

Statistical Data Analysis
Problem sheet 1
Due Monday, 9 October 2023

Submit your solutions as a single pdf file, named `YourName_stat_prob_sheet_1.pdf`.
Remember to put in the pdf file: full name, college, degree programme (PhD, MSci, MSc, ...).

Exercise 1 [6 marks]: A sample of particles produced in high-energy proton-proton collisions are known to be either electrons (e) or pions (π). The prior probabilities for the particles to be of these types are $\pi(e) = 0.01$ and $\pi(\pi) = 0.99$.

The signals produced by the particles as they interact with a detector result in one of three possible outcomes labeled A , B and C . The probabilities for these outcomes given that the particle is either e or π are:

$$\begin{array}{ll} P(A|e) = 0.01 & \text{and} \quad P(A|\pi) = 0.980 \\ P(B|e) = 0.1 & P(B|\pi) = 0.019 \\ P(C|e) = 0.89 & P(C|\pi) = 0.001. \end{array}$$

(a) [3] A single particle is observed resulting in data outcome A . What is the probability that the particle is a pion?

(b) [3] What is the probability for a particle to be an electron given data outcome C ?

Exercise 2 [14 marks]: Consider the joint pdf for the continuous random variables x and y

$$f(x, y) = \begin{cases} \frac{1}{\pi R^2} & x^2 + y^2 \leq R^2, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [6] Find the marginal pdf $f_x(x)$ and by symmetry also $f_y(y)$ (remember to give the relevant ranges for the variables). Make a rough sketch of the result for $f_x(x)$.

(b) [6] Find the conditional pdfs $f(x|y)$ and by symmetry $f(y|x)$ (remember to give the relevant ranges for the variables). Make a rough sketch of the result for $f(x|y)$.

(c) [1] Show explicitly that the conditional and marginal pdfs satisfy Bayes' theorem.

(d) [1] Are x and y independent? Justify your answer.

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