# Problem Sheet 5: Special Relativity

To be handed in **by the lecture on Tuesday 14th December, 2004**.
The mark obtained will contribute to your final mark in this course. It is expected that you will attempt all questions. Part marks for each question are given.

1) **Lorentz transformations:**

a) Write down the Lorentz coordinate transformations. **[4]**

b) Two events are observed in a frame of reference S to occur at the same space point, the second occurring 1.80s after the first. In a second frame S' moving relative to S the second event is observed to occur 2.35s after the first. What is the difference between the positions of the two events as measured in S'? **[6]**

2) **Time dilation** *(Part of Qu 2, 2004)*:

A beam of identical unstable particles flying at a speed $\beta c$ is sent through two counters separated by a distance $L$. It is observed that $N_1$ particles are recorded at the first counter and $N_2$ at the second counter, the reduction being solely due to the decay of the particles in flight.

a) Show that the lifetime of the particles at rest is given by **[7]**

$$\tau = \frac{L}{\ln(N_1 / N_2)\left(\sqrt{\gamma^2 - 1}\right) c}$$

where the Lorentz factor $\gamma$ is defined as usual.

b) Hence determine the lifetime of muons at rest, knowing that when travelling at a speed $c\sqrt{8}/3$ through the apparatus described above (with $L$=200 m) $N_1$ and $N_2$ were measured to be 10000 and 8983, respectively. **[3]**

3) **Velocity transformation** *(Part of Qu 2, 1998)*:

A spacecraft traveling towards the earth at 0.5$c$ launches a rocket with a speed 0.75$c$, relative to the spacecraft, towards the earth. At what speed is the rocket moving, as seen by an observer on the earth? What would be the speed of the rocket as seen from the earth if the spacecraft was moving away from the earth at a speed of 0.5$c$ when the rocket was launched? **[6]**

4) **Relativistic kinematics** *(Part of Qu 3, 2004)*:

A charged π-meson at rest decays into a muon and a massless neutrino: $\pi^+ \rightarrow \mu^+ + \upsilon_\mu$.

a) Considering conservation of energy and momentum, determine the expression for the total energy of the muon, $E_\mu$, in terms of the mass of the π-meson, $M$, and the mass of the muon, $m$. **[6]**

b) Knowing that the masses of the π-meson and muon are respectively $M$ = 140 MeV/$c^2$ and $m$ = 106 MeV/$c^2$, calculate the energy of the muon. **[3]**

(PLEASE TURN OVER)

5) **Threshold energy** *(Part of Qu 4, 1999)*:
Antiprotons can be produced by colliding an accelerated beam of protons with a target of stationary protons to create proton-antiproton pairs according to the reaction: $p + p \rightarrow p + p + p + \overline{p}$. What is the minimum kinetic energy (threshold energy) of a proton in the incident beam for this reaction to occur? How much kinetic energy per particle would be required if two equally energetic protons collided head on? **[8]**