Experimental tests of QCD

Glen Cowan

Royal Holloway, University of London

g.cowan@rhul.ac.uk

Topical Seminar on Frontier of Particle Physics 2004:
QCD and Light Hadrons

Beijing, 26–30 September, 2004

Outline

(i) QCD in $e^+e^- \to$ hadrons

- general theoretical and experimental picture
- Monte carlo models
- defining observables

(ii) QCD with jets and event-shapes

- parton spin
- α_s from jets and event-shapes
- running of α_s
- flavour independence of α_s, measuring m_b

(iii) α_s from total cross sections and BRs: R_l, R_τ

(iv) Scaling violations in fragmentation functions

(v) Properties of four-jet events

(vi) Internal structure of quark and gluon jets

(vii) Conclusions and outlook

Why study QCD? (experimentalist's view)

(i) To see if QCD is Nature's theory of strong interactions?

Sure, but most already convinced.

(ii) To measure its free parameters?

Yes: α_s, quark masses,
needed to understand bigger picture.

(iii) To help search for physics beyond the SM?

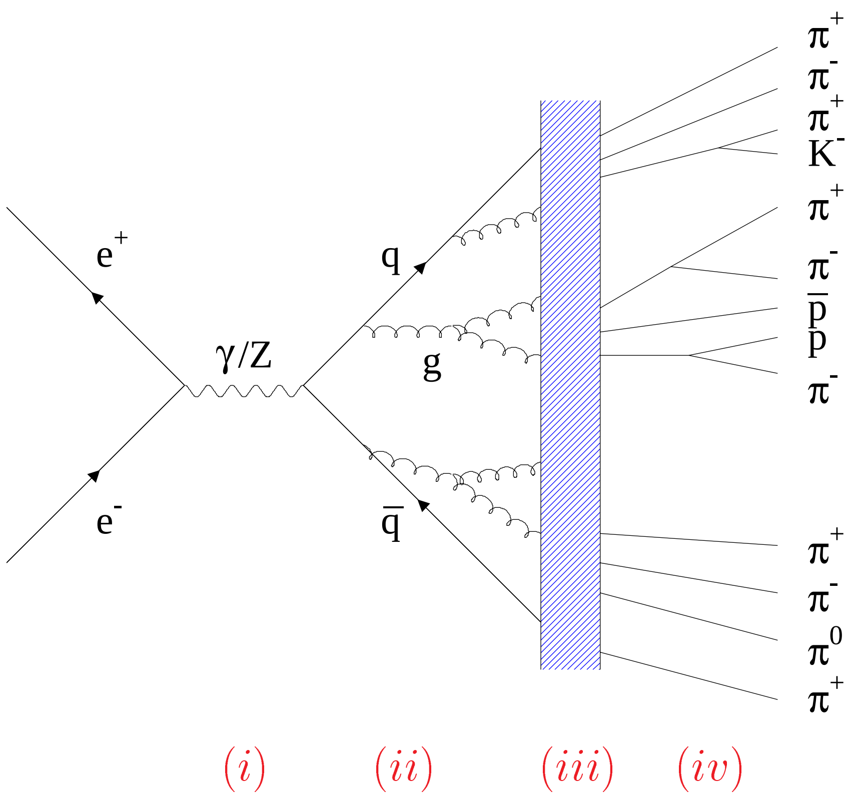
Yes: understand 'QCD background',
deviation from firm QCD prediction → new physics

(iv) Test validity of QCD calculations?

Yes, but not usually driving concern.

(v) Help understand non-perturbative QCD?

Ball (mostly) in theorists' court.



(i) electroweak

(ii) perturbative QCD

(iii) hadronization (non-perturbative QCD)

(iv) resonance decays

Usually define observable to be sensitive to only one of the above.

SPEAR (1972) $E_{cm} = 8$ GeV

PETRA (1978) 14 GeV $< E_{cm} < 44$ GeV

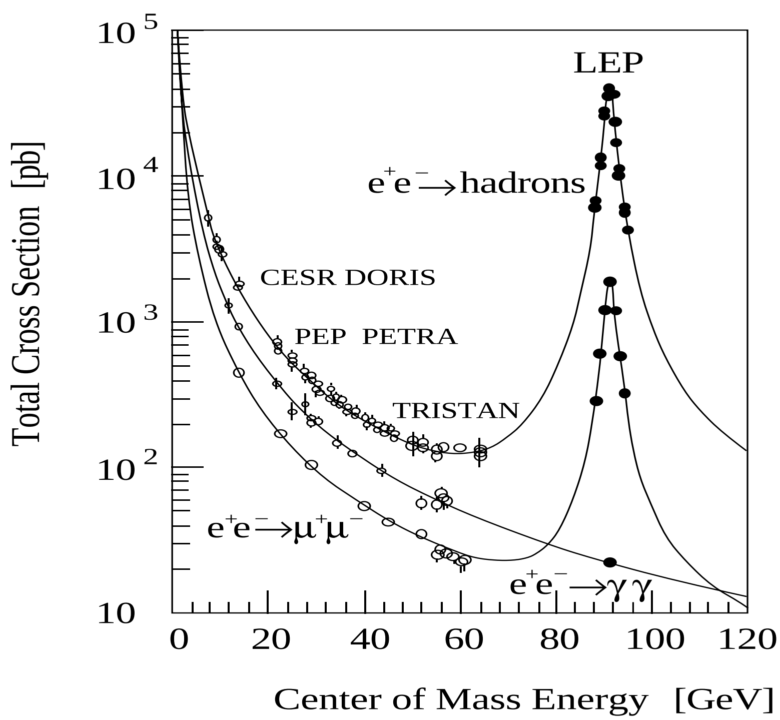
PEP (1980) $E_{cm} = 29$ GeV

TRISTAN (1987) $E_{cm} = 64$ GeV

SLC (1989) $E_{cm} = 91$ GeV

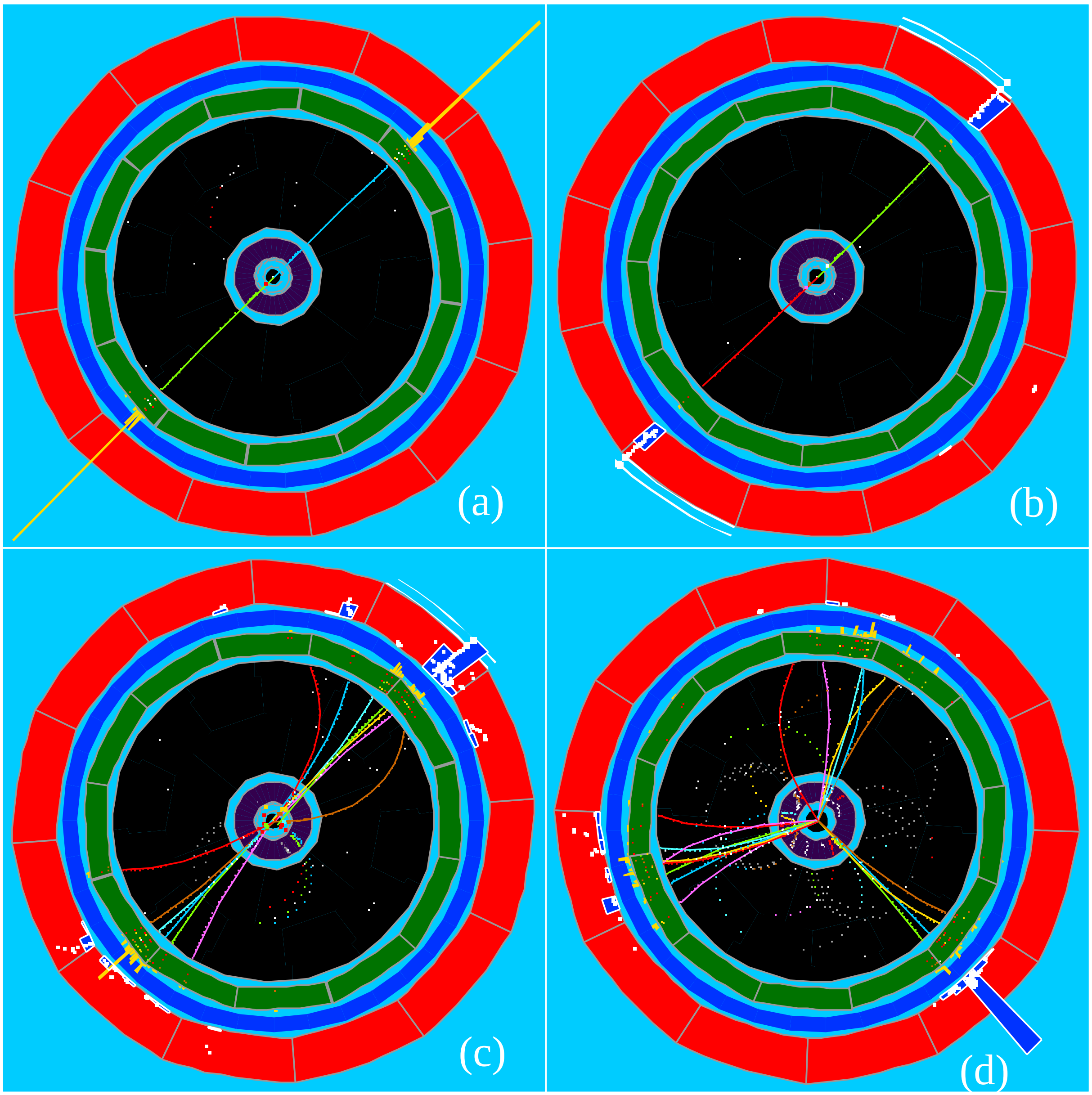
LEP I (1989) $E_{cm} = 91$ GeV

ca. 4×10^6 hadronic events each for ALEPH, DELPHI, L3, OPAL at $E_{cm} \approx M_Z$.



e^+e^- reactions at LEP

$e^+e^- \to e^+e^-$ $\qquad$ $e^+e^- \to \mu^+\mu^-$

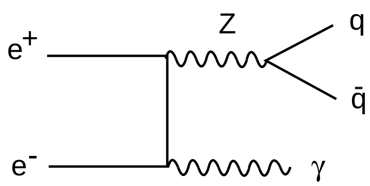


$e^+e^- \to$ hadrons (two-jet) $\qquad$ $e^+e^- \to$ hadrons (three-jet)

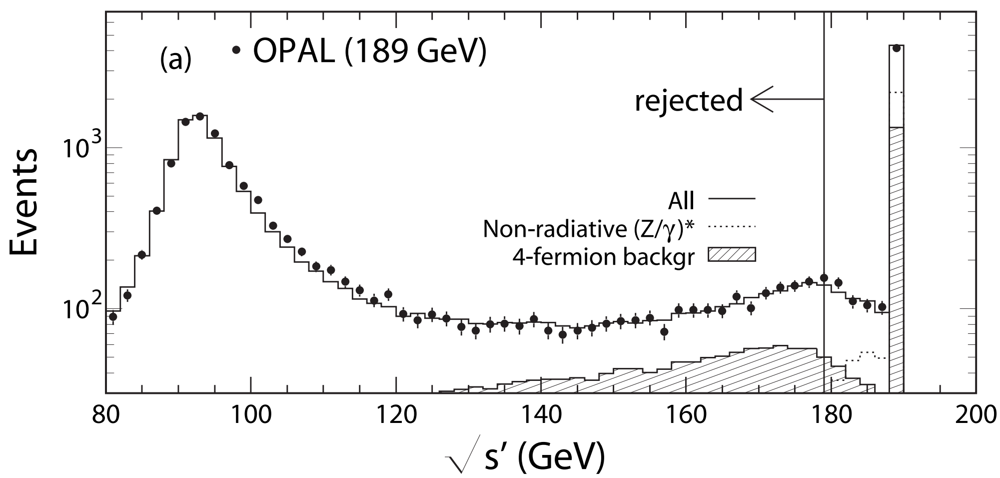
Hadronic events from e^+e^- annihilation data at LEP 2

LEP II (1996 – 2000) $\quad 130\ \text{GeV} < E_{\text{cm}} < 208\ \text{GeV}$

For $\sqrt{s} > M_{\text{Z}}$, many events with initial state photon radiation:



$$m_{q\bar{q}} \equiv \sqrt{s'} \approx M_{\text{Z}}$$



Require $m_{q\bar{q}}$ close to $\sqrt{s}$ for QCD studies.

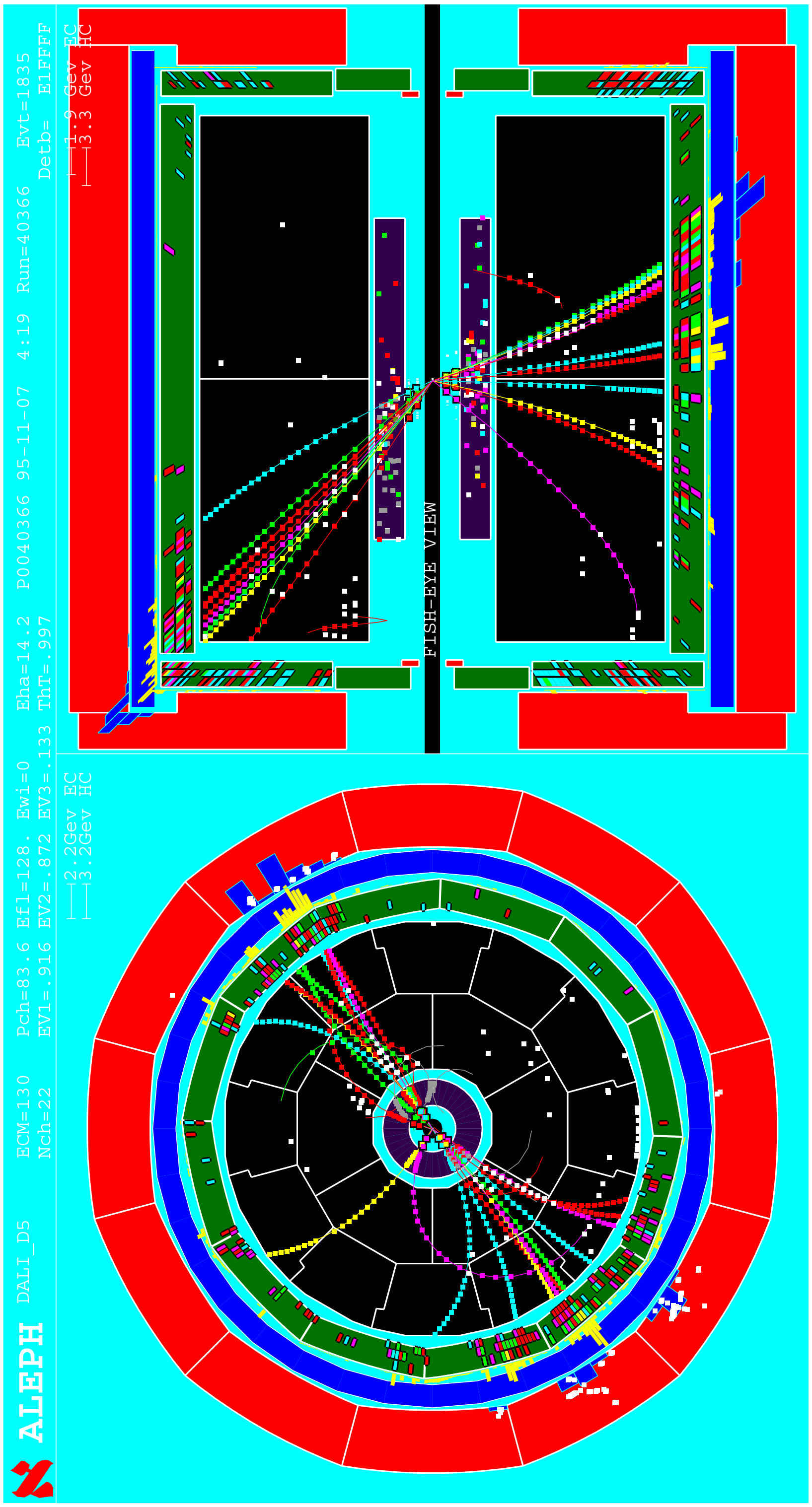
Also reject background from $e^+e^- \rightarrow W^+W^-(ZZ) \rightarrow$ hadrons.

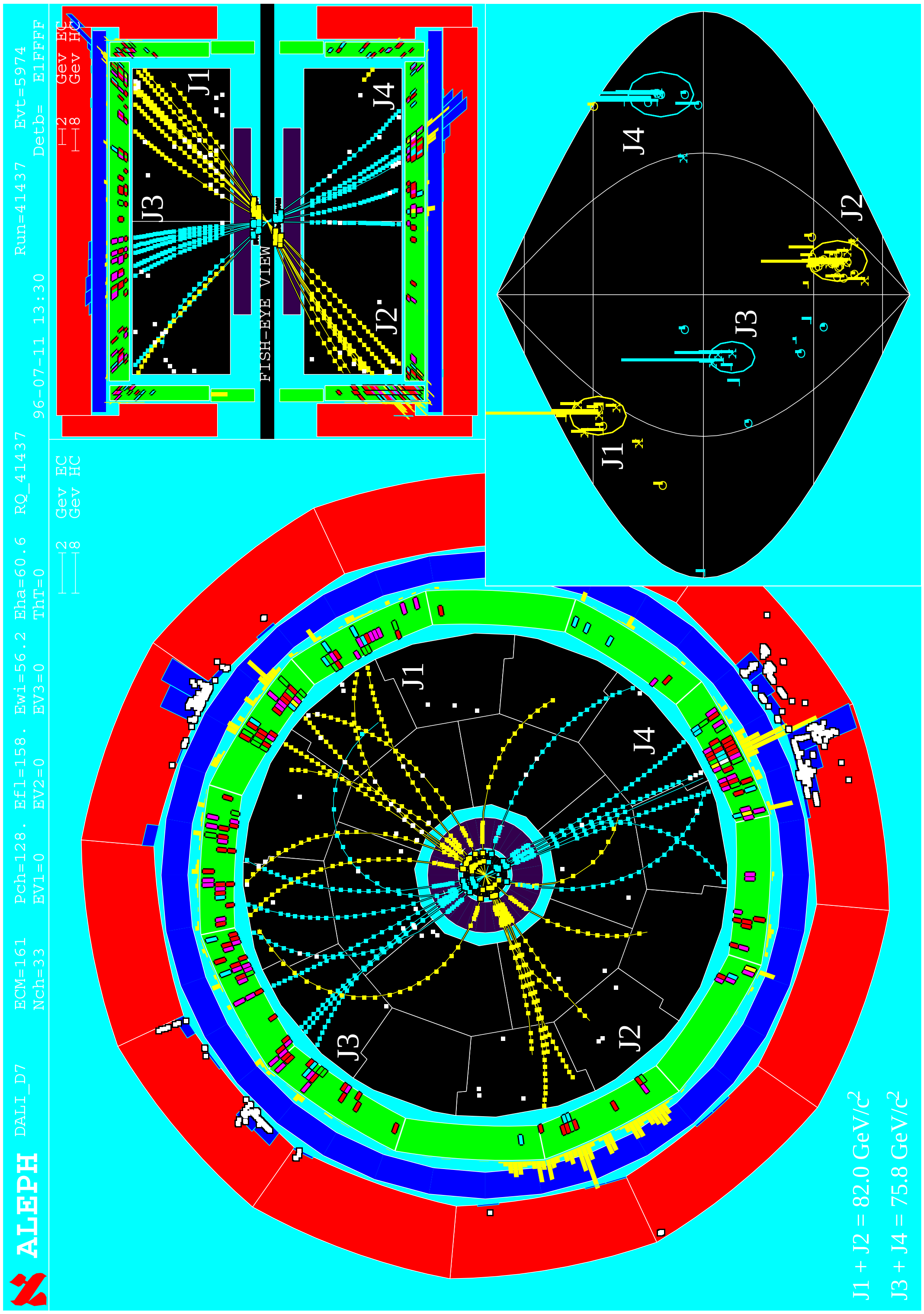
Much smaller data sample than LEP I but:

important for tests of QCD E_{cm} dependence;

theoretical uncertainties smaller at higher E_{cm} .

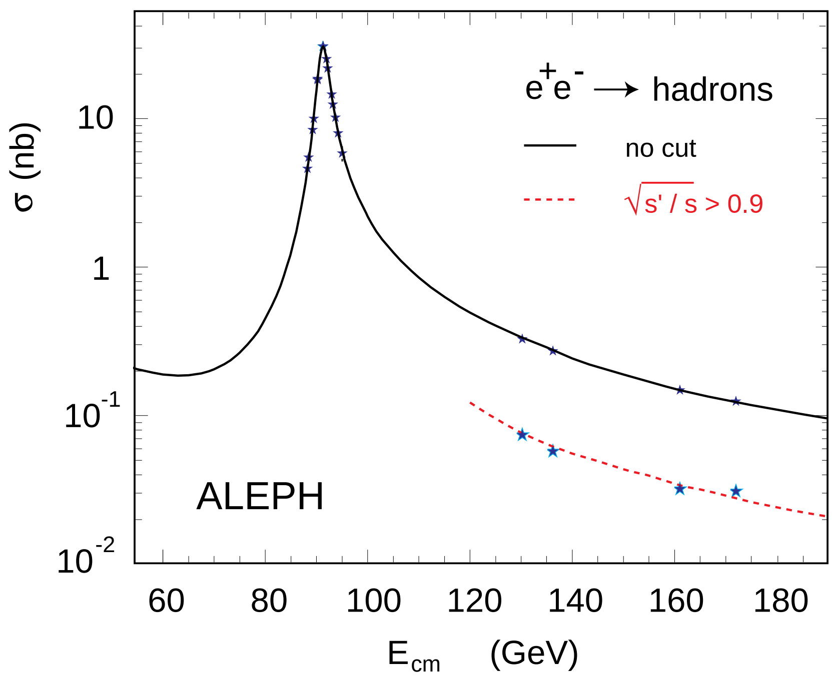
A ‘radiative return’ event: $e^+e^- \to \gamma Z$ with $Z \to$ hadrons




ALEPH
DALI_D7
ECM=161 Pch=128. Efl=158. Ewi=56.2 Eha=60.6 RQ_41437 Run=41437 Evt=5974
Nch=33 EV1=0 EV2=0 EV3=0 ThT=0 96-07-11 13:30 Detb= E1FFFF
2 Gev EC
8 Gev HC
FISH-EYE VIEW
J1
J2
J3
J4
J1 + J2 = 82.0 GeV/c2
J3 + J4 = 75.8 GeV/c2

E_{cm} (GeV)	Approx. events per LEP experiment
91.2	4×10^6 (plus $\sim$ 400 000 from SLD)
133	800
161	300
172	200
183	1200
189	3000
200	3000
206	3000

S. Bethke, hep-ex/0406058



$Z\gamma$, ZZ, WW events rejected

Monte Carlo models: perturbative phase

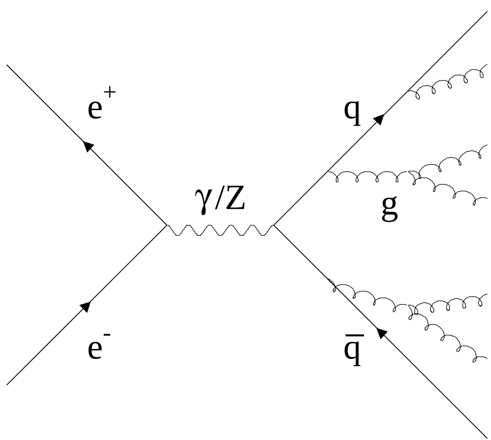
Use random numbers to select a partonic final state and generate all momentum vectors

Usually based on $\mathcal{O}(\alpha_s)$ QCD combined with 'parton shower': leading-log approx. (valid in limit of collinear gluon radiation)

+ angular ordering

+ (sometimes) next-to-leading logs

+ ...



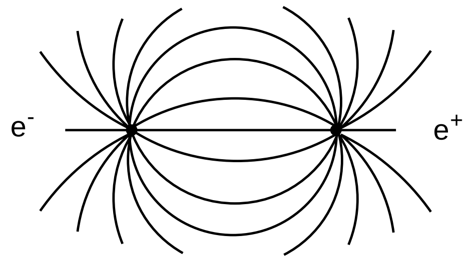
→ generates set of partons for each event

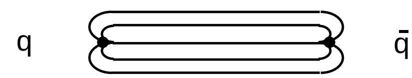
This *is* perturbative QCD but not at its most accurate.
(α_s in MC $\neq$ α_s in $\overline{\text{MS}}$ scheme)

MC with $\mathcal{O}(\alpha_s^2)$ matrix element also available, but without parton shower (max of 4 partons in event)

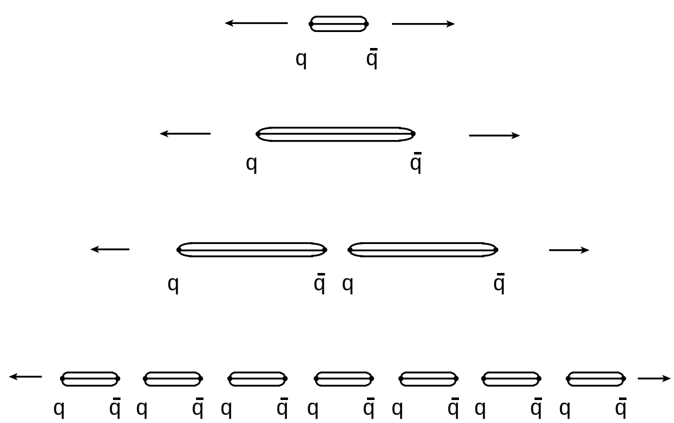
QCD inspired models (e.g. string) convert partons into hadrons

In contrast to electric charges, 'chromoelectric' field between $q\bar{q}$ pair confined to narrow flux tube (string):





$q\bar{q}$ production in flux tube → string breaks → mesons



MC generates flavours of $q\bar{q}$ pairs ($u : d : s \approx 1 : 1 : 0.3$)

space–time location of breaks → momenta of hadrons

Gluons → momentum carrying kinks in string

→ 'Lund family' of models: JETSET, ARIADNE, PYTHIA . . .

Cluster model (program HERWIG):

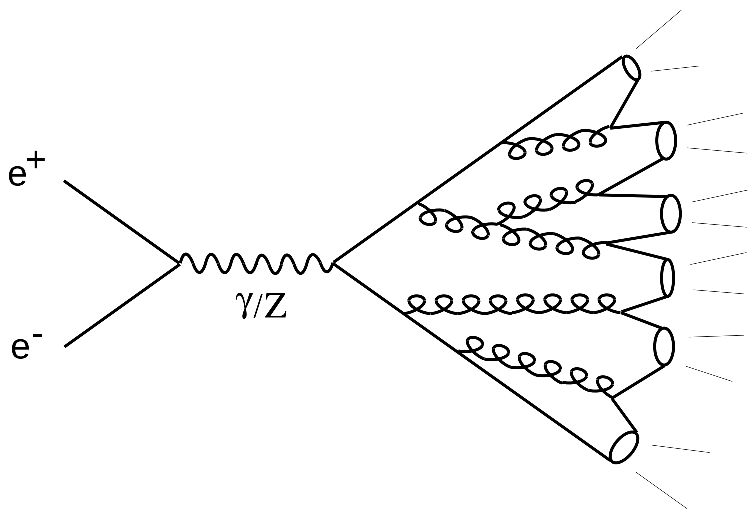
parton shower ends with virtual mass of all partons $= Q_0$;

gluons split into $q\overline{q}$ pairs;

neighbouring q and $\overline{q}$ form colour neutral clusters;

clusters (usually) decay isotropically into two hadrons;

exceptions allowed for very light and very heavy clusters



Parameters:

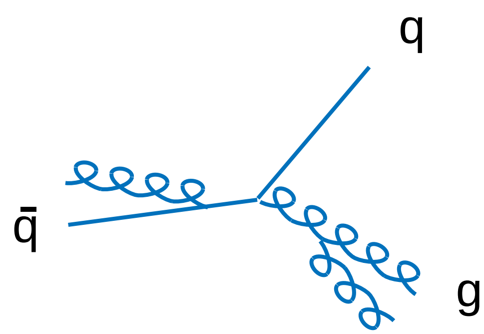
$\Lambda_{\rm QCD}$, Q_0, quark masses,

parameters for treatment of very light/heavy clusters

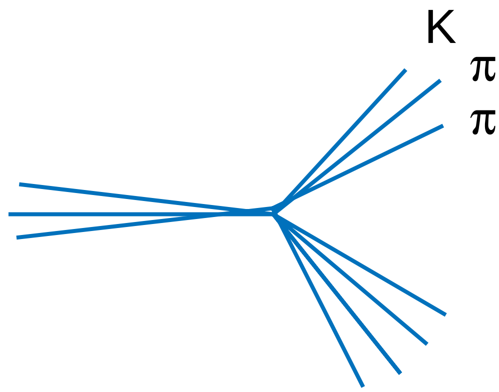
and other tweaks mainly related to flavour production.

Comparing theory and experiment

Need to compare QCD prediction . . .



with measurement . . .



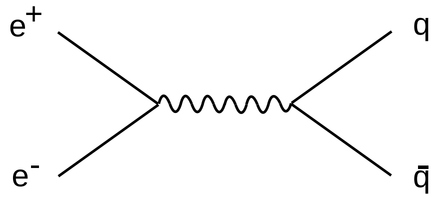
using appropriately defined jet rates, event-shape variables

infrared, collinear safe;

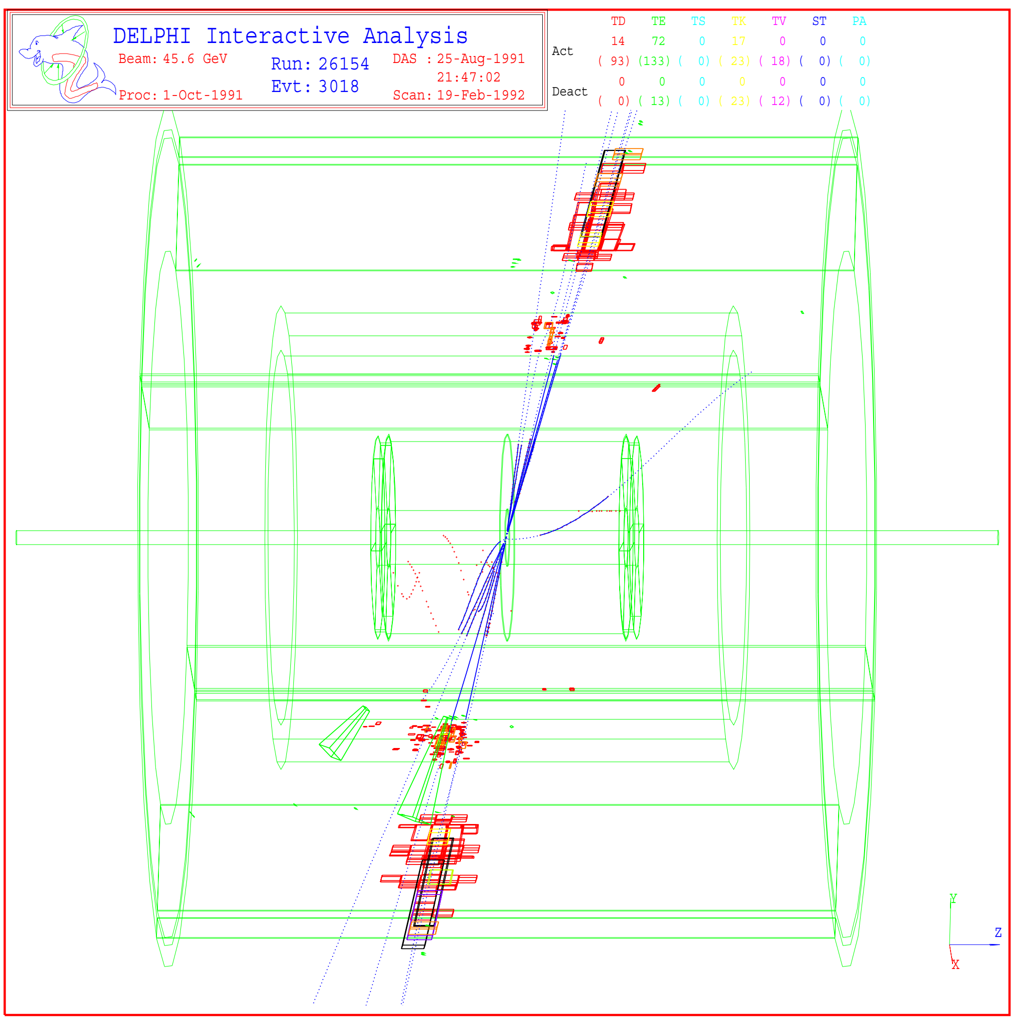
not overly sensitive to hadronization effects

Two-jet events

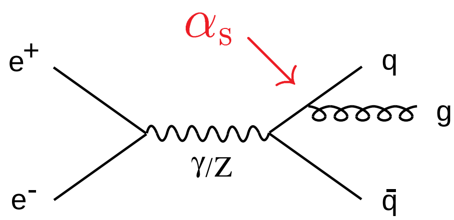
$e^+e^- \to q\bar{q}$ leads to two back-to-back jets of hadrons



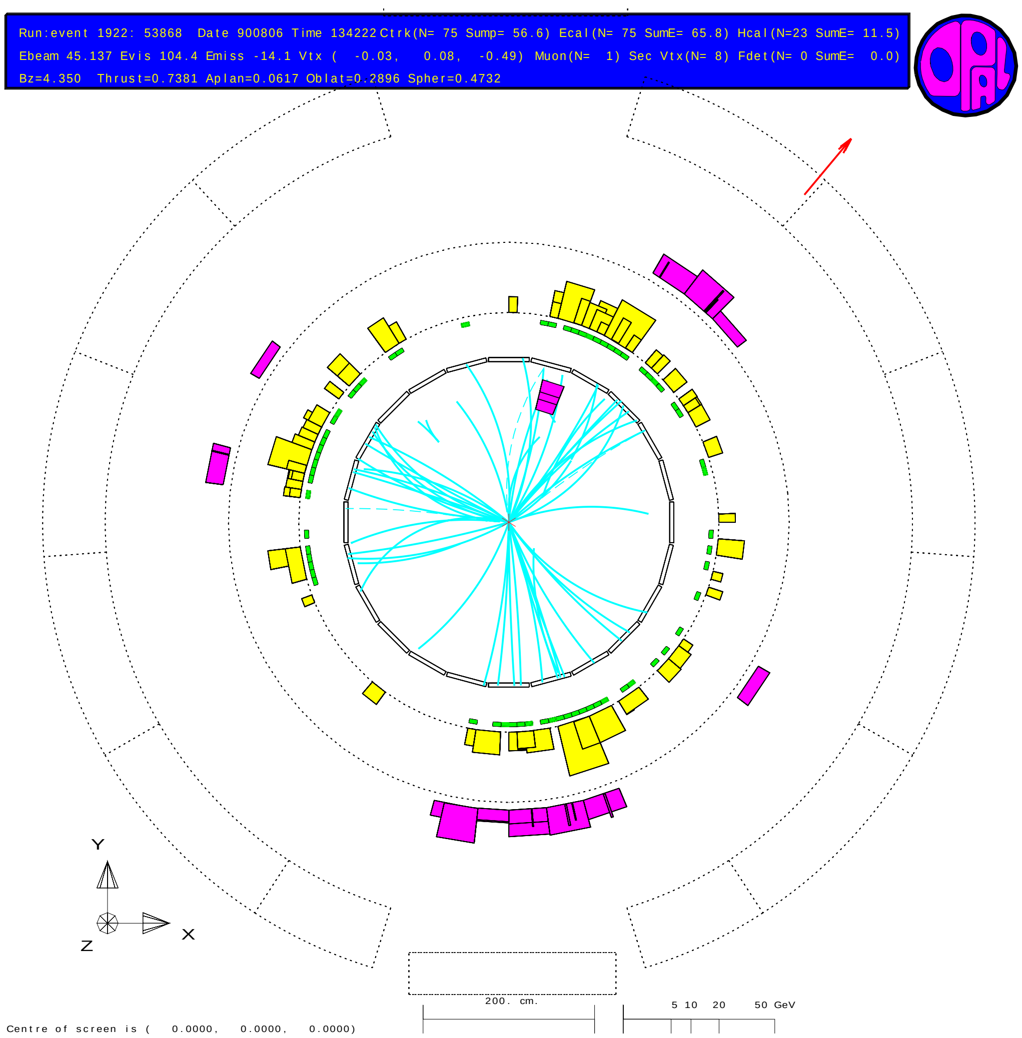
$\to$ angular distribution of jets depends on quark spin



Bremsstrahlung-like gluon radiation (cf. $e^+e^- \to \mu^+\mu^-\gamma$)

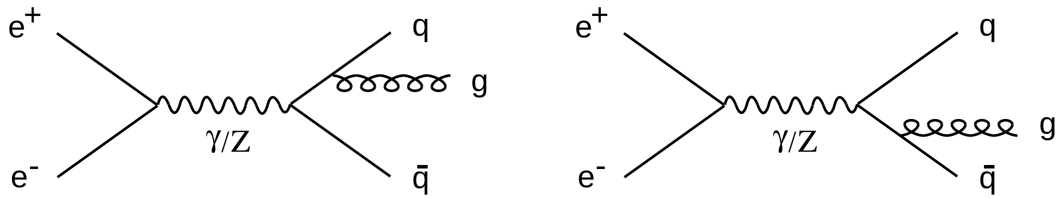


Additional jets $\to$ rate sensitive to strong coupling α_s



$e^+e^- \to q\bar{q}g$ to $\mathcal{O}(\alpha_s)$

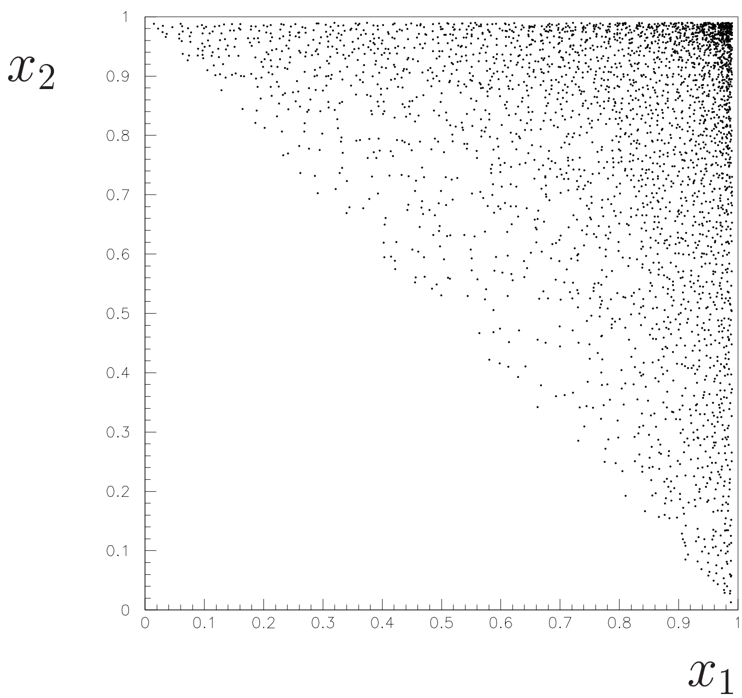
At leading order we have the amplitudes:



Define $x_i = 2E_i/E_{\rm cm}$, with $i = q,\ \bar{q},\ g$ (or 1, 2, 3).

Energy conservation: $x_1 + x_2 + x_3 = 2$

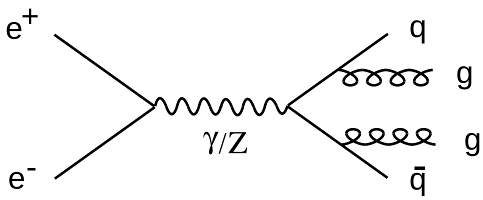
$$\rightarrow \qquad \frac{d\sigma}{dx_1\,dx_2} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

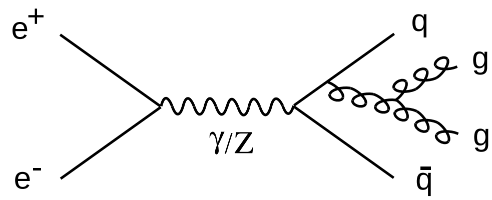


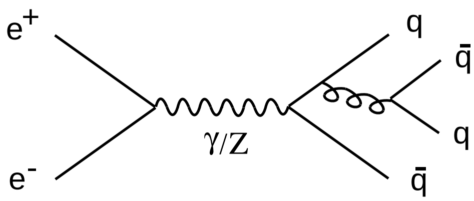
Divergences when $x_1 \to 1$ or $x_2 \to 1$ (collinear) and for both $x_1, x_2 \to 1$ (infrared)

$e^+e^- \to$ hadrons to $\mathcal{O}(\alpha_s^2)$

In addition to $\mathcal{O}(\alpha_s^2)$ corrections to $e^+e^- \to q\bar{q}$, $q\bar{q}g$ we now have four-parton final states $q\bar{q}gg$ and $q\bar{q}q\bar{q}$:

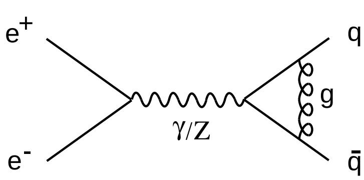


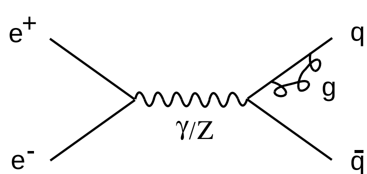


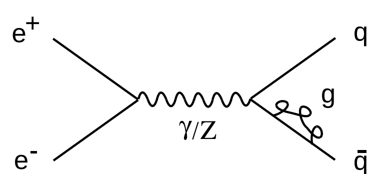


Use $\mathcal{O}(\alpha_s^2)$ matrix elements to compute, e.g., 2-, 3-, 4-jet rates.

Cancel divergences for infrared, collinear gluons against negative dirvergences from virtual corrections, e.g., in







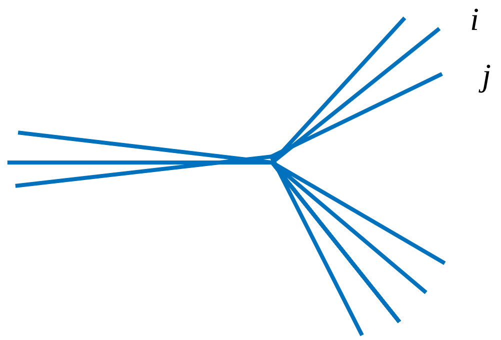
Finite prediction for observables at $\mathcal{O}(\alpha_s^2)$ (program `EVENT`).

Amplitudes now all calculated to $\mathcal{O}(\alpha_s^3)$ but difficult to assemble pieces (soon?)

Glen Cowan
Royal Holloway, University of London

Defining jets

Clustering algorithms: for every pair, compute 'distance' y_{ij}



← partons or hadrons

e.g. $$y_{ij} = \begin{cases} \dfrac{2E_iE_j(1-\cos\theta_{ij})}{s} & \text{(JADE)} \\[2ex] \dfrac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{s} & \text{(Durham)} \end{cases}$$

(*i*) Find pair with smallest y_{ij}

(*ii*) if less than a given y_{cut}, replace i, j with pseudoparticle:

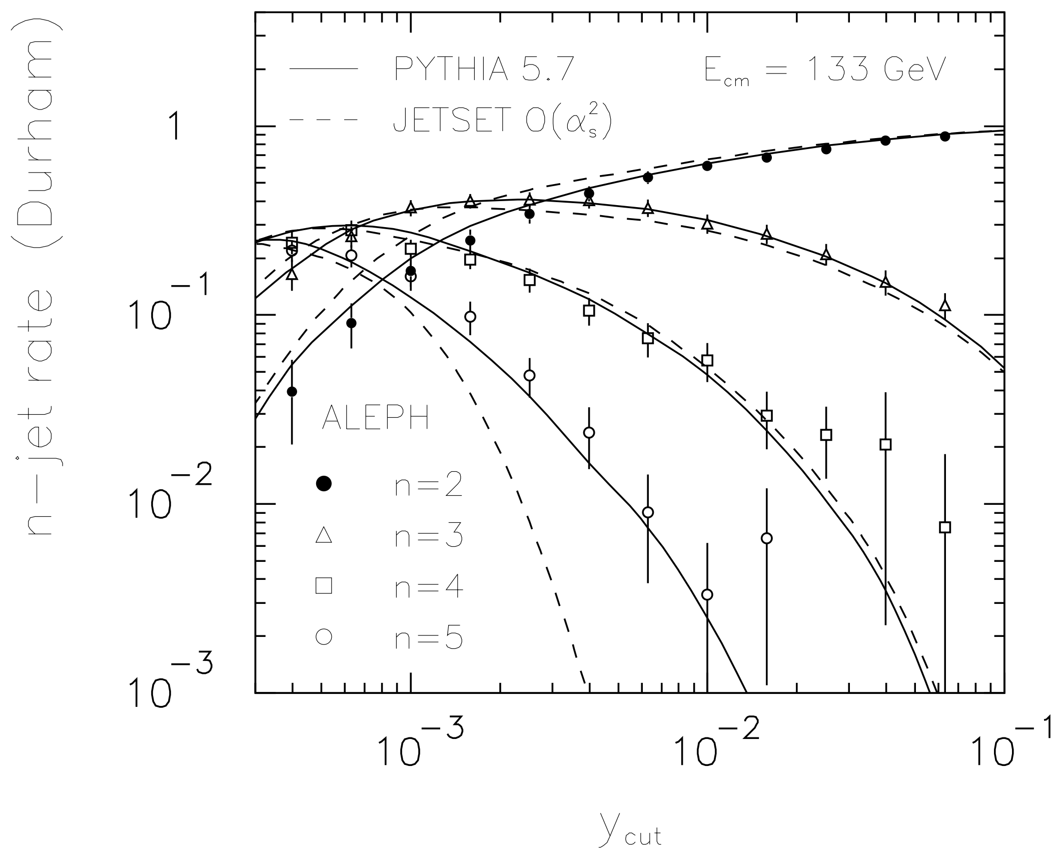
$$p^\mu = p_i^\mu + p_j^\mu \quad \text{('E' scheme)}$$

(*iii*) iterate until all $y_{ij} > y_{\mathrm{cut}}$

remaining pseudoparticles → jets

Other jet definitions also used, e.g., cone algorithm.

Relative rate of finding n jets for $n = 2, 3, 4, 5$

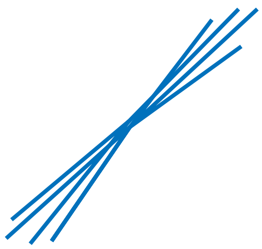


Parton shower based model (PYTHIA) gives good description of multijet rates

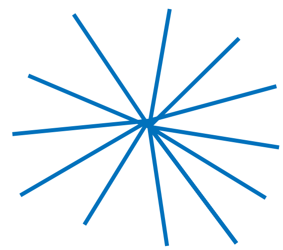
$\mathcal{O}(\alpha_s^2)$ based model has at most 4 partons in final state, falls short for rates of $n \geq 5$ jets

Event-shape variables

Thrust: $$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_{\mathrm{T}}|}{\sum_i |\vec{p}_i|}$$



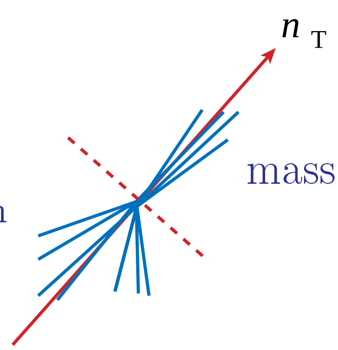
$T \approx 1$



$T \approx 1/2$

Heavy jet mass: divide event into hemispheres with thrust axis

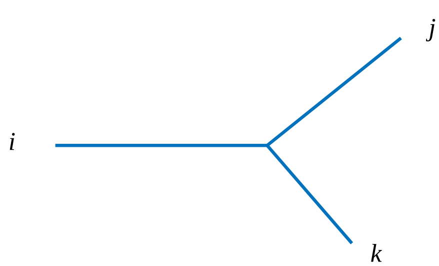
mass M_{h}



mass M_{l}

Often use $\rho = M_{\mathrm{h}}^2/s$

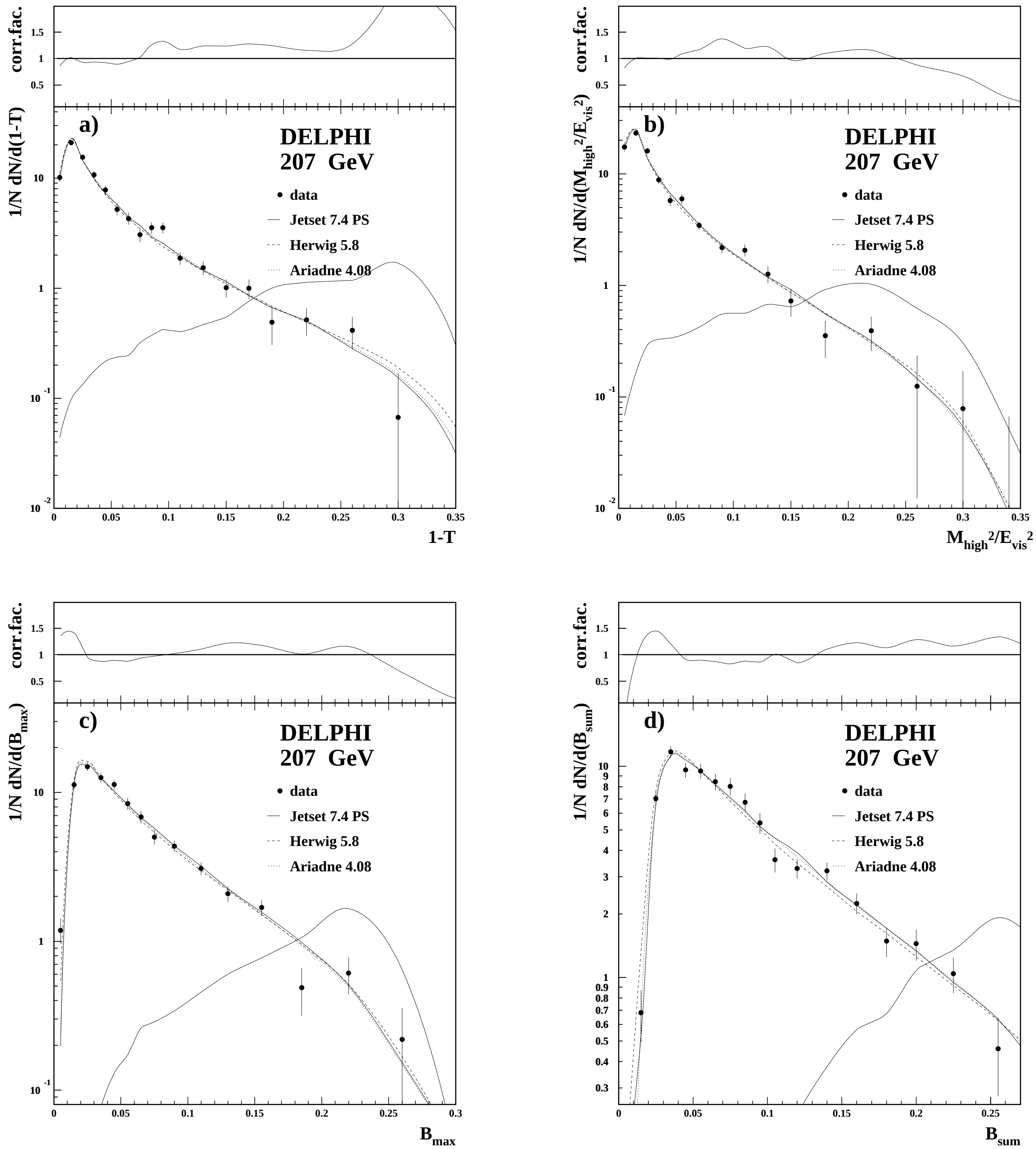
y_3: cluster event to three jets



$y_3 = \min(y_{ij})$

Measurements of event-shape distributions

DELPHI hep-ex/0406011, accepted by Eur. Phys. J. C

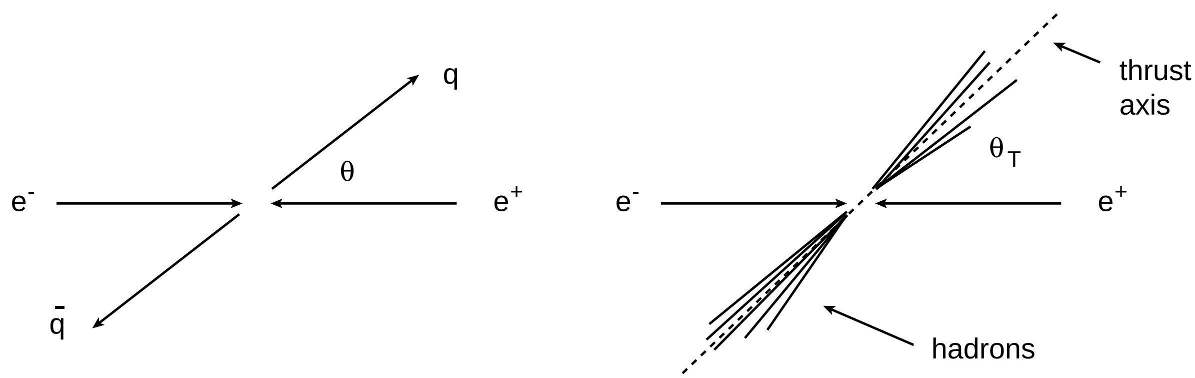


WW and ZZ background subtracted

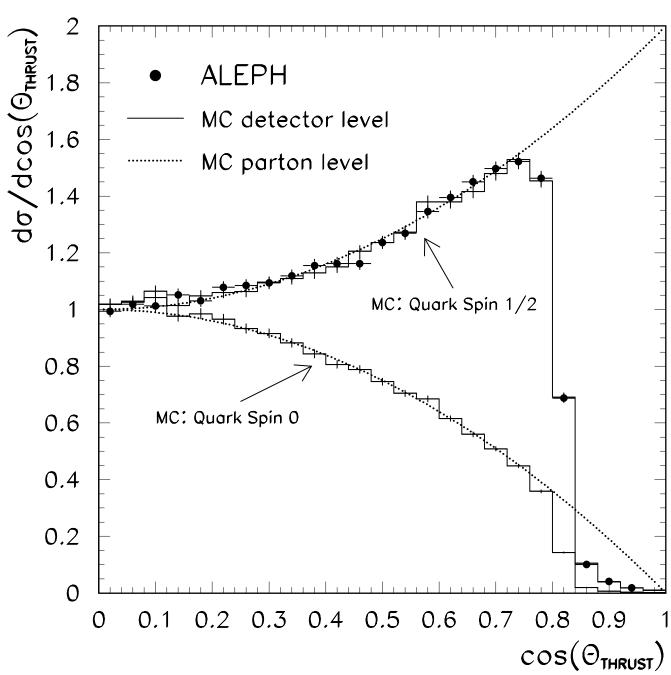
Spin of quarks

distribution of outgoing quark's angle relative to incoming e^-

$$\frac{d\sigma}{d\cos\theta} \sim \begin{cases} 1+\cos^2\theta & \text{spin-}\frac{1}{2}\text{ quarks} \\ 1-\cos^2\theta & \text{spin-0 quarks} \end{cases}$$



estimate θ with angle of thrust axis (doesn't distinguish q direction)



→ quarks have spin $\frac{1}{2}$

Spin of gluons

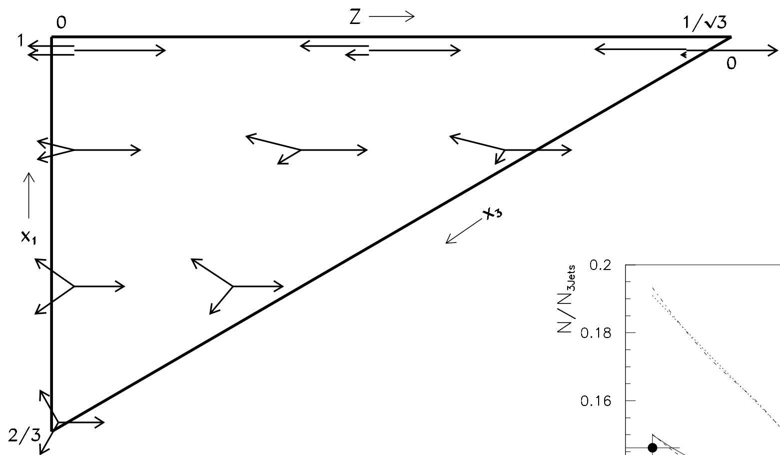
For spin-1 gluon (QCD): $$\frac{d\sigma}{dx_q\,dx_{\bar{q}}} \sim \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

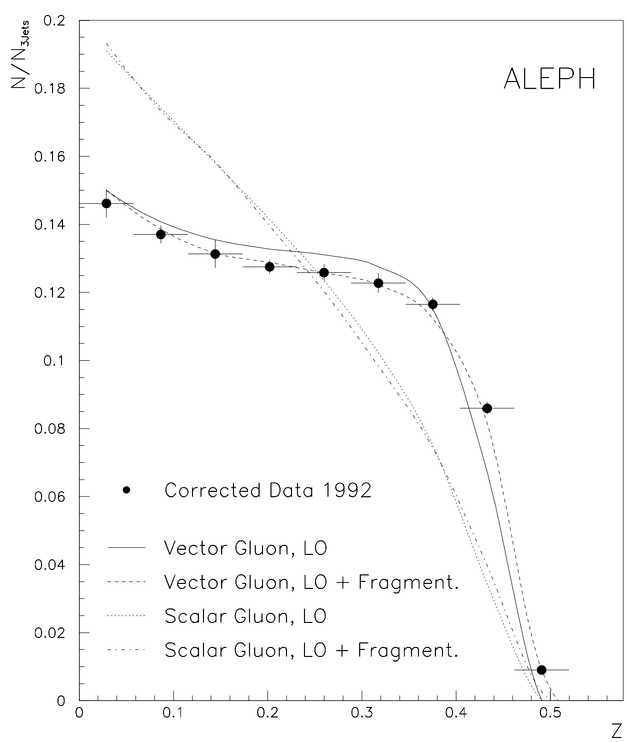
For spin-0 model: $$\frac{d\sigma}{dx_q\,dx_{\bar{q}}} \sim \frac{x_g^2}{(1-x_q)(1-x_{\bar{q}})} + \text{const}$$

Experimentally difficult to distinguish between q, $\bar{q}$, g jets;

→ order the x_i by energy: $x_1 > x_2 > x_3$

Define $z = (x_2 - x_3)/\sqrt{3}$





Select 3-jet events,

jet energy → x_i → z

Good agreement with QCD (spin-1 gluon)

Perturbative QCD predictions for event shape variables

Currently computable (ERT + **EVENT2**) to next-to-leading order:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = A(y)\frac{\alpha_s(\mu)}{2\pi} + \left[B(y) + 2\pi b_0 A(y)\ln\left(\frac{\mu^2}{s}\right)\right]\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2$$

All amplitudes for NNLO computed; assembling the pieces difficult

N.G.— summer 2005?

For small y , $\ln y$ terms dominate at all orders

→ LL, NLL resummed predictions. Some ambiguities:

Avoiding double counting when combining NLO with LL & NLL

→ 'R, $\ln R$ matching schemes'.

Definition of log to resum

→ $\ln y \to \ln(x_L y)$, e.g., $2/3 < x_L < 3/2$.

Need modifications to satisfy kinematic limits.

Incomplete cancelation of μ dependence from missing higher orders

Variation of μ $\sim$ measure of uncertainty due to

missing higher order terms, e.g., $\frac{1}{2} \leq \frac{\mu}{\sqrt{s}} \leq 2$

Non-perturbative corrections:

MC hadronization models (JETSET, HERWIG, . . .), or

Power law corrections ($\sim 1/Q$)

The renormalization scale μ

- μ reflects an ambiguity of perturbation theory

  not a QCD parameter

- Suppose we measure $\alpha_s(\mu)$ with some μ,

  Use RGE: $\alpha_s(\mu) \to \alpha_s(M_Z)$

  resulting $\alpha_s(M_Z)$ still depends on chosen μ

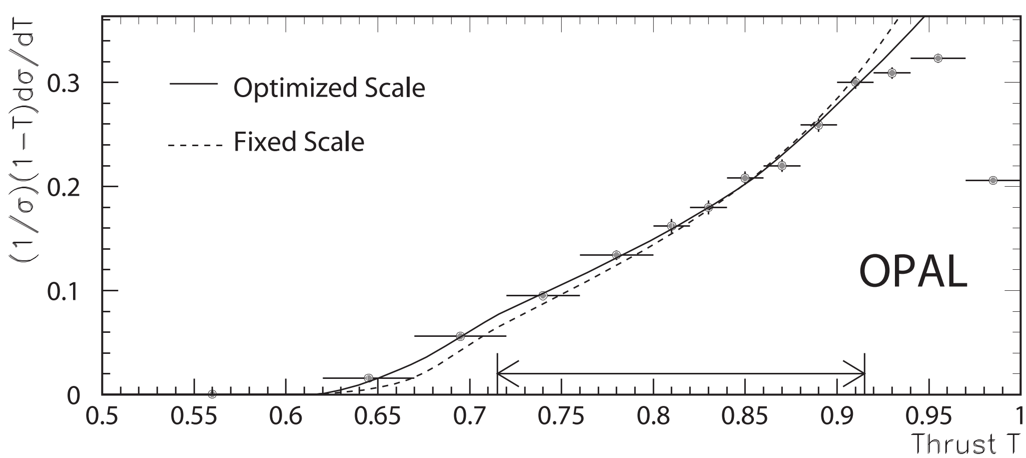
  since μ dependence only cancels to $\mathcal{O}(\alpha_s^2)$

- Higher order coefficients will contain $\sim \left[\ln\left(\frac{\mu^2}{s}\right)\right]^n$

  $\to$ $\mu^2 \approx s$ gives some hope that series is converging.

- But ... at $\mathcal{O}(\alpha_s^2)$, data best described with $\mu^2 \approx 0.002s$ (!?!)

  $\to$ need higher order terms



Glen Cowan
Royal Holloway, University of London

Resumming large logs

Consider cumulative distribution $R(y) = \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy'}dy'$

$$
\begin{aligned}
\ln R(y) &= \alpha_s(G_{12}\ln^2 y + G_{11}\ln y + \ldots) \\
&+ \alpha_s^2(G_{23}\ln^3 y + G_{22}\ln^2 y + \ldots) \\
&+ \alpha_s^3(G_{34}\ln^4 y + G_{33}\ln^3 y + \ldots) \\
&+ \alpha_s^4(G_{45}\ln^5 y + G_{44}\ln^4 y + \ldots) \\
&+ \ldots
\end{aligned}
$$

↑ leading logs ↑ next-to-leading logs

Large logs dominate for $y \to 0$ (two-jet region)

LL and NLL summed to all orders for several variables (including $1-T$, y_3, M_h^2/s)

Matching $\mathcal{O}(\alpha_s^2)$ and (N)LL parts:

subtract double-counted part using R or $\ln R$? (difference $\mathcal{O}(\alpha_s^3)$)

Data no longer prefer small μ

estimate (roughly) magnitude of missing higher orders by varying μ, e.g., $-1 < \ln(\mu^2/s) < 1$

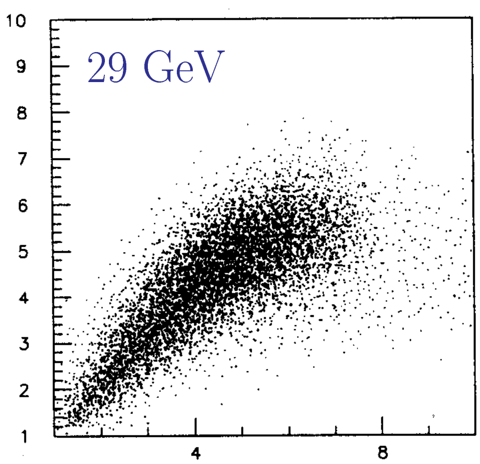
Hadronization corrections

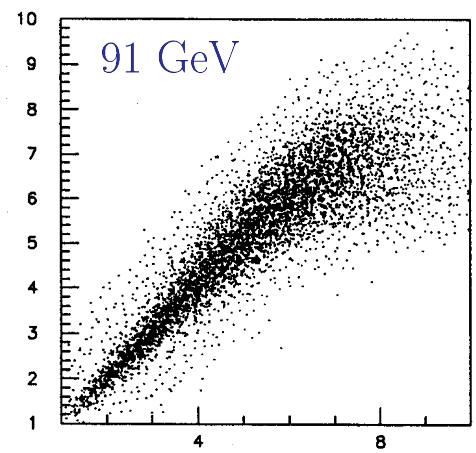
$$\left(\frac{d\sigma}{dy}\right)_{\rm had} (\text{bin } i; \alpha_{\rm s}) = \sum_j \left(\frac{d\sigma}{dy}\right)_{\rm QCD} (\text{bin } j; \alpha_{\rm s}) \cdot P_{ij}$$

$$P_{ij} = P\left(\begin{matrix}\text{hadron level} \\ \text{in bin } i\end{matrix} \middle| \begin{matrix}\text{parton level} \\ \text{in bin } j\end{matrix}\right)$$ ← from MC model

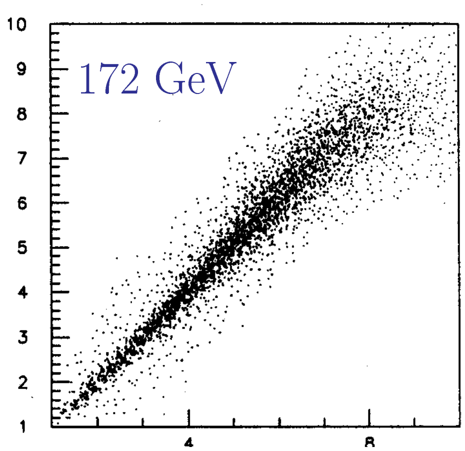
e.g. for $-\ln y_3$ hadron vs. parton level (JETSET):

hadron level





parton level

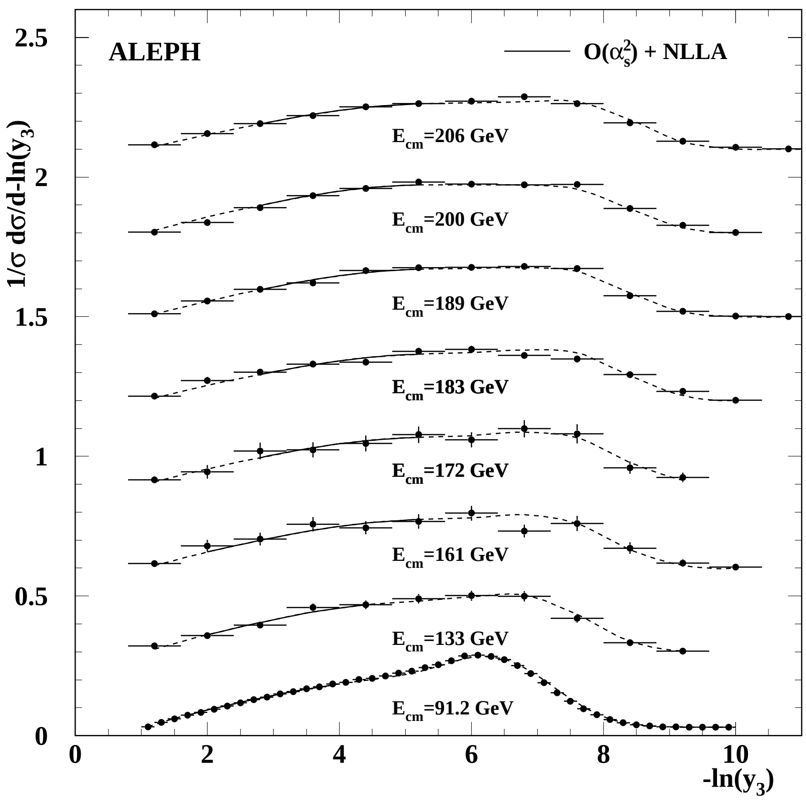


Estimate uncertainty in $\alpha_{\rm s}$ by variation of model, e.g., JETSET, HERWIG, ARIADNE, . . .

Hadronization error small compared to perturbative uncertainty.

Fitting α_s with event-shape distributions

ALEPH, Eur. Phys. J. C35 (2004) 457



E_{cm}	$\alpha_s(E_{cm})$
206	0.1024 ± 0.0039
200	0.1091 ± 0.0039
189	0.1080 ± 0.0039
183	0.1058 ± 0.0053
172	0.1078 ± 0.0098
161	0.1118 ± 0.0089
133	0.1180 ± 0.0064
91.2	0.1180 ± 0.0042

Fit ranges (solid curves) chosen to minimize α_s error.

Compromise between statistics, theoretical uncertainty, . . .

Total error dominated by:

theory at LEP I,

usually theory at LEP II (stat. error big at 161, 172 GeV)

At LEP II, all χ^2 values good;

at LEP I, poor for ρ and B_W (ALEPH).

Estimating α_s uncertainty

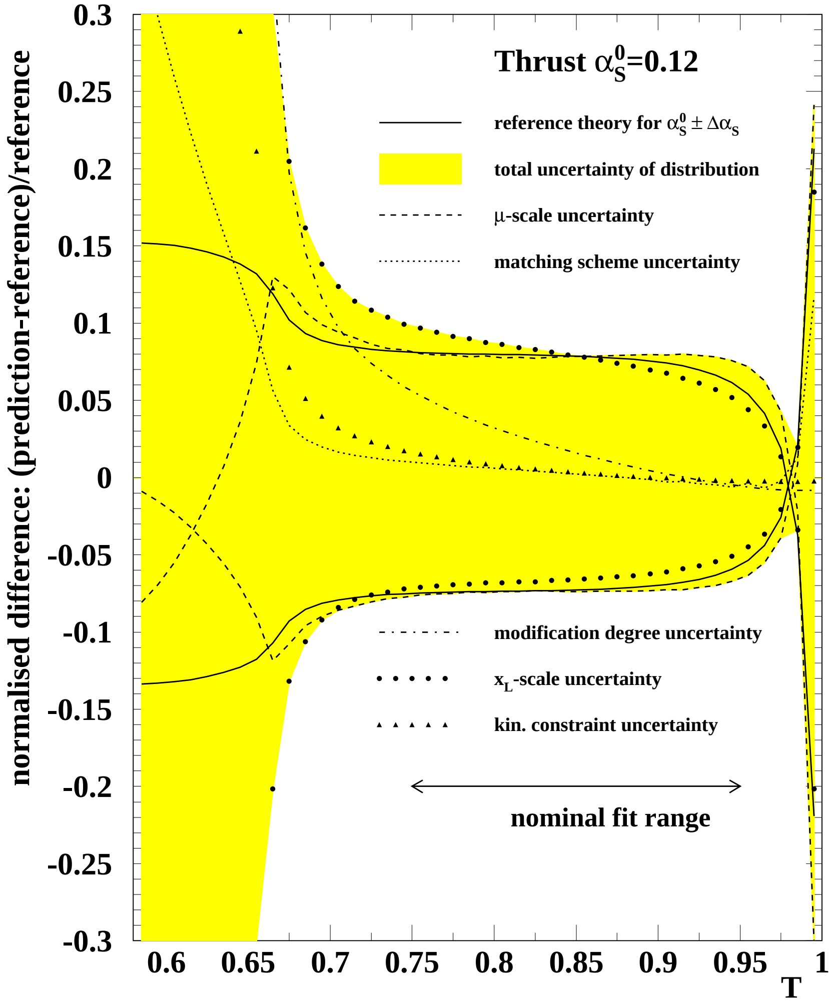
Studied by LEP QCD Working Group

Jones et al., JHEP 12 (2003) 007; hep-ph/0312016

Perturbative theory error dominates (missing higher orders).

Vary theory: μ, x_L, NLO+NLLA matching, kinematic constraints

Relative change in distribution → uncertainty band.



Then use nominal theory; vary α_s so that prediction stays in band → $\Delta\alpha_s$ (theory) .

Combining α_s measurements from event shapes

LEP QCD Working Group procedure

`lepqcd.web.cern.ch/LEPQCD/annihilations`

Variables: T, M_h^2/s, C, y_3, B_W, B_T

4 LEP experiments $\times$ all LEP I/II E_{cm} → 194 α_s values

First attempt:

Estimate full covariance matrix (stat., sys., theory, hadronization)

→ negative weights, sensitive to poorly known correlations

Second attempt:

For weights, zero correlations from theory, hadronization;

error of average not smaller than that of some individual

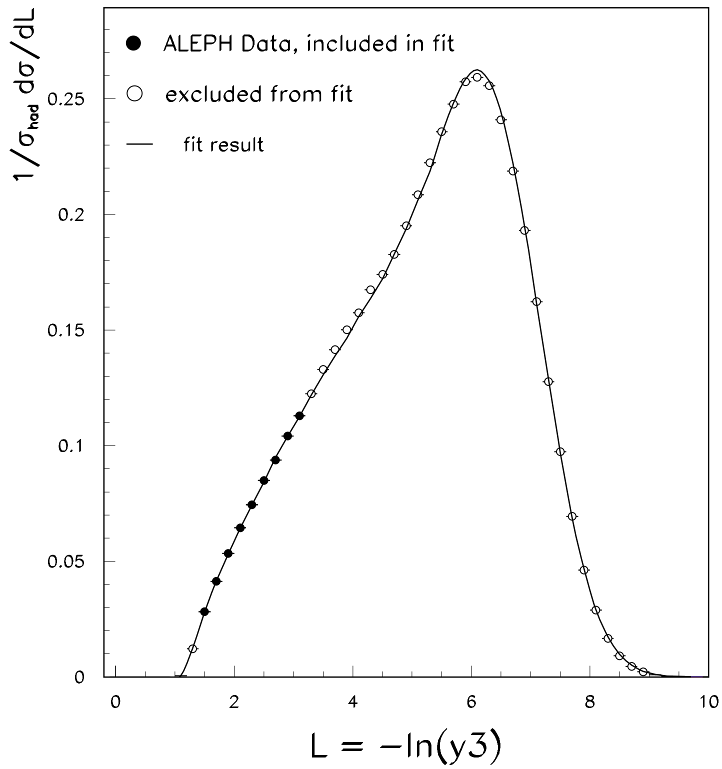
measurements, but result more 'robust'.

LEP QCD Group results preliminary ('almost final')

Fitting α_s to event-shape distributions

Example with y_3:

restrict fit range
to three-jet region;
data also well described
outside fit range



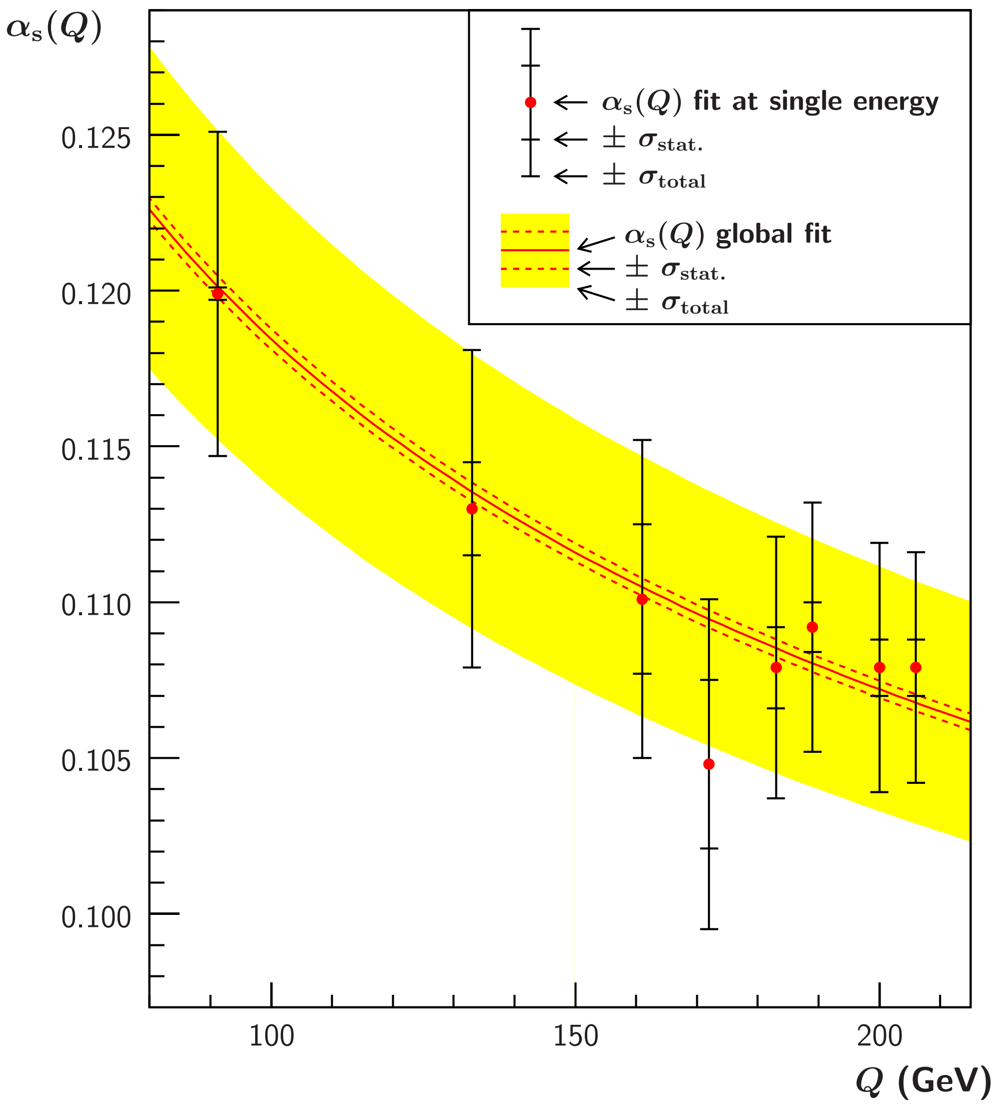
$$\alpha_s(M_Z) = 0.1195 \pm 0.0002\ (\text{stat.})\ \ \pm 0.0038\ (\text{sys.})$$

Systematic error usually dominated by theory (as here)

hadronization corrections: try different models.

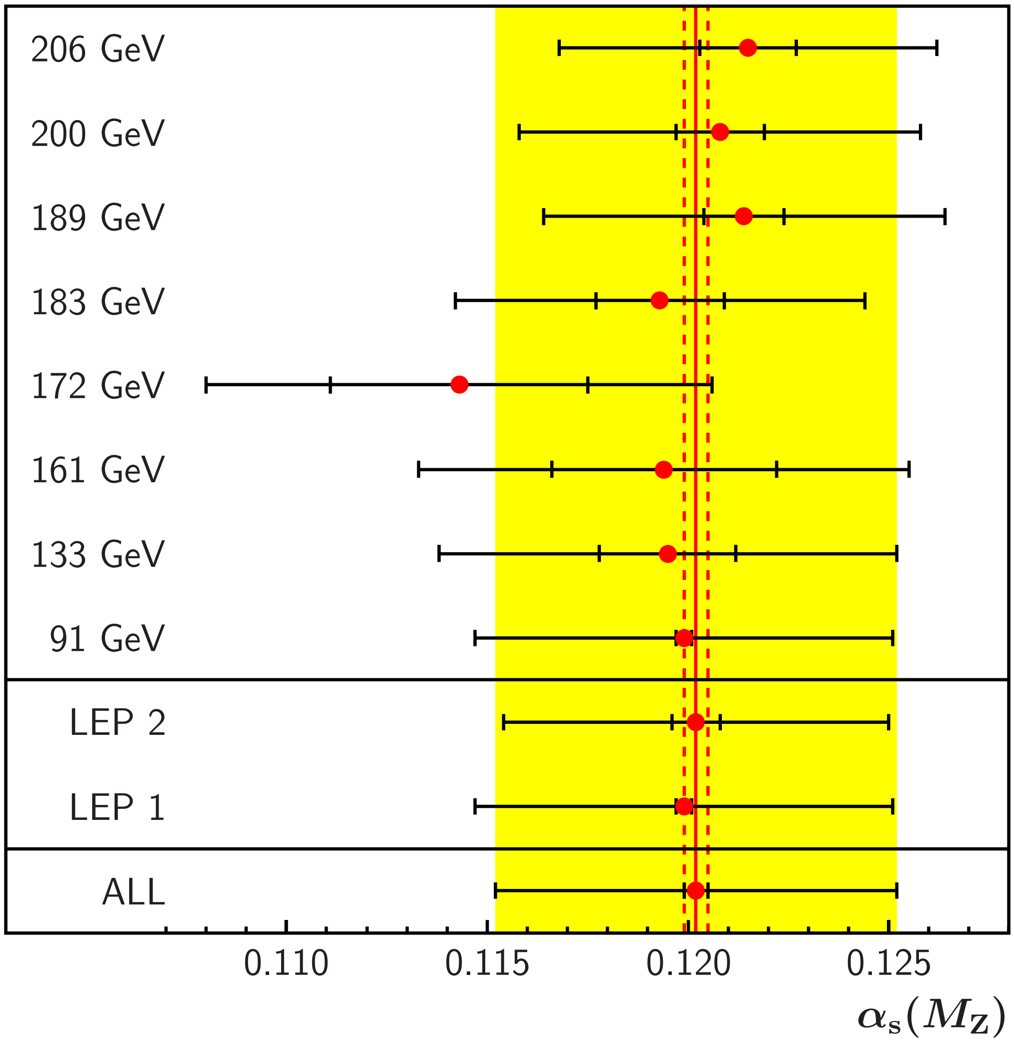
missing higher orders: try varying μ in 'reasonable range',
vary matching scheme to combine NLLA and $\mathcal{O}(\alpha_s^2)$ parts.

$\alpha_s(E_{cm})$ compared to QCD prediction for running (three-loop RGE)



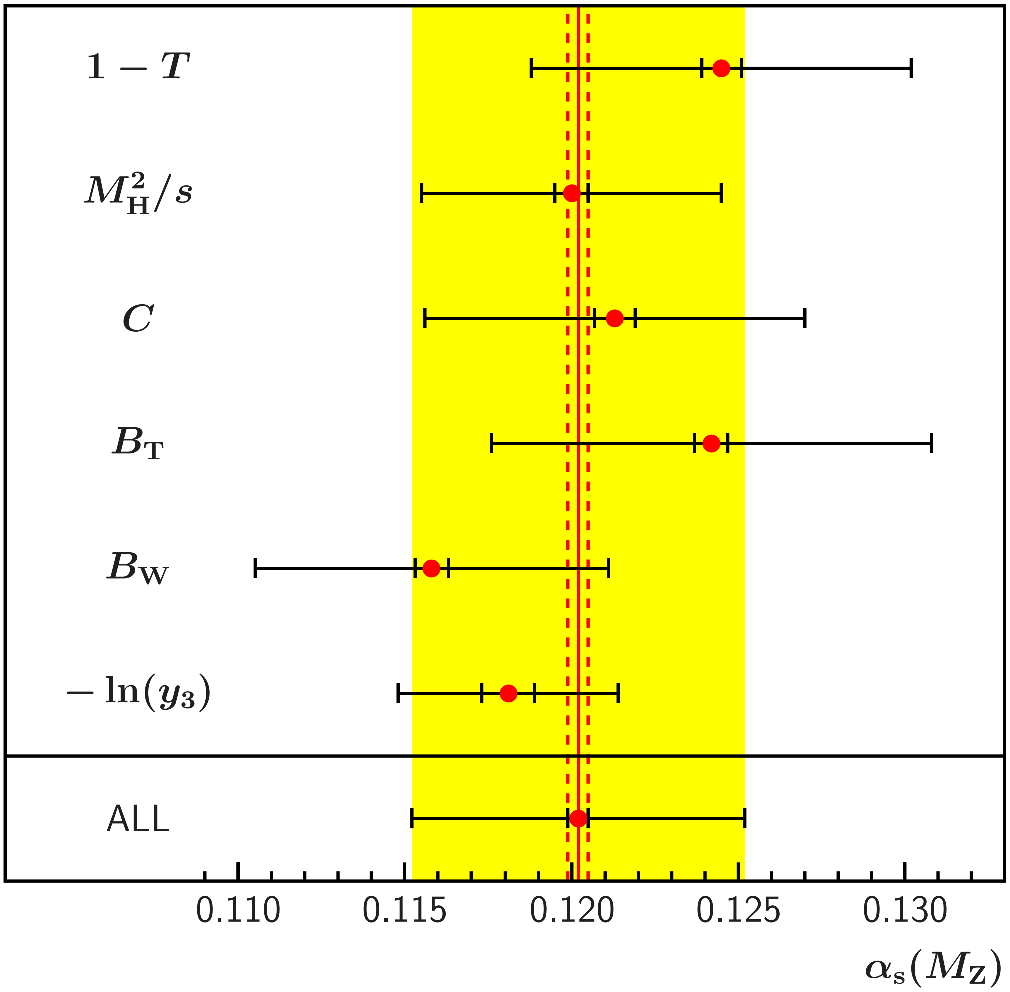
$$
\begin{aligned}
\alpha_s(M_Z) = 0.1202 &\pm 0.0003 \text{ (stat.)} \\
&\pm 0.0009 \text{ (exp.)} \\
&\pm 0.0013 \text{ (hadronization)} \\
&\pm 0.0047 \text{ (pert. theory)}
\end{aligned}
$$

Evolve values to $\alpha_s(M_Z)$ from each centre-of-mass energy range

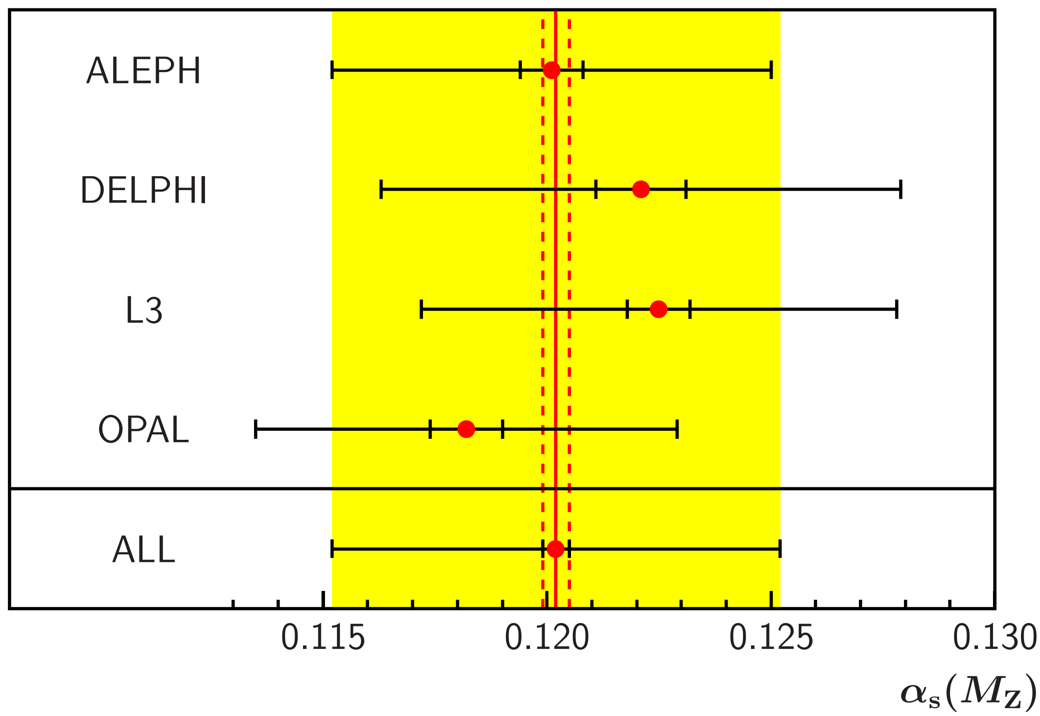


	$\alpha_s(M_Z)$	stat.	exp.	had.	theo.
LEP I	0.1199	$\pm\, 0.0002$	$\pm\, 0.0008$	$\pm\, 0.0017$	$\pm\, 0.0048$
LEP II	0.1202	$\pm\, 0.0006$	$\pm\, 0.0010$	$\pm\, 0.0010$	$\pm\, 0.0046$
All E_{cm}	0.1202	$\pm\, 0.0003$	$\pm\, 0.0009$	$\pm\, 0.0013$	$\pm\, 0.0047$

$\alpha_s(M_Z)$ from individual observables, all E_{cm}



$\alpha_s(M_Z)$ from individual experiments, all observables and E_{cm}



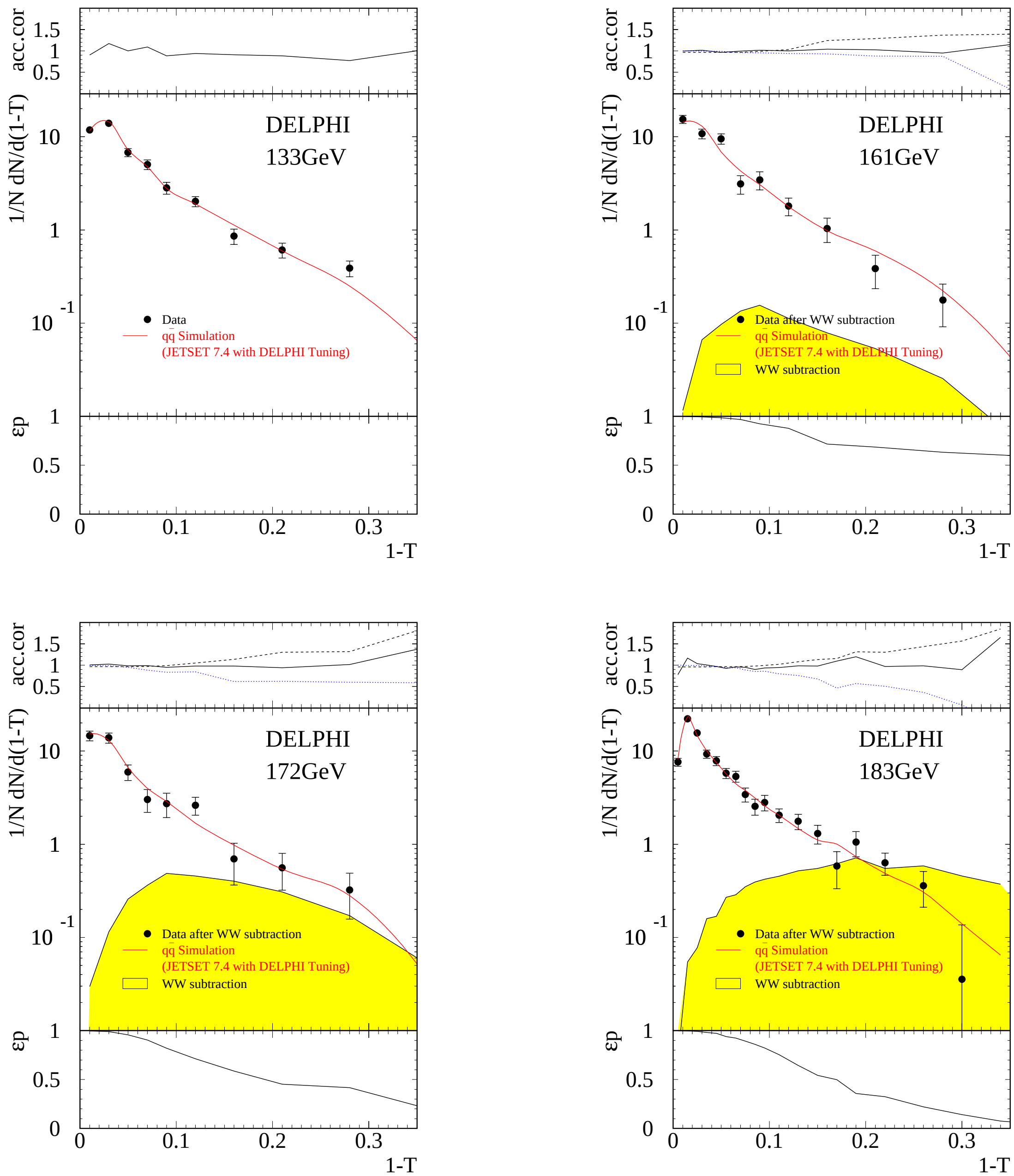
Event-shape distributions at different E_{cm}

Many systematic uncertainties in α_s common to all E_{cm}

→ not a problem for studying running of α_s,

but ... LEP II has hadronic events from $e^+e^- \to W^+W^-$

initial state photon radiation, low statistics ...

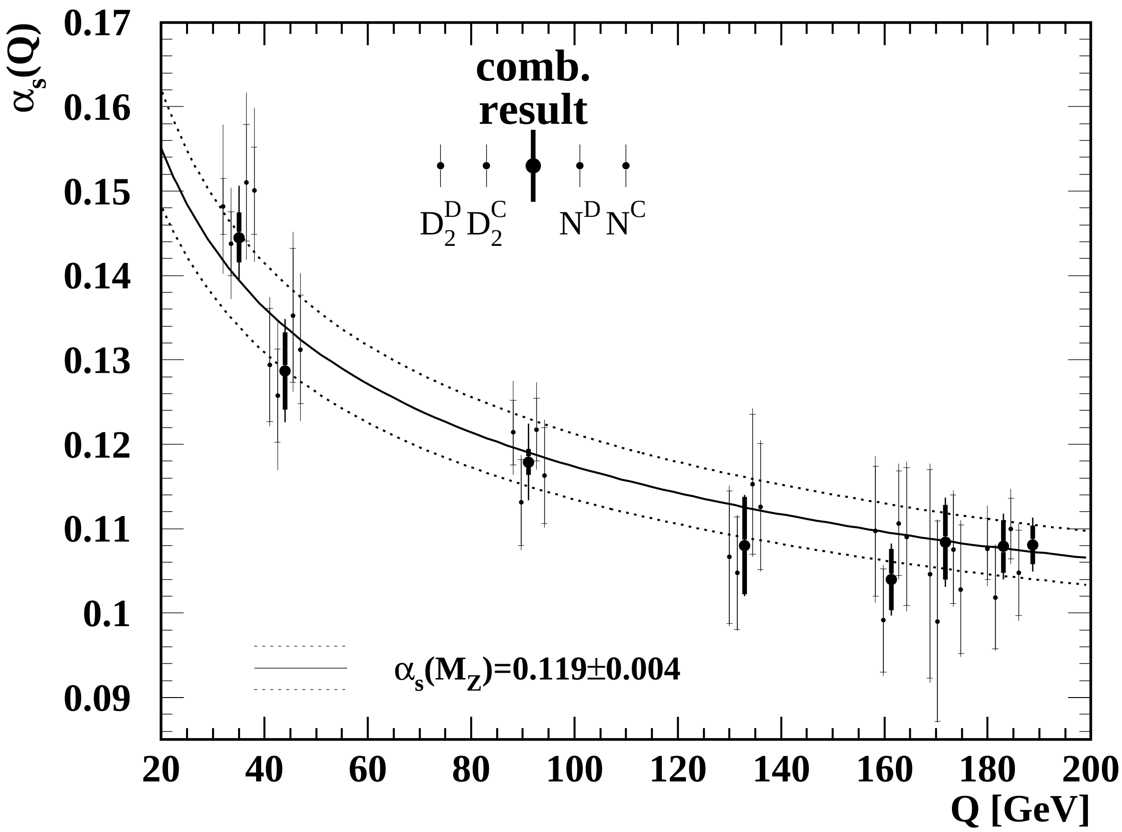


Glen Cowan
Royal Holloway, University of London

Event-shape distributions at different E_{cm}

Common α_s measurements by JADE and OPAL experiments

Eur. Phys. J. C17 (2000) 19.



Inner error bars – uncorrelated errors (e.g. stat.)

Outer error bars – total uncertainty

→ good agreement with predicted running.

Power law corrections to event shapes

$E_{\rm cm}$ dependence of event shape distributions comes from:

running $\alpha_{\rm s}$ (perturbative)

non-perturbative power law correction (typically $\sim 1/Q$)

Event-shape distribution can be written (Webber, Dokshitzer, et al.):

$$\frac{d\sigma}{dy} = \frac{d\sigma_{\rm PT}}{dy}(y - \mathcal{P}D_y)$$

where e.g. $y = 1 - T$, $M_{\rm h}^2$, ... and

$$\mathcal{P} = \frac{f_y(\alpha_0, \alpha_{\rm s})}{Q} \; ,$$

and D_y, f_y are for suitable y computable quantities and α_0 is a universal parameter.

See e.g.

Dokshitzer, Marchisini, Webber, Nucl. Phys. B 469 (1996) 93.

Dokshitzer, Marchesini, Salam, EpJC 3(1999) 1.

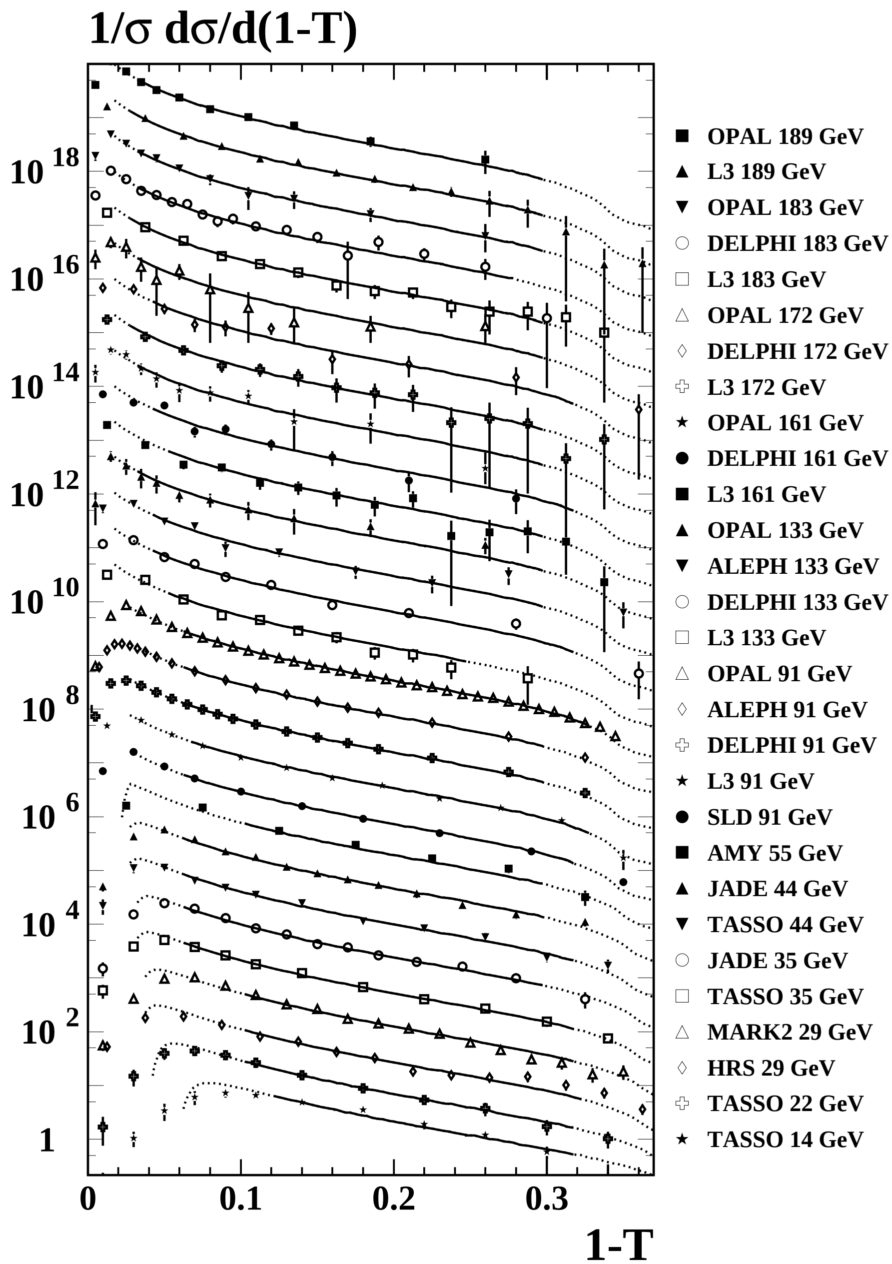
Dokshitzer, Lucenti, Marchesini, Salam, J. High Energy Phys. 5 (1998) 003.

Power law corrections to event shapes (cont.)

Recent study of PETRA, PEP, TRISTAN, SLC, LEP data

Movilla Fernández, Bethke, Biebel, Kluth, EPJ C22 (2001) 1

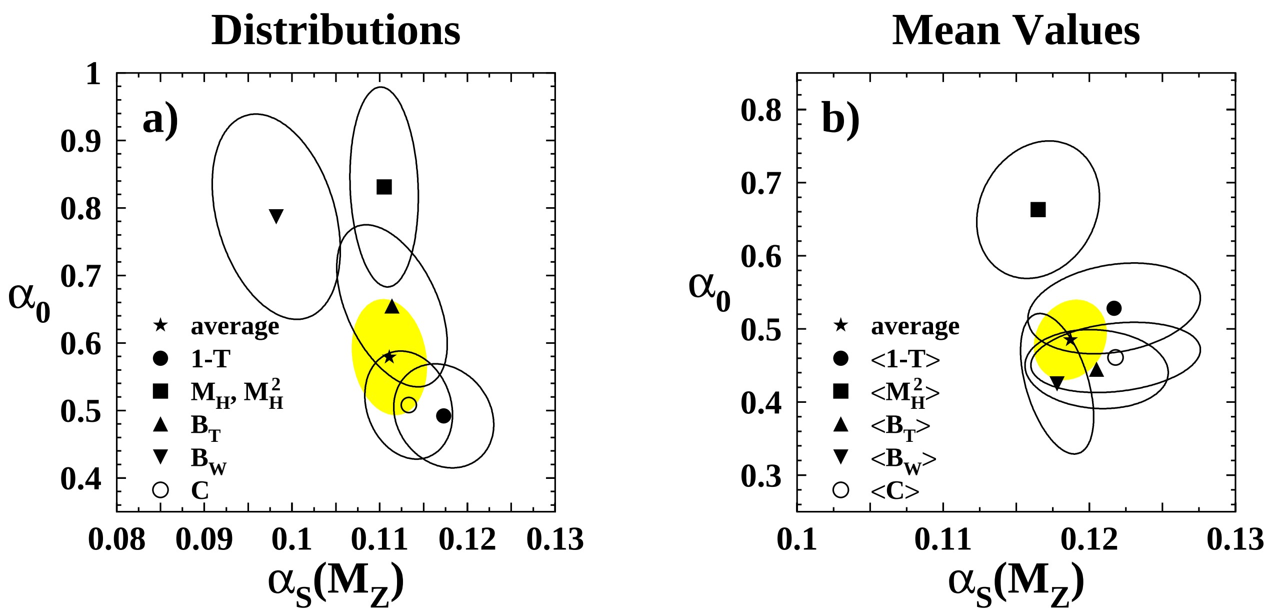
Power law model used instead of hadronization correction from MC



Power law model (Webber, Dokhshitzer, Marchesini):

single (universal) non-perturbative parameter α_0 ;

fit together with α_s using NLO + NLLA QCD



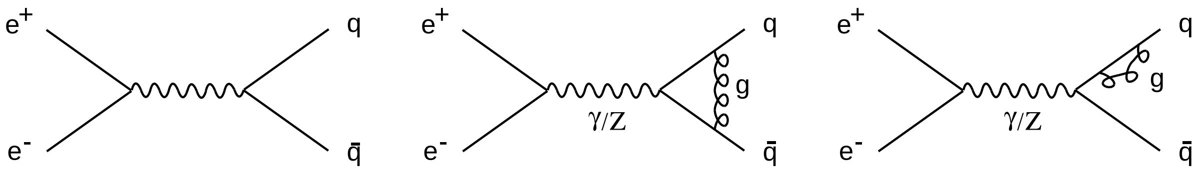
$$\alpha_s(M_Z) = 0.1171^{+0.0032}_{-0.0020}$$

$$\alpha_0(2\ \mathrm{GeV}) = 0.513^{+0.066}_{-0.045}$$

α_s from the total hadronic cross section

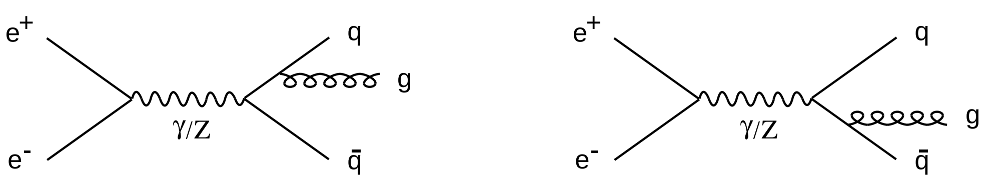
$$\sigma(e^+e^- \to \text{ hadrons}) = \sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) + \ldots$$

$e^+e^- \to q\bar{q}$



→ ultraviolet divergences

$e^+e^- \to q\bar{q}g$



→ infrared, collinear divergences

The divergences (almost) cancel, leaving a finite correction

$$\sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) = \sigma_0\left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

works at every order in perturbation theory

α_s from R_l and R_τ

$$R_l = \frac{\Gamma(\mathrm{Z} \to \ \text{hadrons})}{\Gamma(\mathrm{Z} \to l\bar{l})} \to \text{(almost) same as } \sigma_{\text{had}}:$$

$$= 19.934 \left[1 + 1.045 \frac{\alpha_s}{\pi} + 0.44 \left(\frac{\alpha_s}{\pi}\right)^2 - 15 \left(\frac{\alpha_s}{\pi}\right)^3\right]$$

LEP EW WG (hep-ex/0312023) measures $R_l = 20.767 \pm 0.025$

$$\alpha_s(M_\mathrm{Z}) = 0.1226 \pm 0.0038 \text{ (exp) } {}^{+0.0033}_{-0.0000} \text{ } (M_\mathrm{H})^{+0.0028}_{-0.0005} \text{ (QCD)}$$

Or using global electroweak fit,

$$\alpha_s(M_\mathrm{Z}) = 0.1200^{+0.0031}_{-0.0029} \text{ (exp)} \qquad \text{(QCD error not estimated)}$$

Similarly, decay of virtual W from τ sensitive to QCD corrections

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau \text{ hadrons})}{\Gamma(\tau^- \to \nu_\tau \mathrm{e}^- \overline{\nu}_\mathrm{e})}$$

ALEPH, OPAL average result (Bethke hep-ex/0004021):

$$\alpha_s(m_\tau) = 0.323 \pm 0.005 \text{ (exp) } \pm 0.030 \text{ (theo)}$$

Evolve to M_Z,

$$\alpha_s(M_\mathrm{Z}) = 0.1181 \pm 0.0007 \text{ (exp) } \pm 0.0030 \text{ (theo)}$$

Scaling violations in inclusive spectra

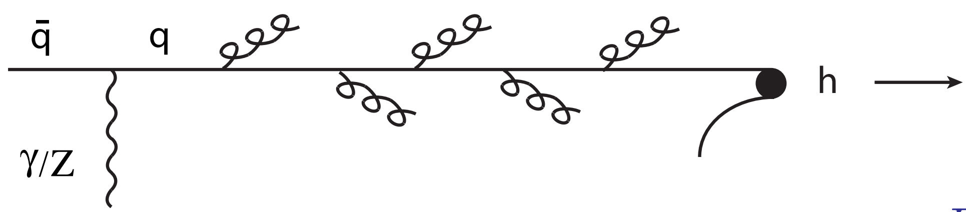
Inclusive cross sections $\frac{1}{\sigma}\frac{d\sigma}{dx}(\mathrm{e^+e^-} \to \mathrm{h} + X)$

with $x = 2E_\mathrm{h}/\sqrt{s}$ not calculable in perturbative QCD, but,

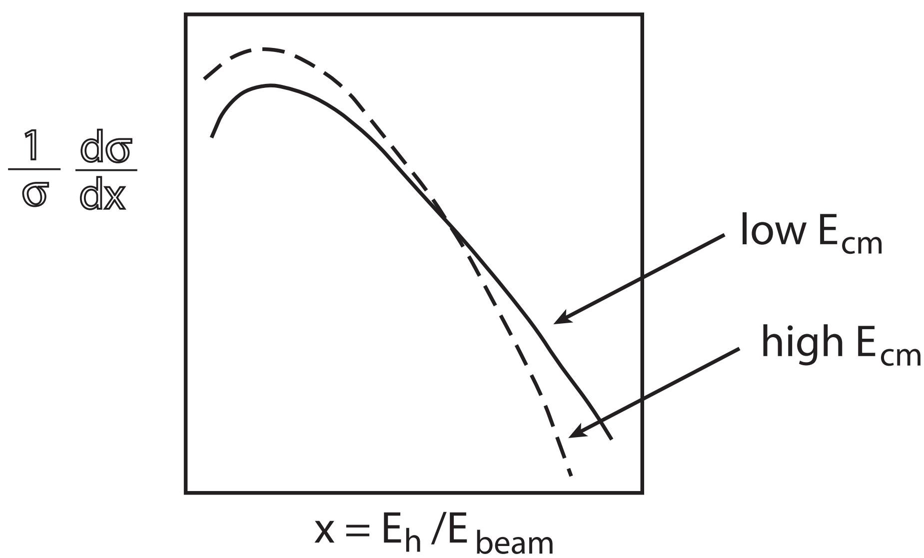
change in Q (here $= E_\mathrm{cm}$) is predicted (DGLAP):

$$\frac{\partial D(x,Q)}{\partial \ln Q} \sim \int_x^1 \frac{dz}{z}\,\alpha_\mathrm{s}\,P_\mathrm{AP}(z)\,D(x/z,Q)$$

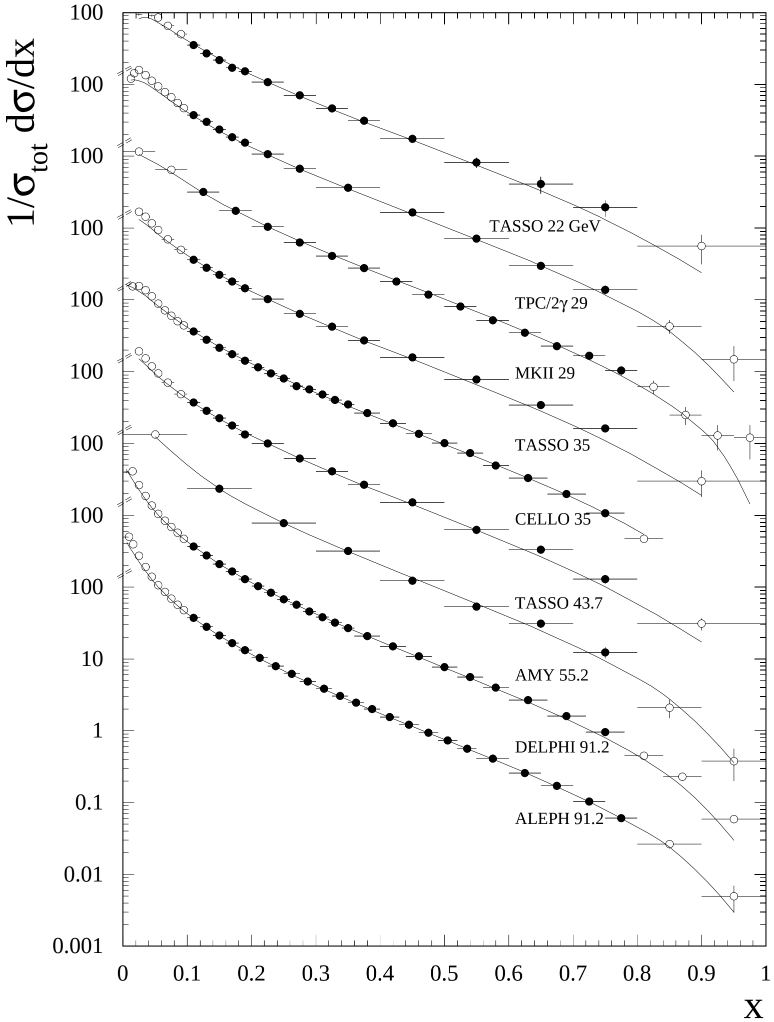
$D(x,Q)$ (fragmentation function) $\sim (1/\sigma)(d\sigma/dx)$



$x = E_q/E_\mathrm{h}$

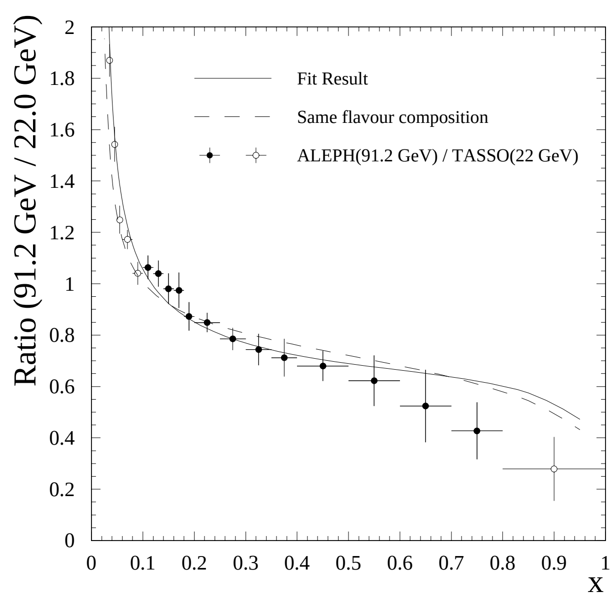


Results on scaling violations and fit of α_s



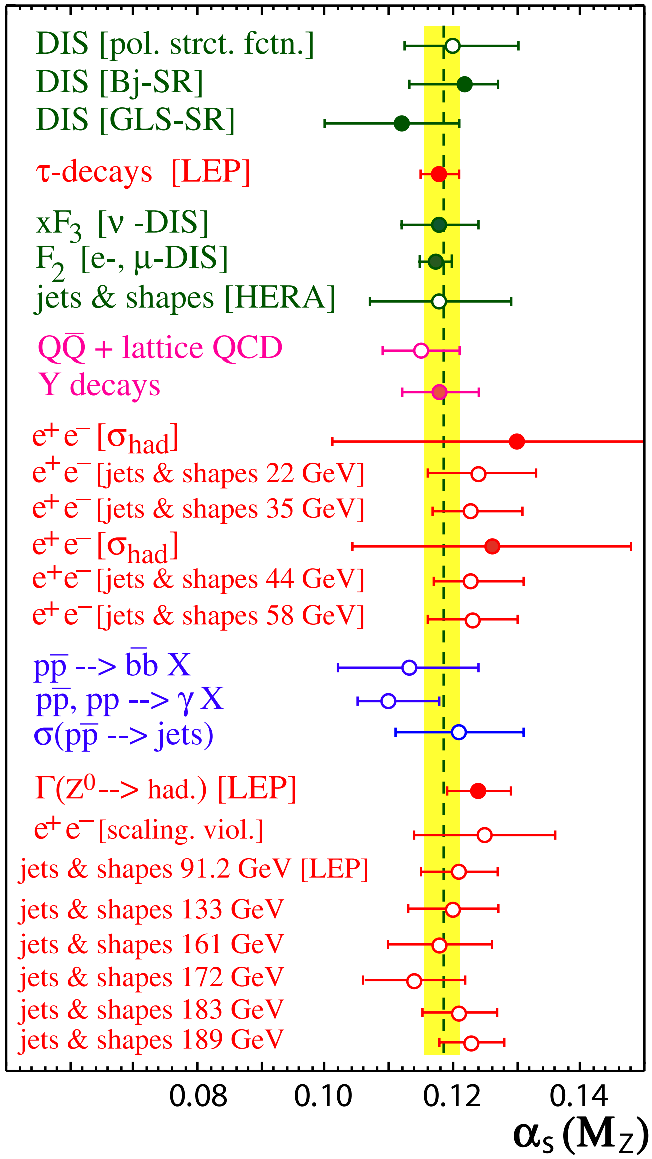
Fit to NLO QCD
ALEPH, PLB 357 (1995) 487

$$\alpha_s(M_Z) = 0.129 \pm 0.009$$



Summary of $\alpha_s(M_Z)$

S. Bethke, hep-ex/0004021

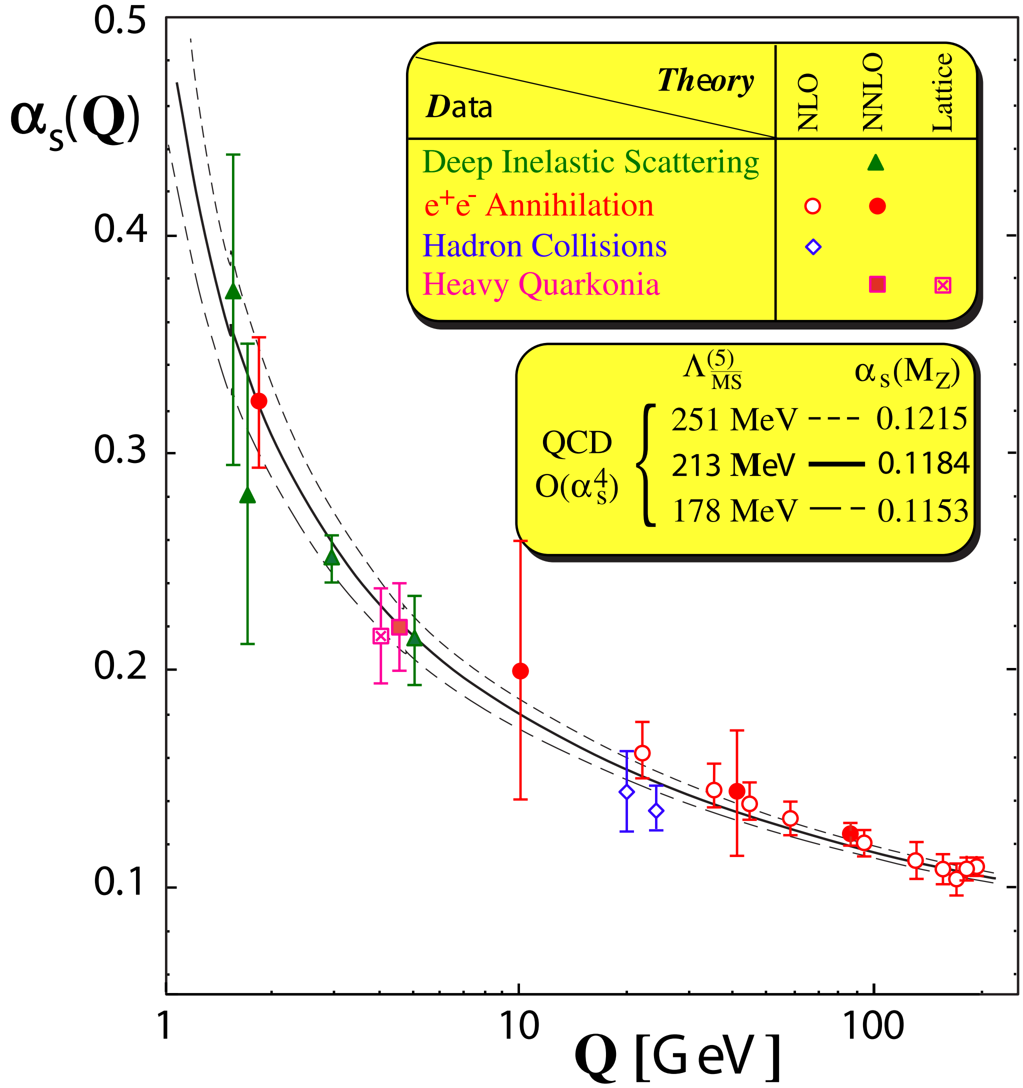


Bethke's average (hep-ex/0004021):

$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

Running of $\alpha_s(M_Z)$

S. Bethke, hep-ex/0004021



Flavour independence of α_s

Does $q\bar{q}g$ coupling depend on quark flavour?

QCD: no (but could be mimicked by new physics)

Select event samples enriched in $b\bar{b}$, $c\bar{c}$ and uds

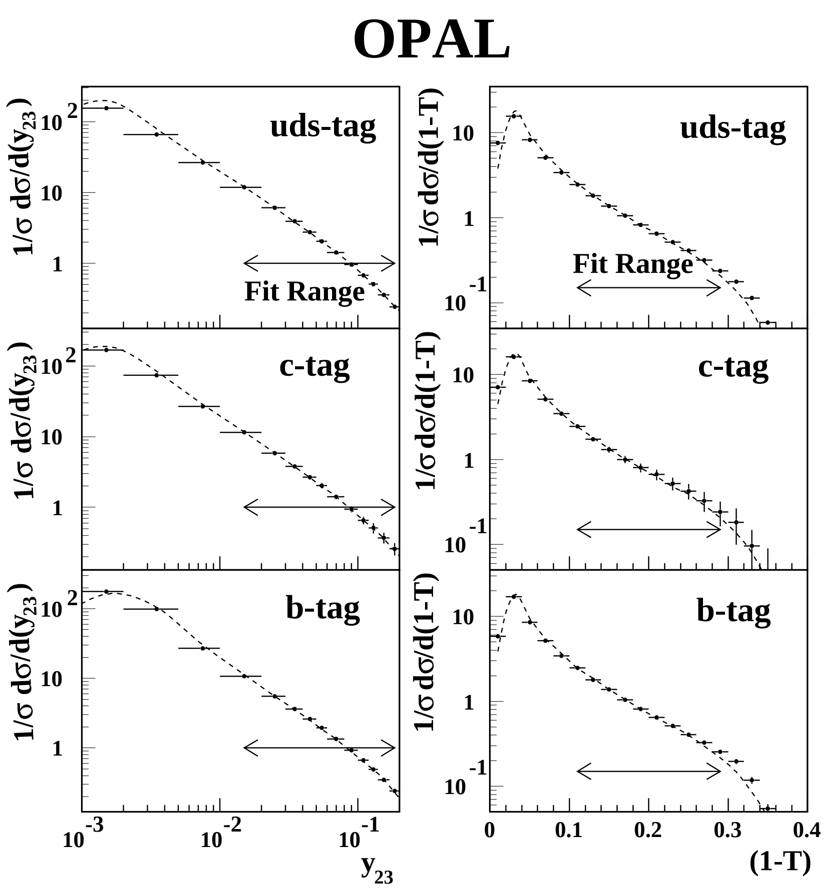
lifetime tag $\rightarrow b$

lifetime antitag $\rightarrow uds$

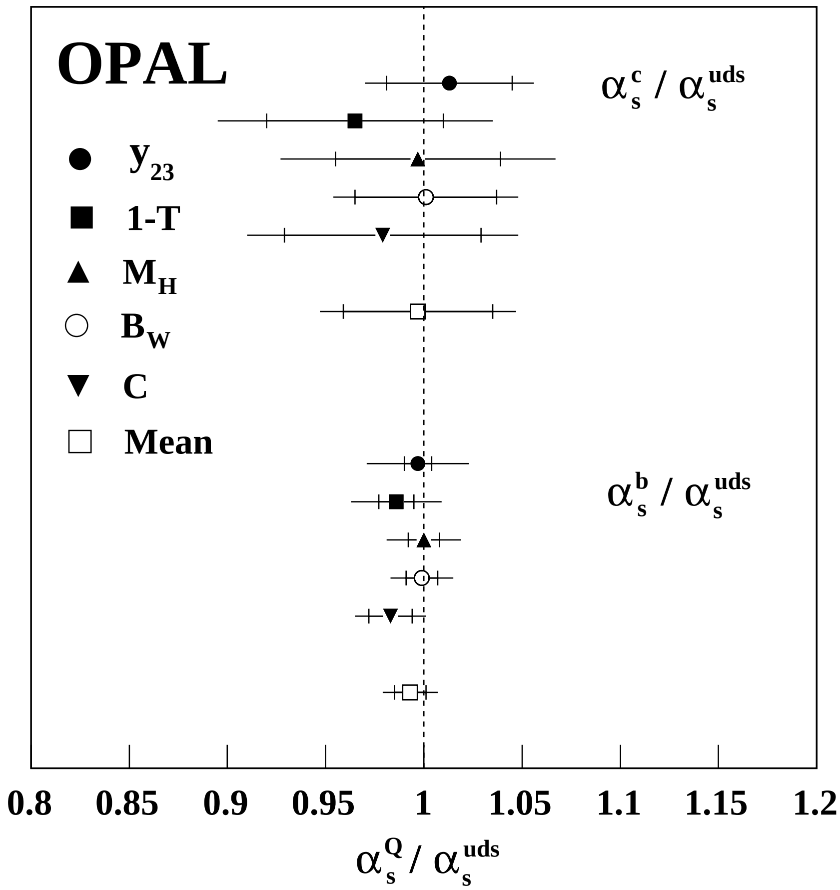
fast D mesons in jets $\rightarrow c$

Fit α_s^{uds}, $\alpha_s^c/\alpha_s^{uds}$, $\alpha_s^b/\alpha_s^{uds}$ as separate parameters,

use $\mathcal{O}(\alpha_s^2)$ including with b, c mass effects.



OPAL, Eur. Phys. J. C11 (1999) 643



Averages:

$$\alpha_s^c/\alpha_s^{uds} = 0.997 \pm 0.038\ (\text{stat.})\ \ \pm 0.030\ (\text{sys.})\ \ \pm 0.012\ (\text{theo})$$

$$\alpha_s^b/\alpha_s^{uds} = 0.993 \pm 0.008\ (\text{stat.})\ \ \pm 0.006\ (\text{sys.})\ \ \pm 0.011\ (\text{theo})$$

$\rightarrow$ α_s flavour independence tested to $\sim$ percent level.

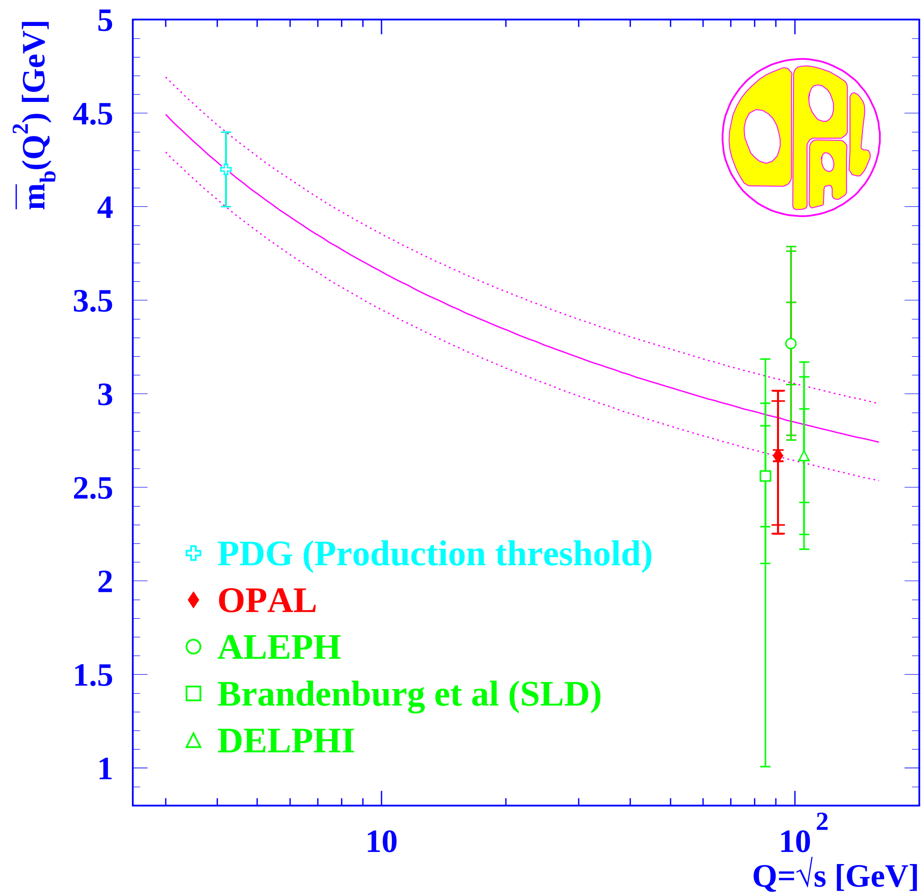
m_b

High b-mass suppresses gluon radiation,

same reason less Bremsstrahlung for muons

$\rightarrow$ event-shapes, jet-rates differ for $b\bar{b}$, uds events

Here *assume* α_s flavour independent, fit m_b

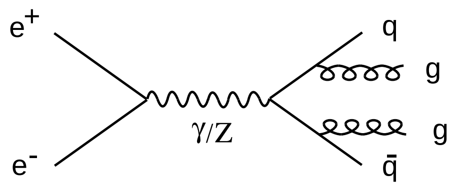


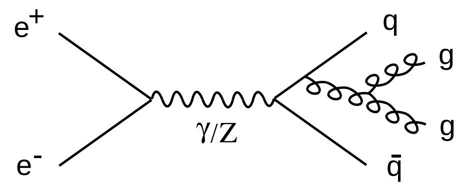
Running of mass observed, e.g., OPAL EPJ C21 (2001) 411:

$$m_b(M_Z) = 2.67 \pm 0.03 \text{ (stat) } \pm^{0.29}_{0.37} \text{ (sys) } \pm .19 \text{ (theo)}$$

Four-jet events

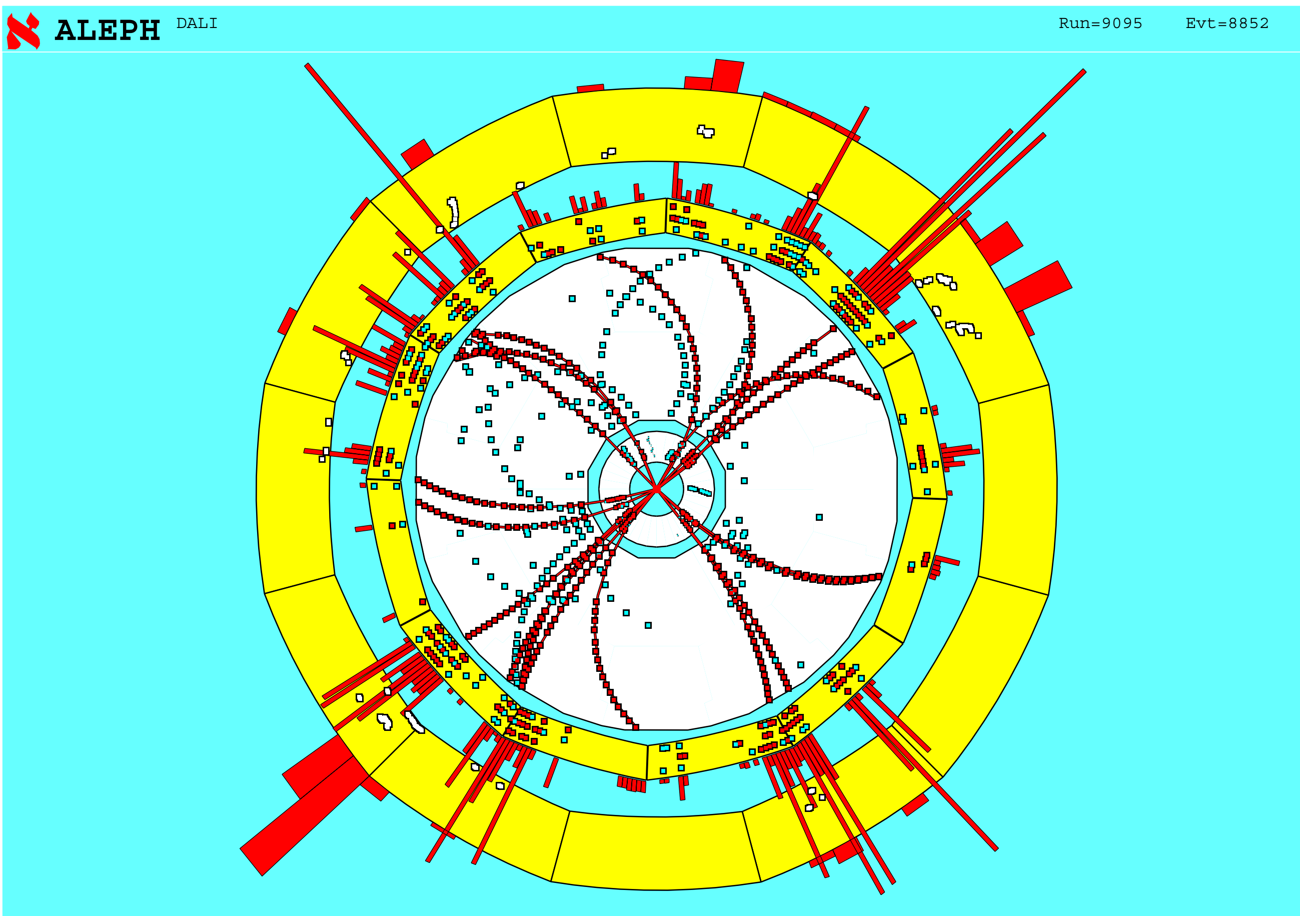
Four-jet properties → triple-gluon vertex, QCD gauge structure





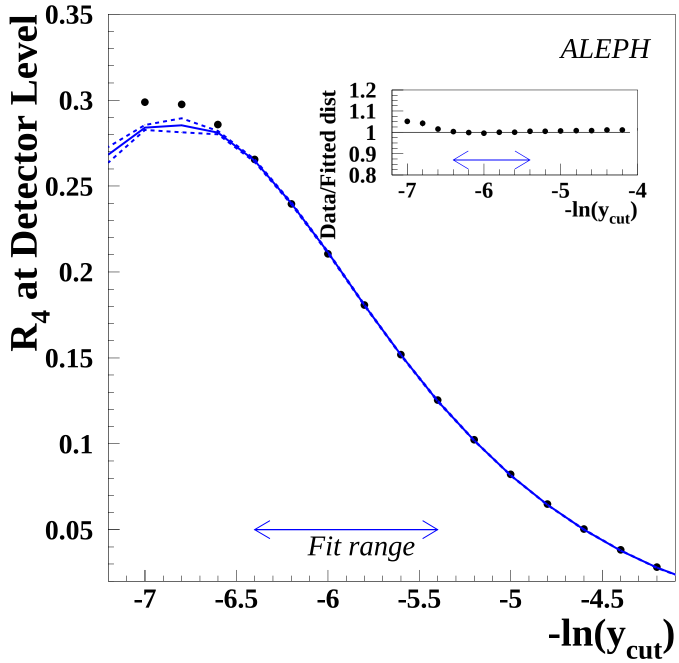
Four-jet matrix element calculated to NLO

Dixon, Signer, Nagy, Trócsányi



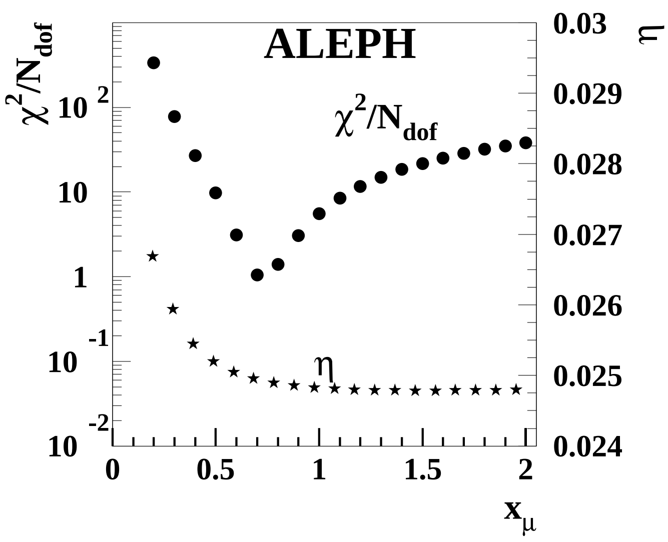
α_s from four-jet rate

ALEPH, Eur. Phys. J. C27 (2003) 1



α_s fitted to four-jet rate using NLO + NLLA QCD

$$\alpha_s(M_Z) = 0.1170 \pm 0.0001 \text{ (stat.)} \ \pm 0.0013 \text{ (sys.)}$$



Error dominated by μ variation; large variation range rejected as χ^2 rapidly deteriorates; not much change in $\eta \propto \alpha_s$

No error contribution estimated from NLO+NLLA matching.

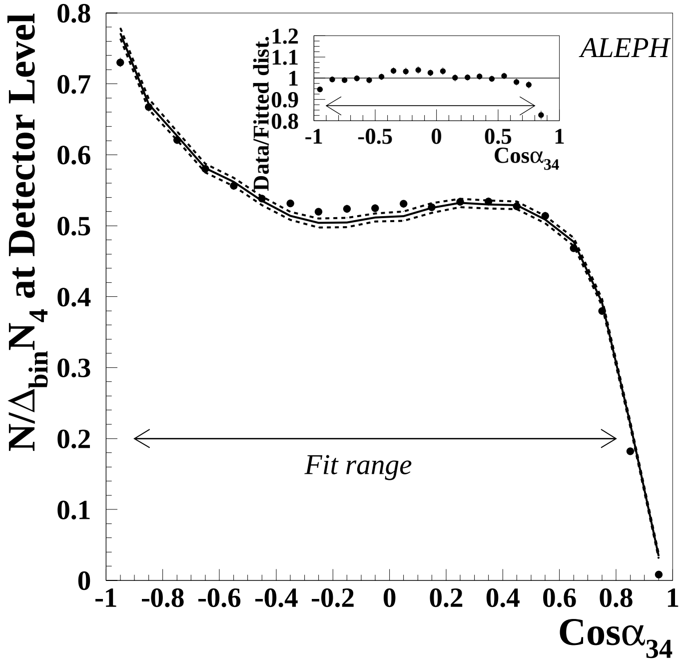
QCD colour factors

Summing over colours, QCD predicts

$$P(q \to qg) \propto C_F = 4/3,$$
$$P(g \to gg) \propto C_A = 3,$$
$$P(g \to q\bar{q}) \propto T_F = 1/2$$

Four-jet angular distributions provide info on colour factors,

e.g., α_{34} = angle between two lowest energy jets.



Recent analyses based on NLO predictions:

ALEPH, Eur. Phys. J. C27 (2003) 1;

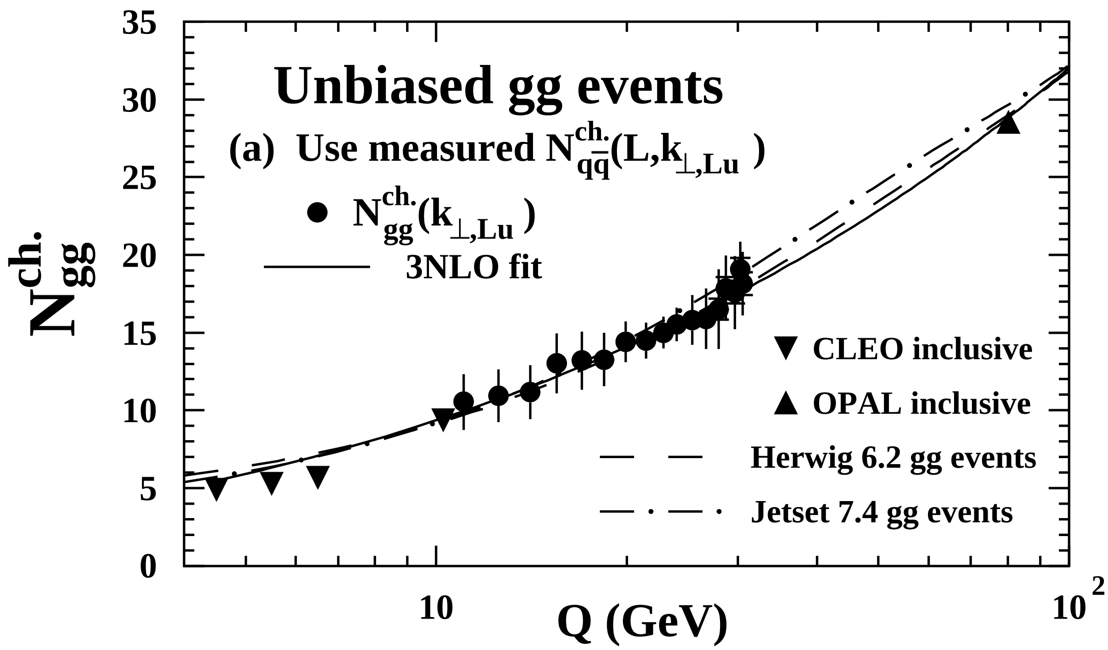
OPAL, Eur. Phys. J. C20 (2001) 601.

Particle multiplicities in quark-, gluon-jets

Many LEP studies; recently e.g. OPAL, Eur. Phys. J. C23 (2002) 597.

'Unbiased' gluon jets through comparison of $q\bar{q}g$ and $q\bar{q}$

$$N_{gg} \sim 2\left[N_{q\bar{q}g} - N_{q\bar{q}}\right]$$



Good agreement with 3NLO QCD prediction

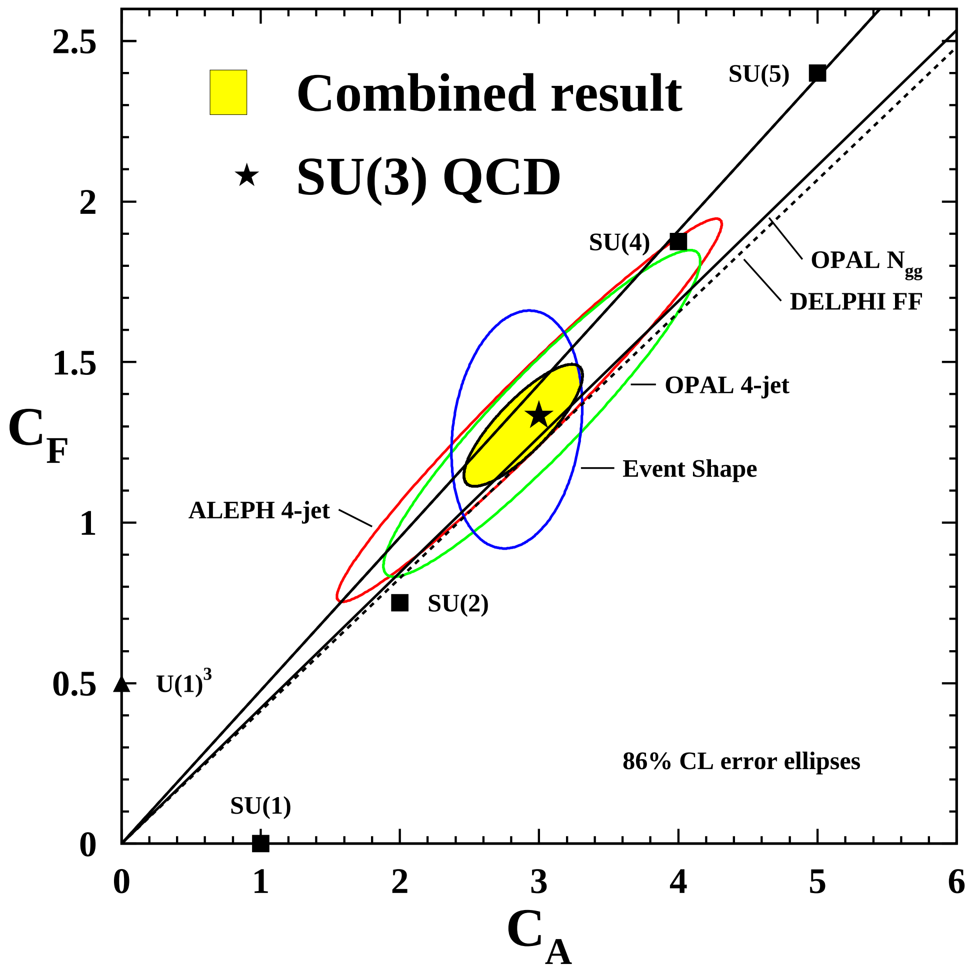
Capella et al., Phys. Rev. D61 (2000) 074009.

Fit ratio of colour factors

$$C_A/C_F = 2.23 \pm 0.14 \quad \text{(QCD: } 9/4)$$

Averages of results on colour factors

S. Kluth, hep-ex/0309070

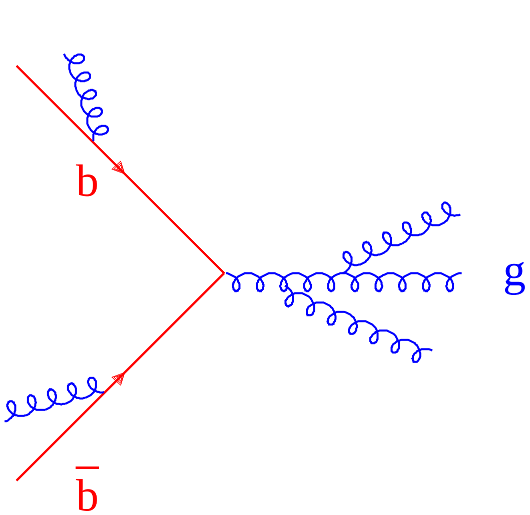


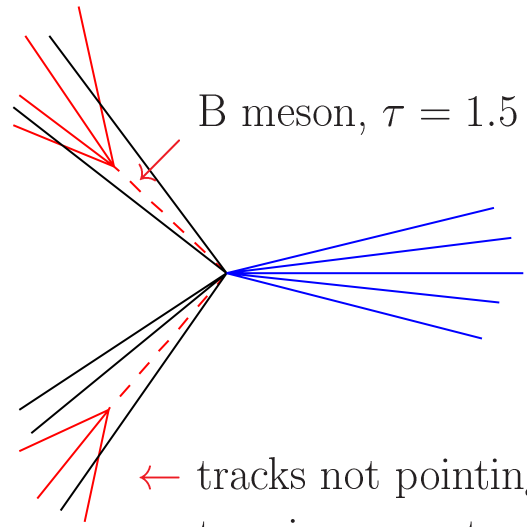
$$C_A = 2.89 \pm 0.03 \text{ (stat.)} \pm 0.21 \text{ (sys.)},$$

$$C_F = 1.30 \pm 0.01 \text{ (stat.)} \pm 0.09 \text{ (sys.)}.$$

Excellent agreement with QCD: $C_A = 3$, $C_F = 4/3$.

Internal structure of quark and gluon jets





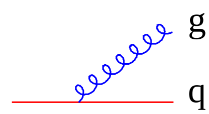
- B hadrons in two jets tags third as gluon jet (purity $p_{\rm g} \approx 94\%$)
- Sample of all jets: 2/3 quark, 1/3 gluon jets ('mixed')
- Properties of pure quark/gluon jets inferred:

$$X_{\rm tag} = (1 - p_{\rm g}) X_{\rm q} + p_{\rm g} X_{\rm g}$$

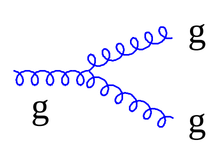
$$X_{\rm mix} = \tfrac{2}{3} X_{\rm q} + \tfrac{1}{3} X_{\rm g}$$

QCD predicts:

$$P(q \to qg) \propto \sum_{\rm colour} \left| \quad \right|^2 \propto C_{\rm F} = 4/3$$

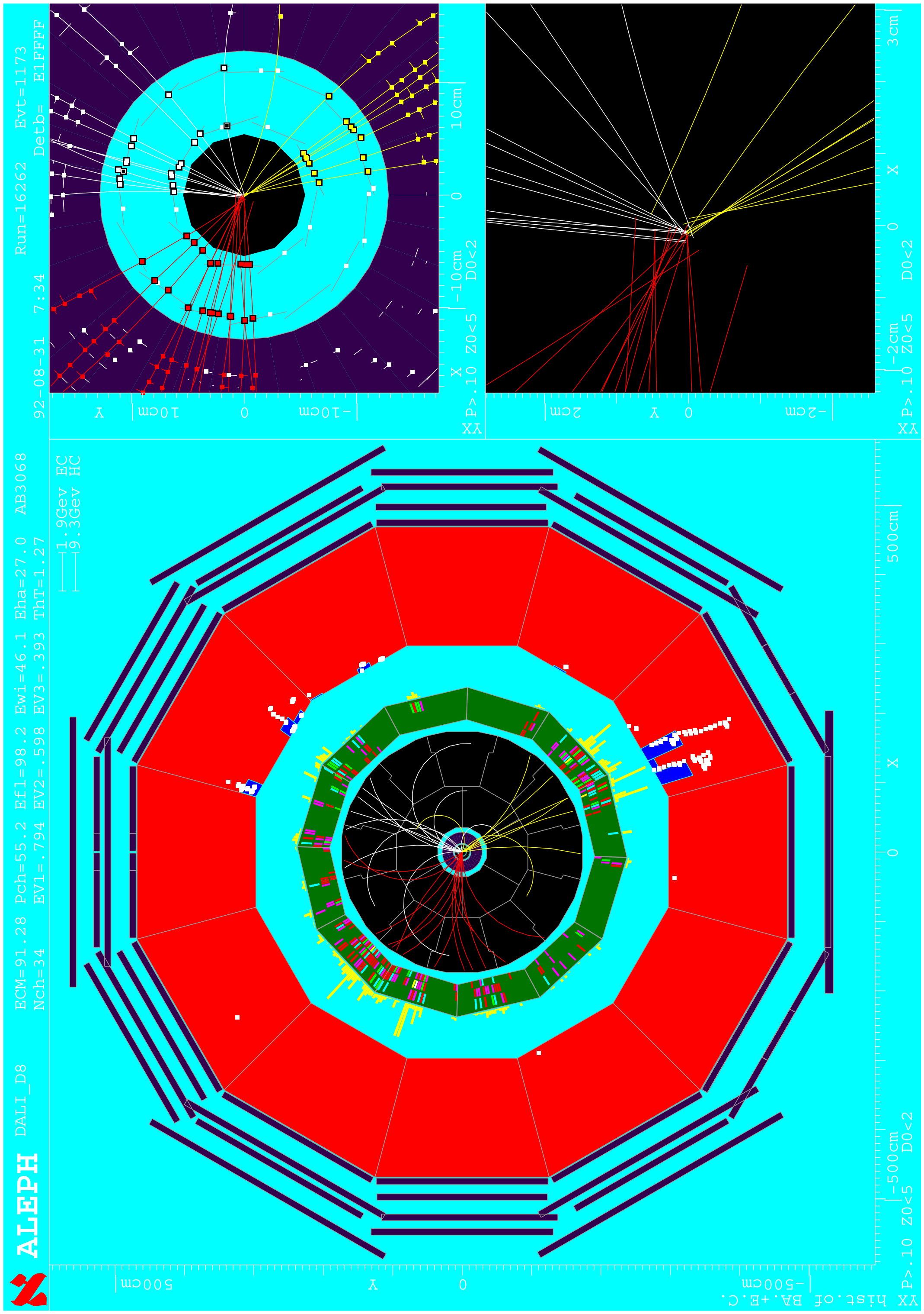


$$P(g \to gg) \propto \sum_{\rm colour} \left| \quad \right|^2 \propto C_{\rm A} = 3$$



→ specific QCD predictions for q/g jet structure

Three-jet event used to obtain gluon jet sample



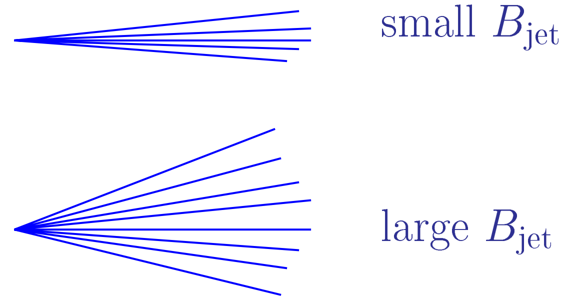
Glen Cowan
Royal Holloway, University of London

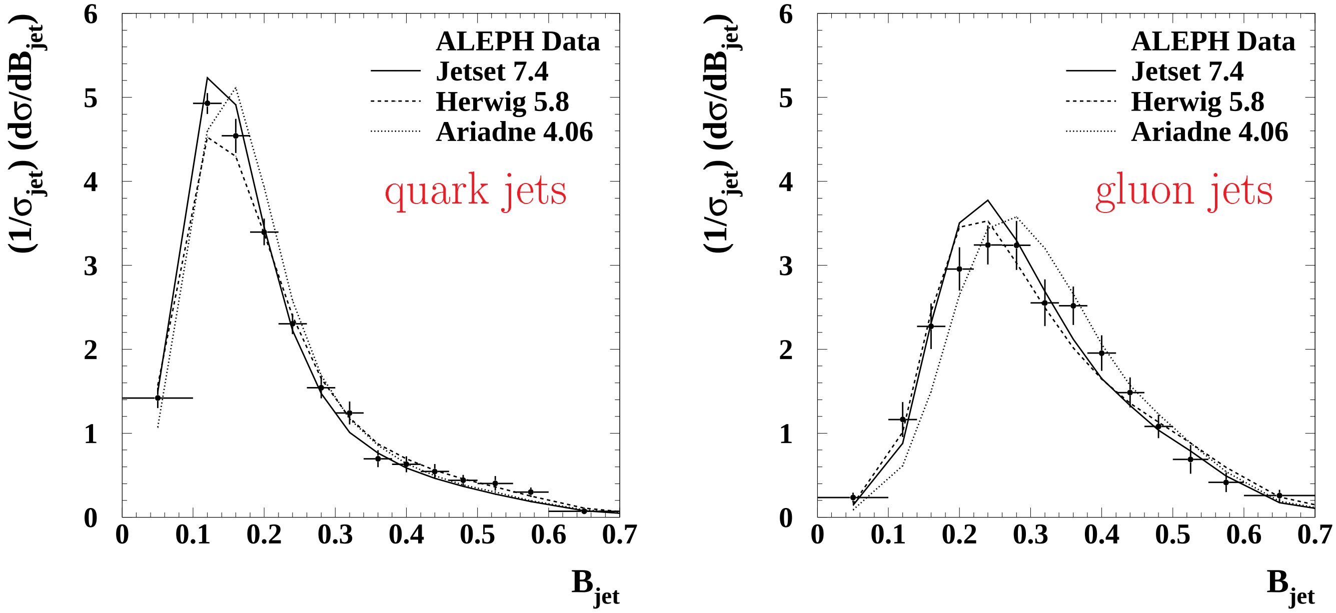
Some results on quark/gluon jets

Distributions of variables describing jet shape:

$$B_{\text{jet}} = \frac{\Sigma_i \, |p_{\perp\, i}|}{\Sigma_i \, |p_i|}$$

'jet broadening'





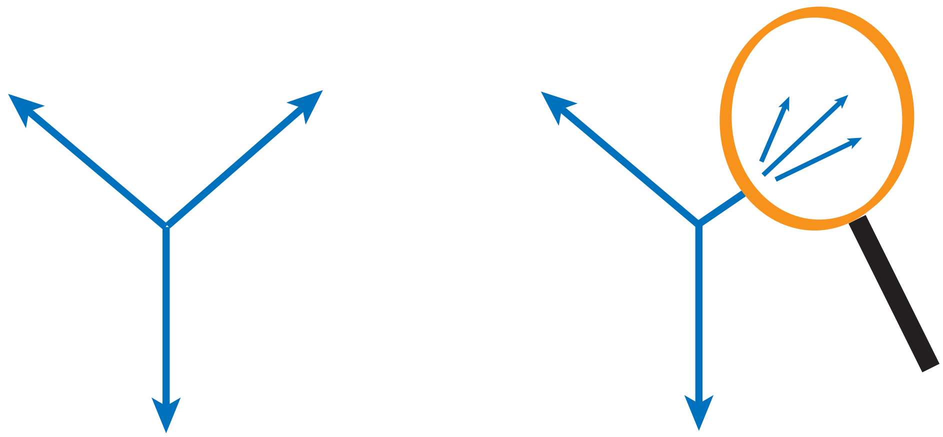
→ Gluon jets have higher $\langle B_{\text{jet}} \rangle$

→ Good agreement with QCD predictions

→ Clear evidence for higher colour charge of gluon at high B_{jet}

Select jets from three-jet events

Cluster particles in jets using finer resolution to form ‘subjets’



$y_{\text{cut}} \equiv y_1 = 0.1$ $\qquad$ $y_{\text{cut}} \equiv y_0 < y_1$

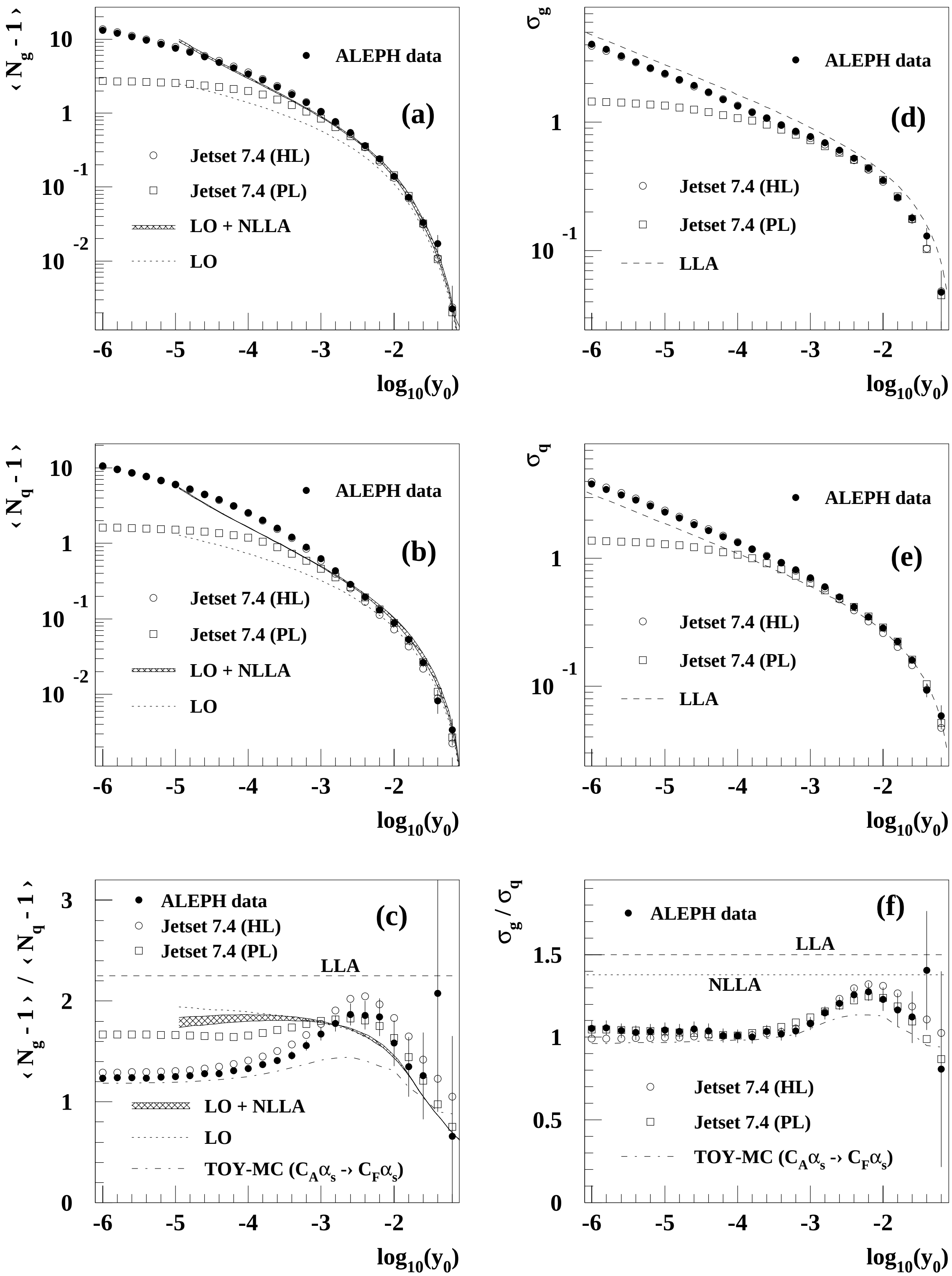
In limit $y_0 \to 0$, all hadrons resolved. Expect perturbative QCD properties to be evident for $k_{\text{t}} \sim \sqrt{y_0} E_{\text{cm}} >$ several GeV.

Leading log QCD prediction for gluon/quark particle multiplicities:

$$N_g / N_q = C_{\text{A}} / C_{\text{F}} = 9/4$$

but expect significant nonperturbative effects for small y_0 .

Subjet properties (ALEPH, Eur. Phys. J., C 17 (2000) 1-18)



Summary

Several $\times 10^2$ QCD publications from LEP/SLC

$\alpha_s(M_Z)$ from LEP:

Event shapes (LEP II)	0.1202 ± 0.0048
τ decays	0.1181 ± 0.0031
Global EW fit	0.1200 ± 0.0030
Four-jet rate	0.1170 ± 0.0013

Not all uncertainties (esp. theoretical) well understood but picture nevertheless very consistent.

QCD colour factors measured at $\sim 7\%$ level;
excellent agreement with SU(3) QCD

Internal structure of quark and gluon jets in good
agreement with QCD predictions

LEP/SLC data sets will continue to be of value for some time as better QCD predictions become available.

We should now use our knowledge of QCD as a tool to help discover new physics, e.g., at the LHC.