PH5260 – Particle Physics / 2006/07

Post Graduate Problem Set 2

To be handed in at the tutorial on Thursday, November 23, 2006.

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1. Fermi motion. The way we make a target of protons and neutrons (nucleons) at rest in the Lab frame is to just use, say, a brick. However the nucleons in a brick are confined inside nuclei, and thus, quantum-mechanically, cannot be at rest. This zero-point motion is called "Fermi motion".

a. Nuclei are typically 5 fm or so in size. Use an uncertainty principle argument to estimate the typical momentum of a nucleon in a nucleus.

Consider 200 GeV muons ($E_\mu \gg m_\mu$=106 MeV) incident on the brick. Your answer for part **a.** should be very small compared with 200 GeV. Yet, very annoyingly, Fermi motion can have a sizeable effect.

b. Consider the nucleon to be moving parallel or antiparallel to the muon beam, with the momentum from part **a**. What effect does this have on the centre-of-mass energy (in percent)?

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2. Reconsider problem 2 from Problem Sheet 1, on the determination of the lab energy spectrum of the photons from $\pi^0 \to \gamma\gamma$ decay. This problem can also be solved using a **Monte Carlo technique**.

a. Write down the fluxogram for a computer program designed to do this MC calculation, indicating clearly the several different tasks such a program would have to perform.

b. Write the program and run it. Plot the resulting photon energy spectra for E_π = i) 135 MeV, ii) 500 MeV, iii) 1 GeV.

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3. Consider $e^+e^- \to e^+e^-$ scattering. Verify that the **Mandelstam variables**

$$\begin{aligned} s &= 4(k^2 + m^2) \\ t &= -2k^2(1 - \cos\theta) \\ u &= -2k^2(1 + \cos\theta) \end{aligned}$$

where θ is the centre-of-mass scattering angle and $k = |\vec{k_i}| = |\vec{k_f}|$, where $\vec{k_i}$ and $\vec{k_f}$ are, respectively, the momenta of the incident and scattered electrons in the centre-of-mass frame. Show that the process is physically allowed provided $s \geq 4m^2$, $t \leq 0$ and $u \leq 0$. Note that $t = 0$ ($u = 0$) corresponds to forward (backward) scattering.

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4. Consider plane-wave solutions to the **Dirac equation**, $\psi(x) = a \cdot e^{-ix\cdot p}u(p)$, where a is a normalization factor and $p = (E, \vec{p})$.

a. What are the four independent solutions of the Dirac equation?
(You will have to determine $u(p)$.)

b. Indicate clearly for each solution the particle state that it describes.
(Hint: you may want to recall "helicity")

5. The **Dirac gamma matrices** γ_μ satisfy the anticommutation relations $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}\mathbb{1}$, where $g_{\mu\nu}$ is the Minkowski metric tensor. The matrix γ_5 is defined by $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. Using only the information given here (i.e., independently of any specific representation for γ-matrices), show that:

a. $\gamma_5^2 = \mathbb{1}$

b. $\gamma_5^\dagger = \gamma_5$

c. $\{\gamma_\mu, \gamma_5\} = 0$

(You may also use $\gamma_\mu^\dagger = \gamma_0\gamma_\mu\gamma_0$.)

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6. Use the standard **Feynman rules** to write down the form of the matrix element for the scattering of an electron of momentum $\vec{k}$ and spin s off a charged pion (as a whole) of momentum $\vec{p}$.

At what energies (and why) would you expect this simple form of the matrix element to break down?