Statistical Analysis of Data
Problem sheet #3

Please send by the announced due date to Glen Cowan, Physics Dept., Royal Holloway, University of London, Egham, Surrey, TW20 0EX, or e-mail to `g.cowan@rhul.ac.uk`.

For these exercises you will need a random number generator to produce random values uniformly distributed between zero and one. Simple FORTRAN and C++ examples are given in the files `random.f` and `random.cc` from `alephwww.cern.ch/~cowan/stat/compute.html`. These routines are mainly for pedagogical purposes and simple applications. More sophisticated routines such as `RANMAR` or `RANLUX` can be found in the CERN Program Library. Alternatively, you may implement the solutions using whatever computer tools you prefer, e.g. ROOT.

Exercise 3.1: Using `random.cc` or another random number generator, write a short program to generate 10000 random values uniformly distributed between zero and one, and display the result as a histogram with 100 bins. Optional: try implementing your own random number generator as described in the lecture.

Exercise 3.2 Suppose the independent random variables r_i are uniformly distributed between zero and one. Write a computer program to make histograms of

(a) $x = r_1 + r_2 - 1$

(b) $x = r_1 + r_2 + r_3 + r_4 - 2$

(c) $x = \sum_{i=1}^{12} r_i \quad - 6$

What are the means and variances of the variables histogrammed in (a)–(c)? Comment on the connection between your histograms and the central limit theorem.

Exercise 3.3: Consider a sawtooth p.d.f.,

$$f(x) = \begin{cases} \frac{2x}{x_{\max}^2} & 0 < x < x_{\max} \,, \\ 0 & \text{otherwise} \,. \end{cases} \tag{1}$$

(a) Use the transformation method to find the function $x(r)$ to generate random numbers according $f(x)$. Implement the method in a short computer program and make a histogram of the results. (Use e.g. $x_{\max} = 1$.)

(b) Write a program to generate random numbers according to the sawtooth p.d.f. using the acceptance-rejection technique. Plot a histogram of the results.