PH5260 – Particle Physics / 2006-07

Post Graduate Problem Set 3

To be handed in to PTD by Monday 18^{th} December, 2006.

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1. Consider the two-body decay of the (spinless) charged pion into leptons: $\pi^- \rightarrow \ell^- \bar{\nu}_\ell$.
a. Draw the tree-level diagram for this decay;
b. In the rest frame of the pion, what is the helicity of the emitted $\bar{\nu}_\ell$? ...and the helicity of the ℓ^-? Explain.
c. How do you explain that the electron decay mode is strongly supressed with respect to the muon decay mode?

	fraction (Γ_i/Γ)
$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$	$\approx$100%
$\pi^- \rightarrow e^- \bar{\nu}_e$	1.23×10^{-4}

♣

2. The four-momenta k, p and k', p' describe, respectively, the initial and final state of electron-muon scattering.

Write down the lowest order Feynman diagram for this process and then use the Feynman rules to determine the corresponding matrix element.

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3. The cross-section for electron-muon scattering, in the limit that masses are negligible ("extreme relativistic limit"), is given by

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}[\frac{s^2 + u^2}{t^2}]$$

where s, t and u are the usual Mandelstam variables and α is the fine structure constant.
a. From the above, use "crossing" to obtain the differential cross-section for $e^+e^- \rightarrow \mu^+\mu^-$ annihilation;
b. Show that the angular distribution is of the form $(1 + \cos^2\theta)$, where θ is the angle between the incoming e^- and outgoing μ^- directions.

(p.t.o.)

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4a. Show *in detail* that, for $e^-\mu^+ \rightarrow e^-\mu^+$ scattering, the spin-averaged matrix element-squared can be written as[1]:

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{q^4} L^{\mu\nu}_{elec} L^{muon}_{\mu\nu}$$

where

$$L^{\mu\nu}_{elec} = \frac{1}{2} Tr\{(\not{k}' + m)\gamma^\mu(\not{k} + m)\gamma^\nu\}$$

and

$$L^{\mu\nu}_{muon} = \frac{1}{2} Tr\{(\not{p}' - M)\gamma^\mu(\not{p} - M)\gamma^\nu\}$$

where $\not{p} = \gamma^\mu p_\mu$, and M and m are, respectively, the muon and electron masses.

4b. Hence show that, in the extreme relativistic limit:

$$\frac{\overline{|\mathcal{M}|^2}}{2e^4} = \frac{s^2 + u^2}{t^2}$$

where s, t, u are the usual Mandelstam variables.

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[1]Spin-averaging and $e^-\mu^-$ scattering are discussed in, e.g., Halzen & Martin Sections 6.2 and 6.3.