Chapter 8

The Method of Moments

Exercise 7.1: Consider a random variable x distributed according to a Gaussian p.d.f. of unknown mean μ and variance σ^2, and suppose we have a sample of values $x_1, \ldots, x_n$.

(a) Construct estimators for μ and σ^2 using the method of moments. Use the functions $a_1 = x$, $a_2 = x^2$, so that the expectation values $E[a_i(x)]$ correspond to the first and second algebraic moments of x.

(b) Compute the expectation values of the estimators $\hat{\mu}$ and $\widehat{\sigma^2}$ from (a). Are the estimators biased?

Exercise 7.2: Consider ρ^0 mesons produced in a particle reaction which decay into two charged pions ($\pi^+\pi^-$). The decay angle θ is defined as the angle of the π^+ with respect to the original direction of the ρ, measured in the $\pi^+\pi^-$ rest frame (see Fig. 8.1).

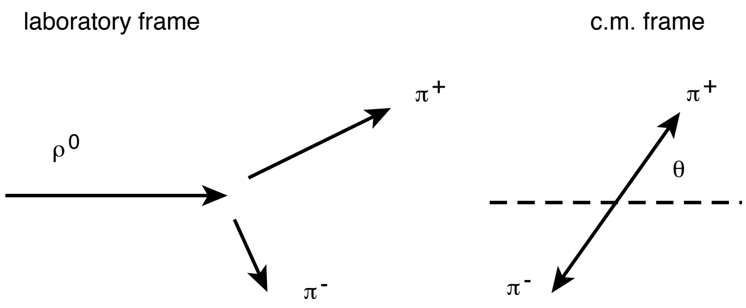

Figure 8.1: The definition of the decay angle θ in the decay $\rho^0 \to \pi^+\pi^-$.

Since the ρ^0 has spin 1 and the pions have spin 0, one can show that the distribution of $\cos\theta$ has the form

$$f(\cos\theta; \eta) = \frac{1}{2}(1-\eta) + \frac{3}{2}\eta\cos^2\theta, \tag{8.1}$$

where the spin-alignment parameter η can take on values in the range $-\frac{1}{2} \leq \eta \leq 1$.

(a) Suppose that n values of $\cos\theta$ have been measured for ρ^0 mesons produced in a certain reaction. Construct an estimator $\hat{\eta}$ for the spin alignment using the method of moments, by using the function $a = x^2$. Why is it not possible to construct an estimator using $a = x$?

(b) Determine the expectation value and variance of $\hat{\eta}$.