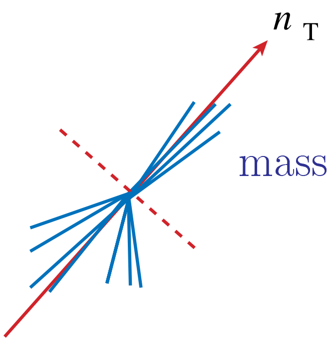
$T \approx 1$



$T \approx 1/2$

Heavy jet mass: divide event into hemispheres with thrust axis

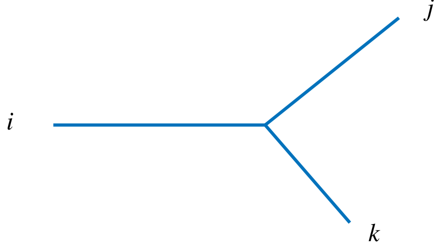
mass $M_{\rm h}$



mass $M_{\rm l}$

Often use $\rho = M_{\rm h}^2/s$

y_3: cluster event to three jets

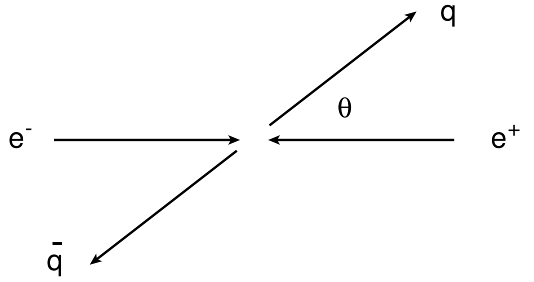


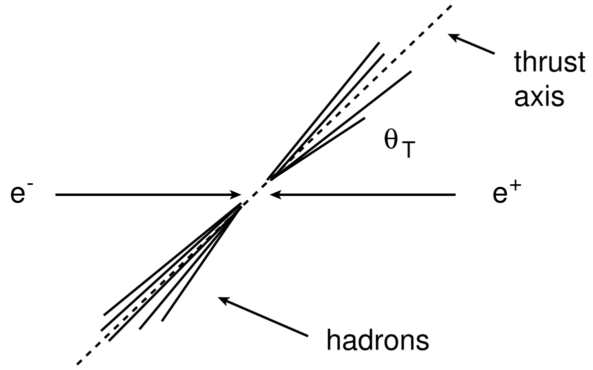
$$y_3 = \min(y_{ij})$$

Spin of quarks

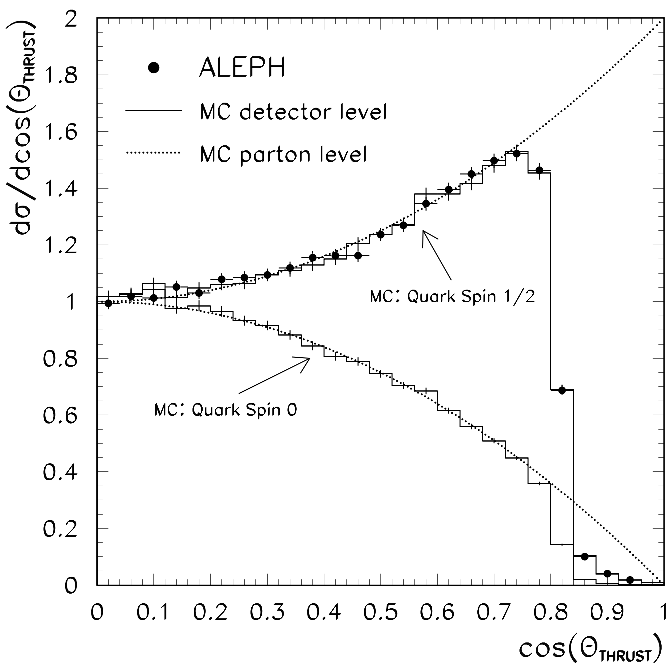
distribution of outgoing quark's angle relative to incoming e^-

$$\frac{d\sigma}{d\cos\theta} \sim \begin{cases} 1+\cos^2\theta & \text{spin-}\frac{1}{2}\text{ quarks} \\ 1-\cos^2\theta & \text{spin-0 quarks} \end{cases}$$





estimate θ with angle of thrust axis (doesn't distinguish q direction)



$\rightarrow$ quarks have spin $\frac{1}{2}$

QCD predictions to $\mathcal{O}(\alpha_s^2)$

Consider variable $y = y_3,\ \rho,\ 1-T,\ \ldots$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = A(y)\frac{\alpha_s(\mu)}{2\pi} + \left[B(y) + 2\pi b_0 A(y)\ln\left(\frac{\mu^2}{s}\right)\right]\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2$$

$A(y),\ B(y)\ =$ computable functions

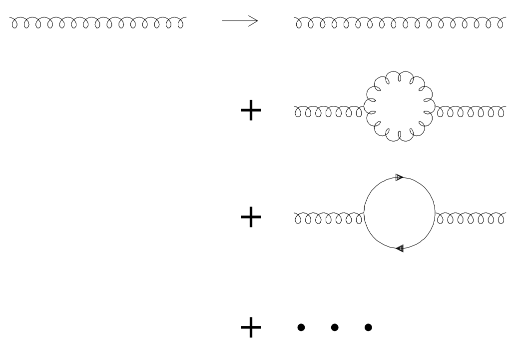
$\mu =$ renormalization scale

$d\sigma/dy$ should be independent of μ (to $\mathcal{O}(\alpha_s^2)$)

$\rightarrow$ this determines the μ dependence of α_s (RGE)

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(Q)} + b_0 \ln\left(\frac{\mu^2}{Q^2}\right), \qquad b_0 = \frac{33 - 2n_f}{12\pi}$$

Equivalent to



This μ business

- μ reflects an ambiguity of perturbation theory

  not a QCD parameter

- Suppose we measure $\alpha_s(\mu)$ with some μ,

  Use RGE: $\alpha_s(\mu) \to \alpha_s(M_Z)$

  resulting $\alpha_s(M_Z)$ still depends on chosen μ

  since μ dependence only cancels to $\mathcal{O}(\alpha_s^2)$

- Higher order coefficients will contain $\sim \left[\ln\left(\frac{\mu^2}{s}\right)\right]^n$

  $\to$ $\mu^2 \approx s$ gives some hope that series is converging.

- But . . . at $\mathcal{O}(\alpha_s^2)$, data best described with $\mu^2 \approx 0.002s$ (!?!)

  $\to$ need higher order terms

Resumming large logs

Consider cumulative distribution $R(y) = \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy'}dy'$

$$
\begin{aligned}
\ln R(y) = {} & \alpha_s (G_{12}\ln^2 y + G_{11}\ln y + \ldots) \\
& + \alpha_s^2 (G_{23}\ln^3 y + G_{22}\ln^2 y + \ldots) \\
& + \alpha_s^3 (G_{34}\ln^4 y + G_{33}\ln^3 y + \ldots) \\
& + \alpha_s^4 (G_{45}\ln^5 y + G_{44}\ln^4 y + \ldots) \\
& + \ldots
\end{aligned}
$$

leading logs next-to-leading logs

Large logs dominate for $y \to 0$ (two-jet region)

LL and NLL summed to all orders for several variables (including $1-T$, y_3, M_h^2/s)

Matching $\mathcal{O}(\alpha_s^2)$ and (N)LL parts:

subtract double-counted part using R or $\ln R$? (difference $\mathcal{O}(\alpha_s^3)$)

Data no longer prefer small μ

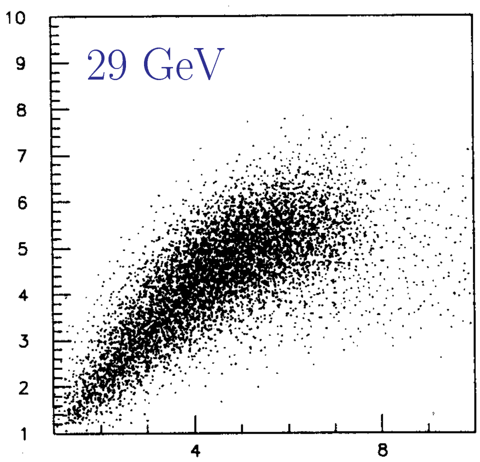
estimate (roughly) magnitude of missing higher orders by varying μ , e.g., $-1 < \ln(\mu^2/s) < 1$

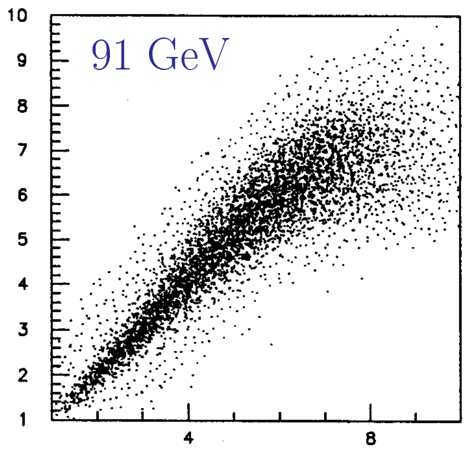
Hadronization corrections

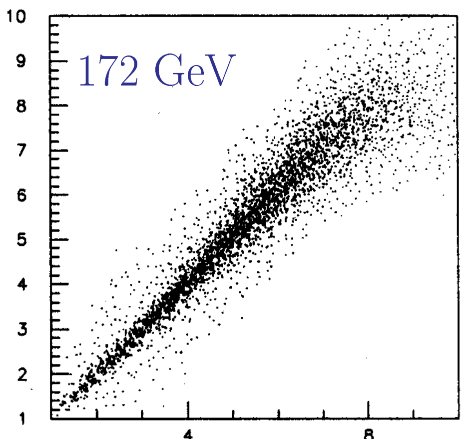
$$\left(\frac{d\sigma}{dy}\right)_{\rm had} (\text{bin } i; \alpha_{\rm s}) \;=\; \sum_j \left(\frac{d\sigma}{dy}\right)_{\rm QCD} (\text{bin } j; \alpha_{\rm s}) \;\cdot\; P_{ij}$$

$$P_{ij} \;=\; P\left(\begin{array}{c|c} \text{hadron level} & \text{parton level} \\ \text{in bin } i & \text{in bin } j \end{array} \right) \qquad \leftarrow \text{from MC model}$$

e.g. for $-\ln y_3$ hadron vs. parton level (JETSET):







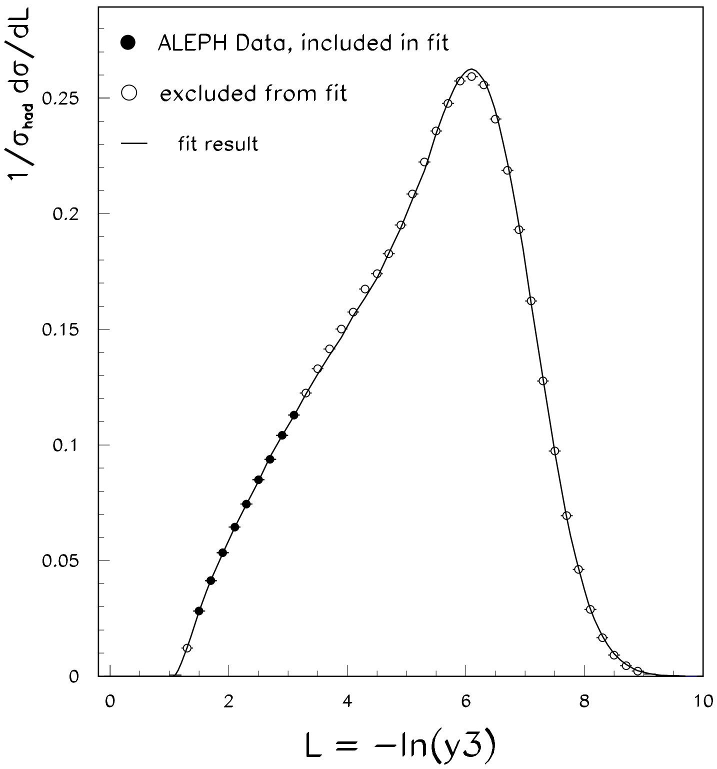
Partons follow hadrons

better with increasing $E_{\rm cm}$

Fitting α_s to event-shape distributions

Example with y_3:

restrict fit range to three-jet region; data also well described outside fit range



$$\alpha_s(M_Z) = 0.1195 \pm 0.0002 \text{ (stat.)} \;\; \pm 0.0038 \text{ (sys.)}$$

Systematic error usually dominated by theory (as here)

hadronization corrections: try different models.

missing higher orders: try varying μ in 'reasonable range',

vary matching scheme to combine NLLA and $\mathcal{O}(\alpha_s^2)$ parts.

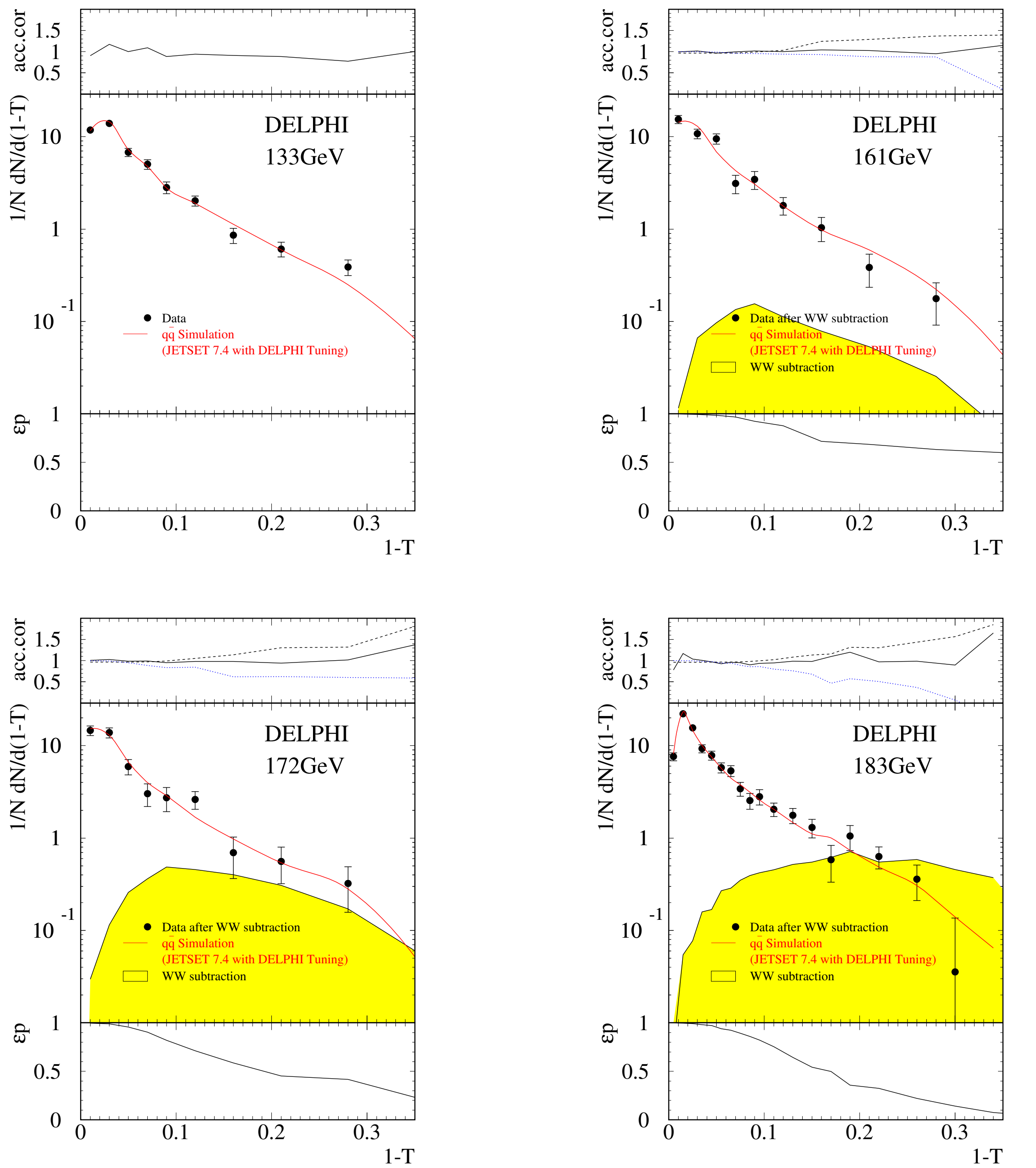
Event-shape distributions at different E_{cm}

Many systematic uncertainties in α_s common to all E_{cm}

$\rightarrow$ not a problem for studying running of α_s,

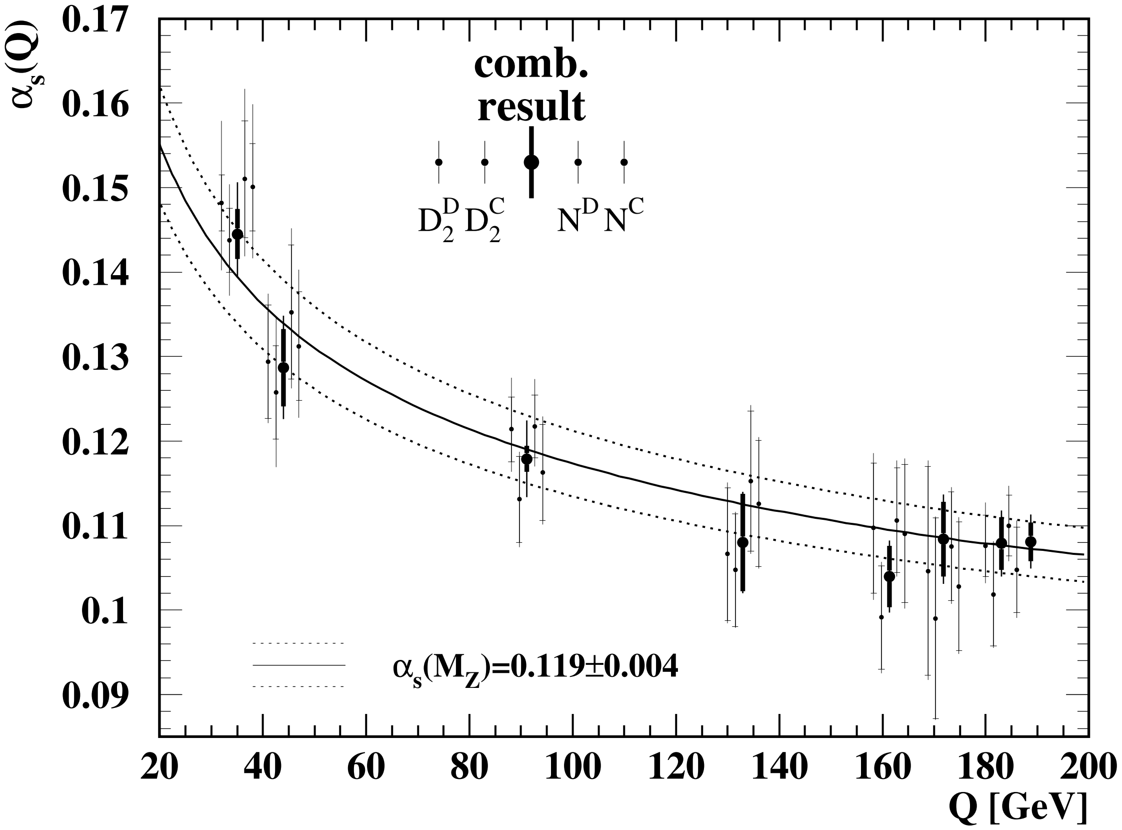
but . . . LEP II has hadronic events from $e^+e^- \rightarrow W^+W^-$

initial state photon radiation, low statistics . . .



Event-shape distributions at different E_{cm}

Common α_s measurements by JADE and OPAL experiments



Inner error bars – uncorrelated errors (e.g. stat.)

Outer error bars – total uncertainty

→ good agreement with predicted running.

Flavour independence of α_s

Does $q\bar{q}g$ coupling depend on quark flavour?

QCD: no. (Still, good to test.)

Select event samples enriched in $b\bar{b}$, $c\bar{c}$ and uds

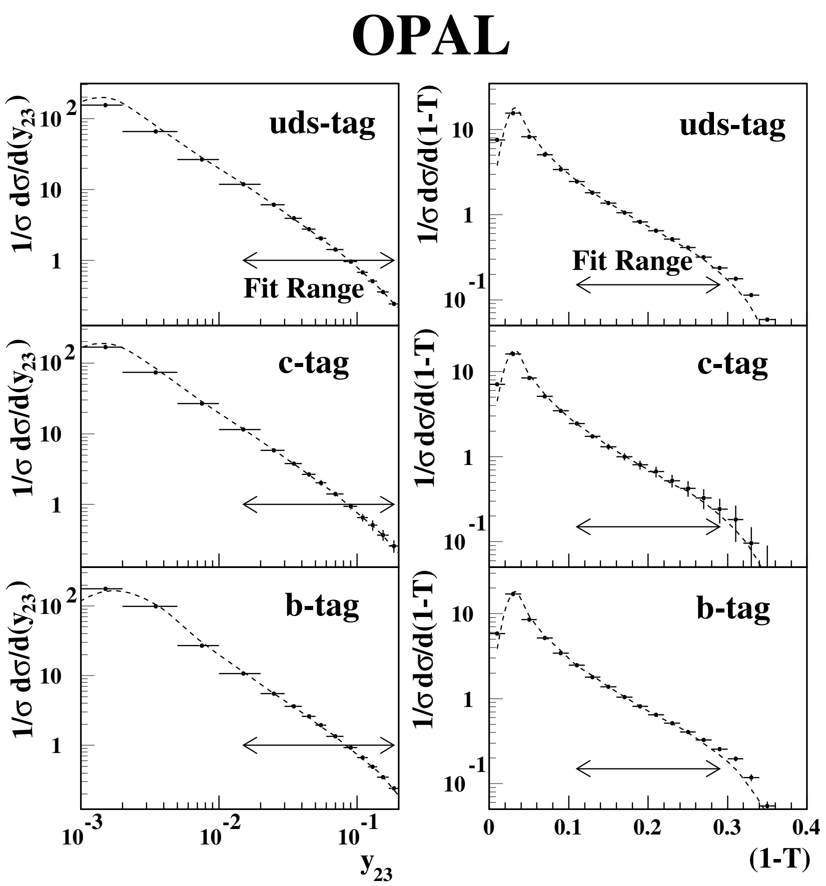
lifetime tag $\rightarrow b$

lifetime antitag $\rightarrow uds$

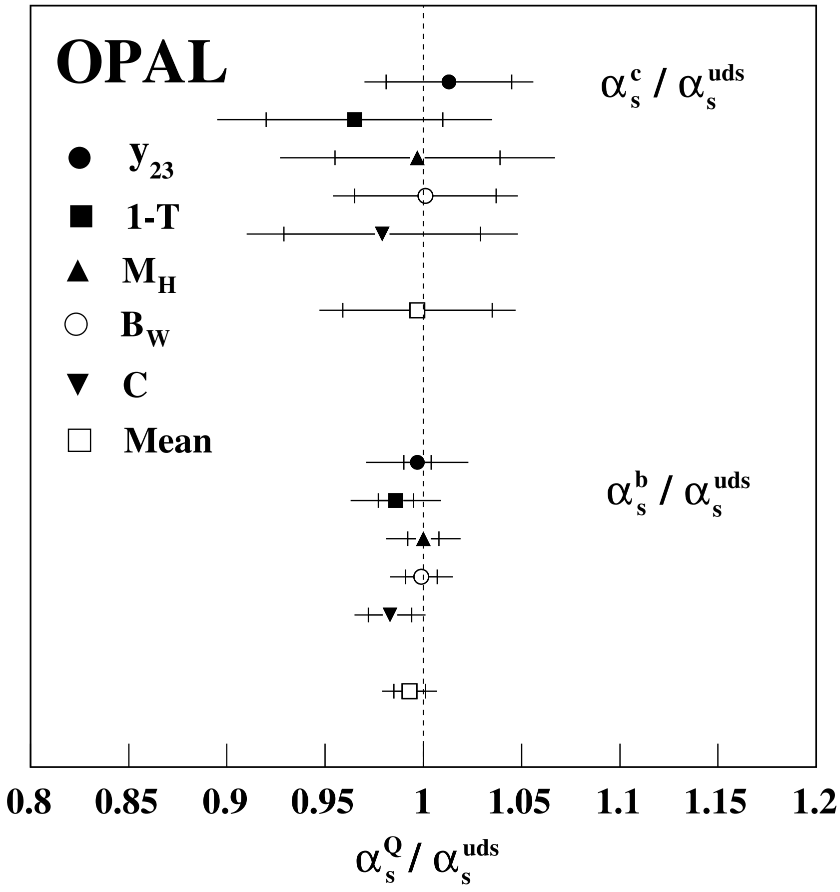
fast D mesons in jets $\rightarrow c$

Fit α_s^{uds}, $\alpha_s^c/\alpha_s^{uds}$, $\alpha_s^b/\alpha_s^{uds}$ as separate parameters,

use $\mathcal{O}(\alpha_s^2)$ including with b, c mass effects.



Flavour independence of α_s: results



Averages:

$$\alpha_s^c/\alpha_s^{uds} = 0.997 \pm 0.038\ (\text{stat.})\ \pm 0.030\ (\text{sys.})\ \pm 0.012\ (\text{theo})$$

$$\alpha_s^b/\alpha_s^{uds} = 0.993 \pm 0.008\ (\text{stat.})\ \pm 0.006\ (\text{sys.})\ \pm 0.011\ (\text{theo})$$

$\rightarrow$ α_s flavour independence tested to $\sim$ percent level.

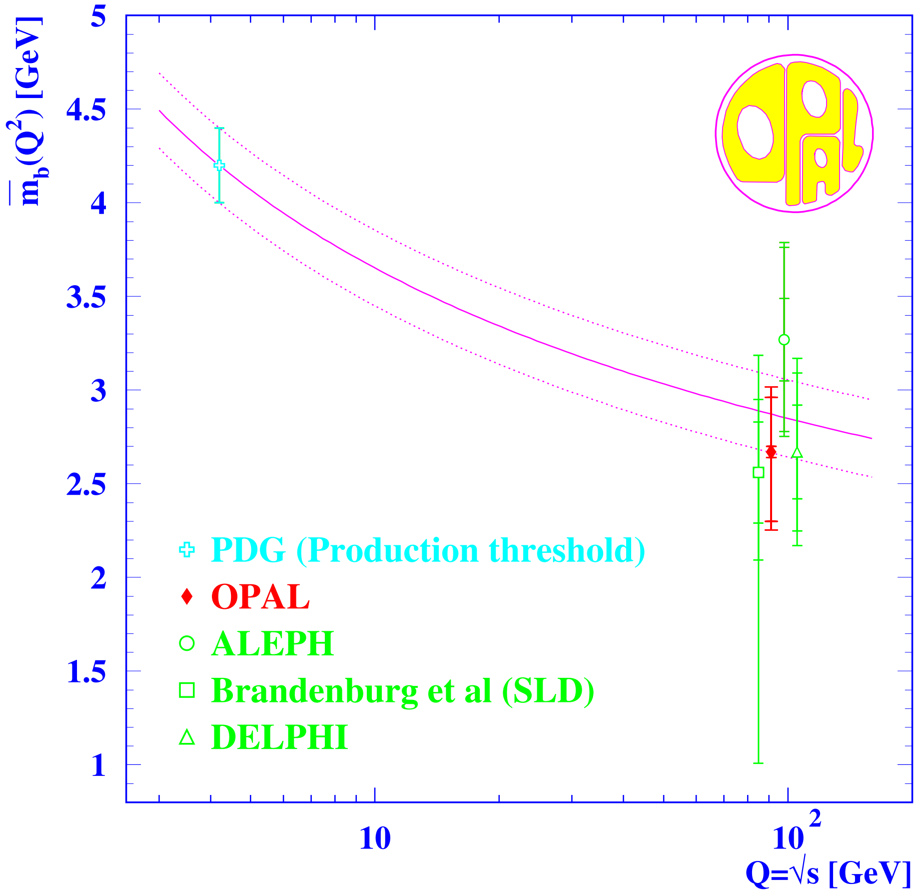
m_b

High b-mass suppresses gluon radiation,

same reason less Bremsstrahlung for muons

$\rightarrow$ event-shapes, jet-rates differ for $b\bar{b}$, uds events

Here *assume* α_s flavour independent, fit m_b



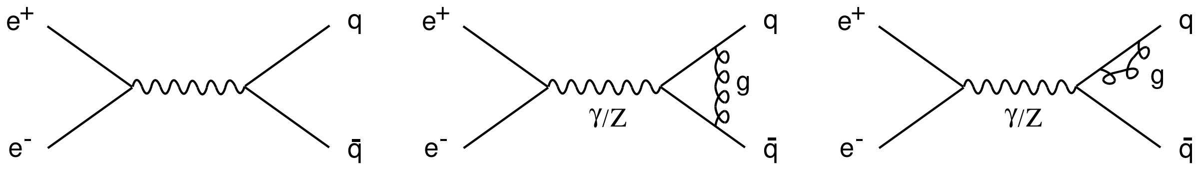
Running of mass observed, e.g., OPAL finds

$$m_b(M_Z) = 2.67 \pm 0.03 \text{ (stat) } \pm^{0.29}_{0.37} \text{ (sys) } \pm .19 \text{ (theo)}$$

α_s from the total hadronic cross section

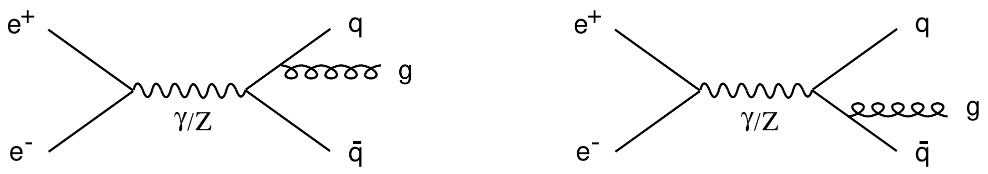
$$\sigma(e^+e^- \to \text{ hadrons}) = \sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) + \ldots$$

$e^+e^- \to q\bar{q}$



$\rightarrow$ ultraviolet divergences

$e^+e^- \to q\bar{q}g$



$\rightarrow$ infrared, collinear divergences

The divergences (almost) cancel, leaving a finite correction

$$\sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) = \sigma_0 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

works at every order in perturbation theory

α_s from R_l and R_τ

$$R_l = \frac{\Gamma(\mathrm{Z} \to \text{ hadrons})}{\Gamma(\mathrm{Z} \to l\bar{l})} \to \text{(almost) same as } \sigma_{\text{had}}:$$

$$= 19.934 \left[1 + 1.045 \frac{\alpha_s}{\pi} + 0.44 \left(\frac{\alpha_s}{\pi} \right)^2 - 15 \left(\frac{\alpha_s}{\pi} \right)^3 \right]$$

LEP measures $R_l = 20.768 \pm 0.024$

$$\to \alpha_s(M_\mathrm{Z}) = 0.124 \pm 0.004 \text{ (stat) } \pm^{0.003}_{0.002} \text{ (theo)}$$

Uncertainties from missing higher orders, Higgs mass

Similarly, decay of virtual W from τ sensitive to QCD corrections

$$R_\tau = \frac{\mathcal{B}(\tau^- \to \nu_\tau \text{ hadrons})}{\mathcal{B}(\tau^- \to \nu_\tau \mathrm{e}^- \overline{\nu}_\mathrm{e})}$$

ALEPH, OPAL average result (Bethke hep-ex/0004021):

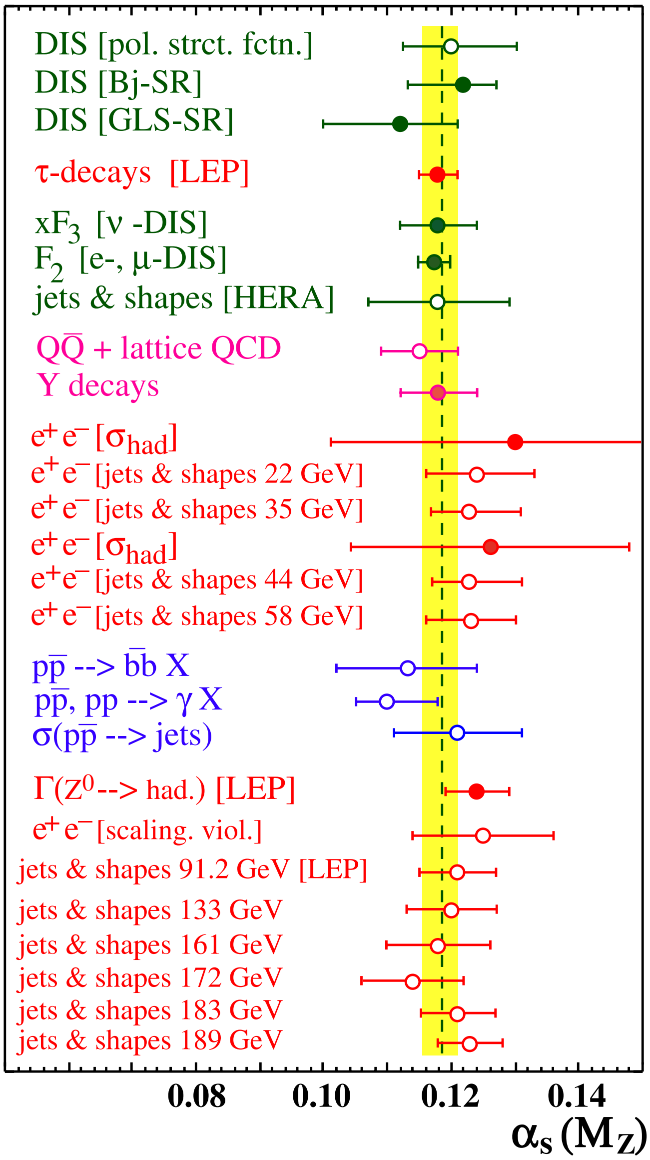
$$\alpha_s(m_\tau) = 0.323 \pm 0.005 \text{ (exp) } \pm 0.030 \text{ (theo)}$$

Evolve to M_Z,

$$\alpha_s(M_\mathrm{Z}) = 0.118 \pm 0.001 \text{ (exp) } \pm 0.003 \text{ (theo)}$$

Summary of $\alpha_s(M_Z)$

S. Bethke, hep-ex/0004021

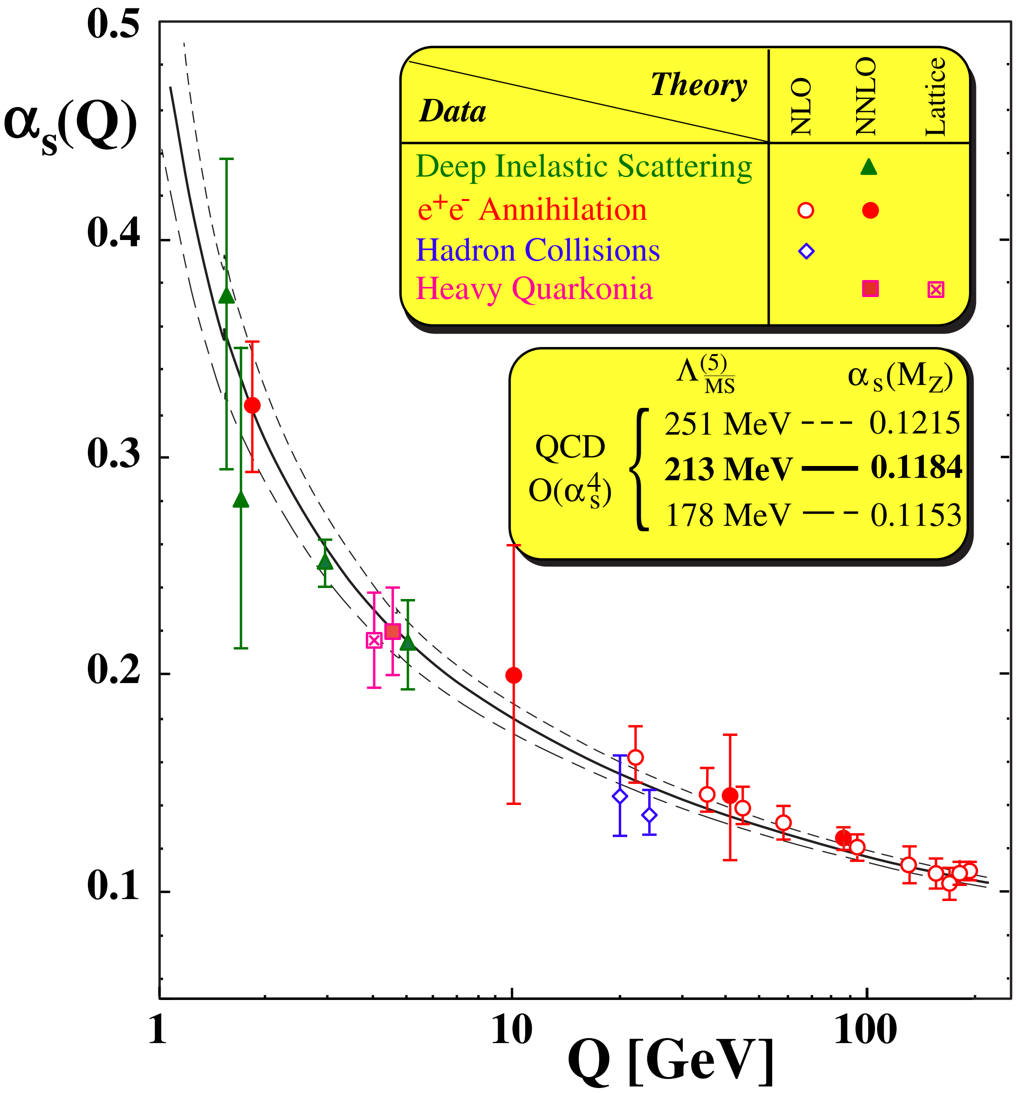


Bethke's average (hep-ex/0004021):

$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

Running of $\alpha_s(M_Z)$

S. Bethke, hep-ex/0004021



Last word?

Many QCD predictions are well verified in e^+e^- collilsions,

consistent $\alpha_s(M_Z)$ values measured each to several percent from:

event-shapes and jet rates, total cross section, τ decays

What we missed

$\gamma\gamma$ physics ($e^+e^- \to e^+e^- +$ hadrons)

QCD with $e^+e^- \to W^+W^- \to$ hadrons

scaling violations of fragmentation functions

four-jet properties ($\to$ Bill Gary)

$\vdots$

Next step:

use our knowledge of QCD to discover new physics!

Many thanks for help in preparing this talk to:

Günther Dissertori

Michael Schmelling