# Studying QCD in $e^+e^-$ collisions

## Part I: jets, event-shapes, $\alpha_s$, etc.

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# Outline

*(i)* QCD in $e^+e^- \to$ hadrons

- general theoretical and experimental picture
- Monte carlo models
- defining observables

*(ii)* QCD with jets and event-shapes

- parton spin
- $\alpha_s$ from jets and event-shapes
- running of $\alpha_s$
- flavour independence of $\alpha_s$, measuring $m_b$

*(iii)* $\alpha_s$ from total cross sections and BRs: $R_l$, $R_\tau$

*(iv)* Conclusions (and what we missed)

# Why study QCD? (experimentalist's view)

(*i*) To see if QCD is Nature's theory of strong interactions?

Sure, but most already convinced.

(*ii*) To measure its free parameters?

Yes: $\alpha_s$, quark masses,
needed to understand bigger picture.

(*iii*) To help search for physics beyond the SM?

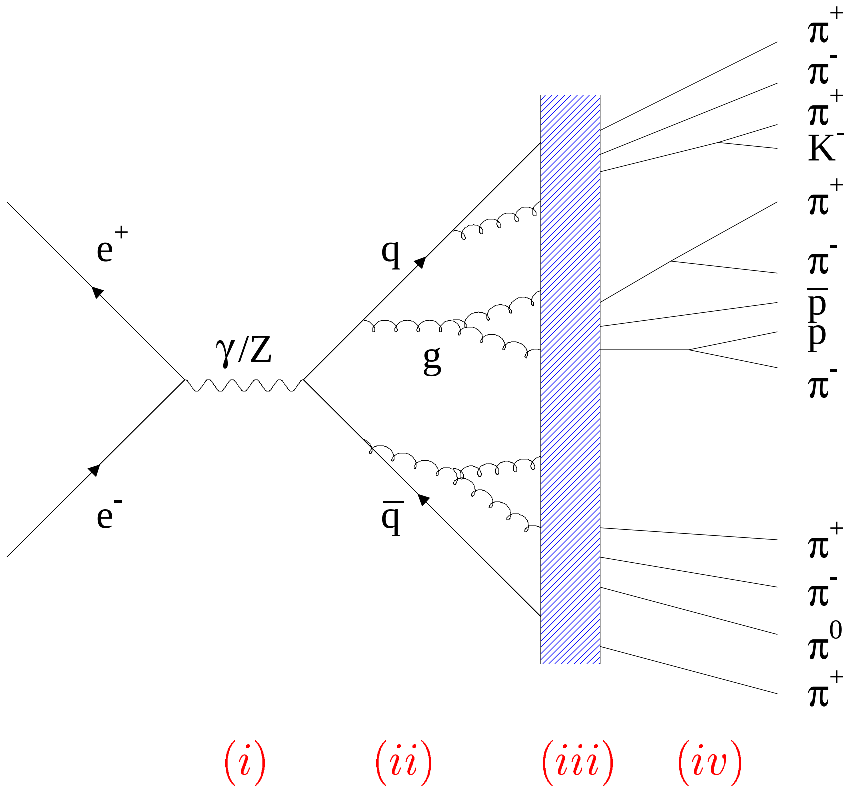
Yes: understand 'QCD background',
deviation from firm QCD prediction $\rightarrow$ new physics

(*iv*) Test validity of QCD calculations?

Yes, but not usually driving concern.

(*v*) Help understand non-perturbative QCD?

Ball (mostly) in theorists' court.




(*i*) electroweak

(*ii*) perturbative QCD

(*iii*) hadronization (non-perturbative QCD)

(*iv*) resonance decays

Usually define observable to be sensitive to only one of the above.

SPEAR (1972) $E_{cm} = 8$ GeV

PETRA (1978) $14 \text{ GeV} < E_{cm} < 44$ GeV

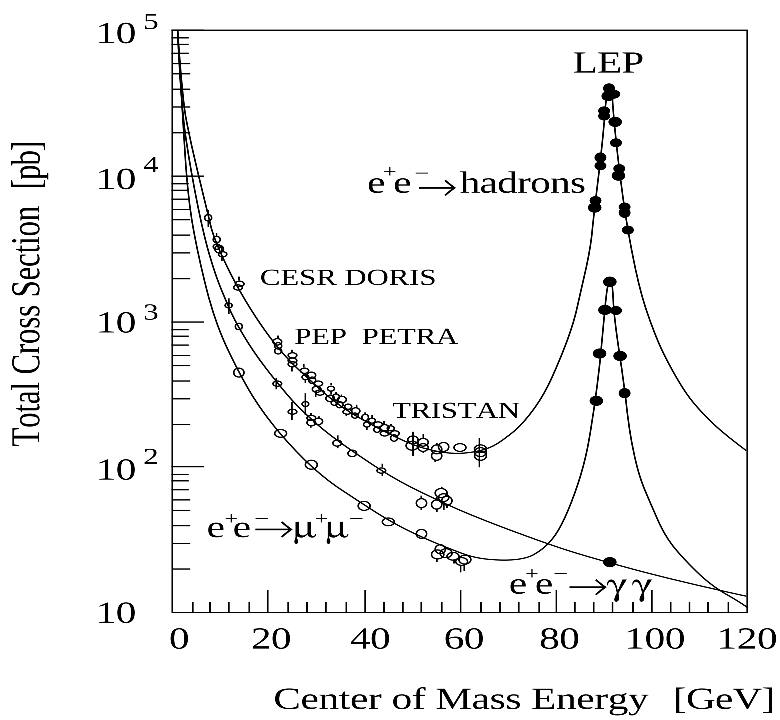
PEP (1980) $E_{cm} = 29$ GeV

TRISTAN (1987) $E_{cm} = 64$ GeV

SLC (1989) $E_{cm} = 91$ GeV

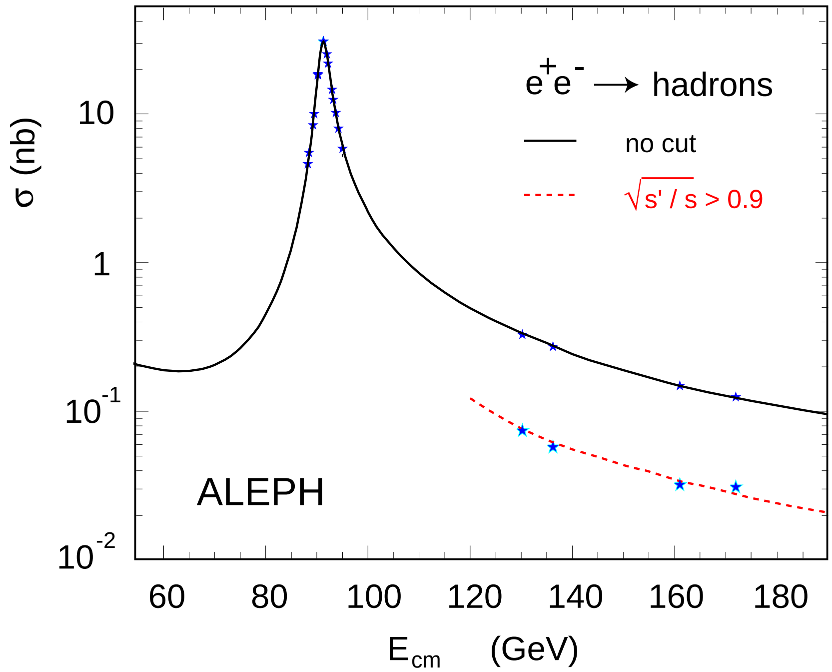
LEP I (1989) $E_{cm} = 91$ GeV

*ca.* $4 \times 10^6$ hadronic events each for ALEPH, DELPHI, L3, OPAL at $E_{cm} \approx M_Z$.



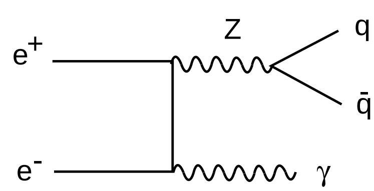
# More $e^+e^-$ annihilation data

LEP II (1996) $130\ \text{GeV} < E_{\text{cm}} < 208\ \text{GeV}$




LEP II hadronic cross section $\sim 10^3$ smaller than at Z peak

Many events with initial state photon radiation: $m_{q\bar{q}} \approx M_Z$




'Background' from $e^+e^- \to W^+W^- \to$ hadrons

Important for tests of QCD $E_{\text{cm}}$ dependence

# Monte Carlo models: perturbative phase

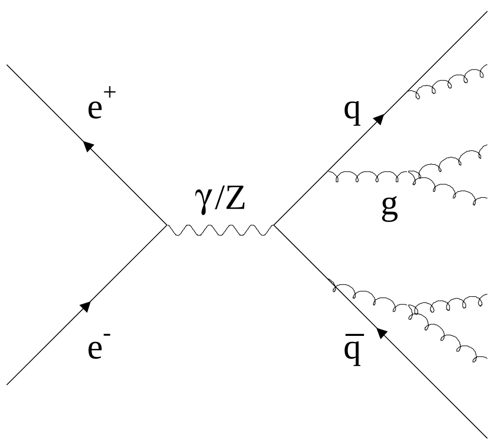
Use random numbers to select a partonic final state and generate all momentum vectors

Usually based on $\mathcal{O}(\alpha_s)$ QCD combined with 'parton shower': leading-log approx. (valid in limit of collinear gluon radiation)

$+$ angular ordering

$+$ (sometimes) next-to-leading logs

$+ \ldots$




$\rightarrow$ generates set of partons for each event

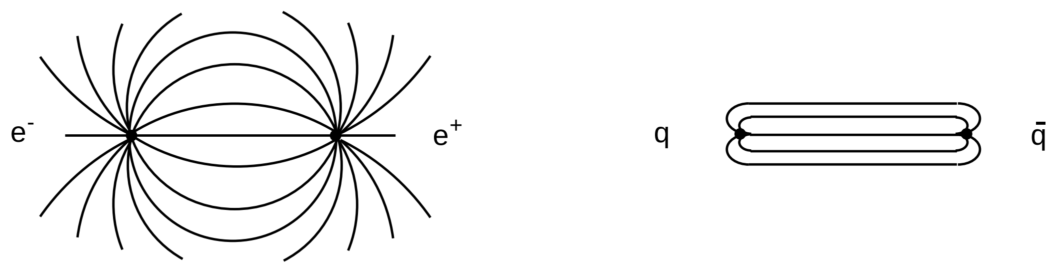
This *is* perturbative QCD but not at its most accurate.
($\alpha_s$ in MC $\neq$ $\alpha_s$ in $\overline{\text{MS}}$ scheme)

MC with $\mathcal{O}(\alpha_s^2)$ matrix element also available, but without parton shower (max of 4 partons in event)

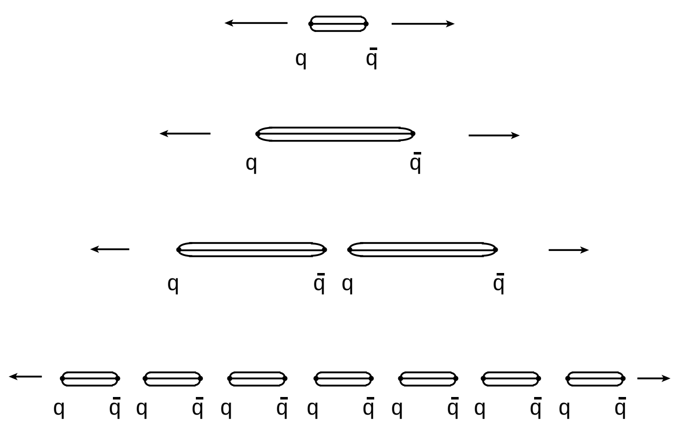
# Monte Carlo models: string hadronization

QCD inspired models (e.g. string) convert partons into hadrons

In contrast to electric charges, 'chromoelectric' field between $q\bar{q}$ pair confined to narrow flux tube (string):




$q\bar{q}$ production in flux tube → string breaks → mesons




MC generates flavours of $q\bar{q}$ pairs ($u : d : s \approx 1 : 1 : 0.3$)

space–time location of breaks → momenta of hadrons

Gluons → momentum carrying kinks in string

→ 'Lund family' of models: JETSET, ARIADNE, PYTHIA ...

# Monte Carlo models: cluster hadronization

Cluster model (program HERWIG):

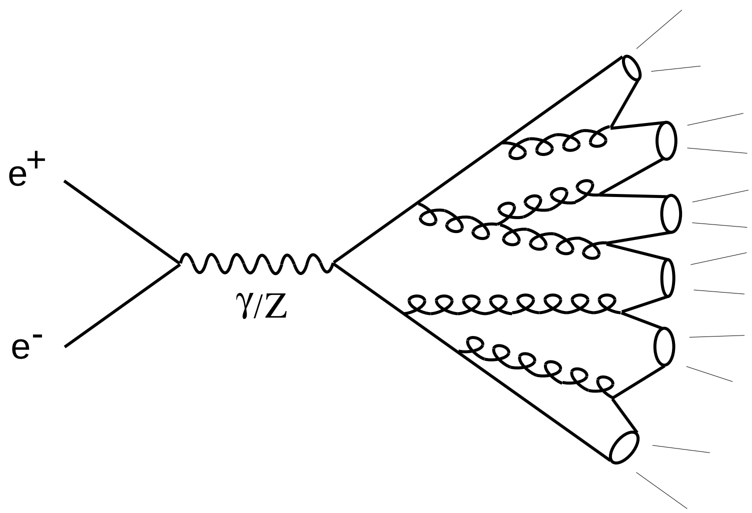
parton shower ends with virtual mass of all partons $= Q_0$;

gluons split into $q\overline{q}$ pairs;

neighbouring $q$ and $\overline{q}$ form colour neutral clusters;

clusters (usually) decay isotropically into two hadrons;

exceptions allowed for very light and very heavy clusters




Parameters:

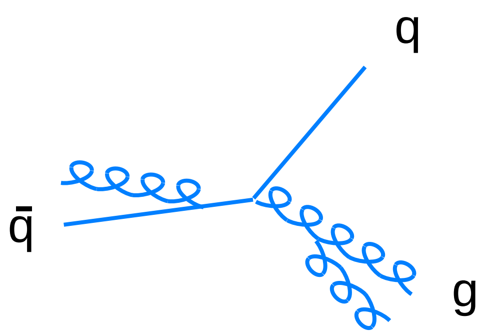
$\Lambda_{\mathrm{QCD}}$, $Q_0$, quark masses,

parameters for treatment of very light/heavy clusters

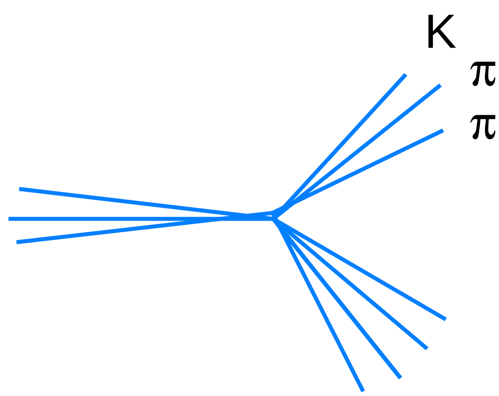
and other tweaks mainly related to flavour production.

# Comparing theory and experiment

Need to compare QCD prediction . . .




with measurement . . .



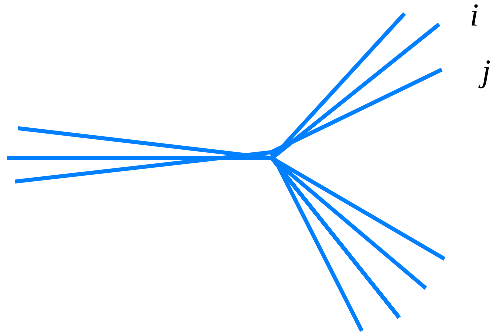

using appropriately defined jet rates, event-shape variables

infrared, collinear safe;

not overly sensitive to hadronization effects

# Defining jets

Clustering algorithms: for every pair, compute 'distance' $y_{ij}$




← partons or hadrons

$$\text{e.g.}\quad y_{ij} = \begin{cases} \dfrac{2E_iE_j(1-\cos\theta_{ij})}{s} & \text{(JADE)} \\[2ex] \dfrac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{s} & \text{(Durham)} \end{cases}$$

(*i*) Find pair with smallest $y_{ij}$

(*ii*) if less than a given $y_{\rm cut}$, replace $i$, $j$ with pseudoparticle:

$p^\mu = p_i^\mu + p_j^\mu$ ('E' scheme)

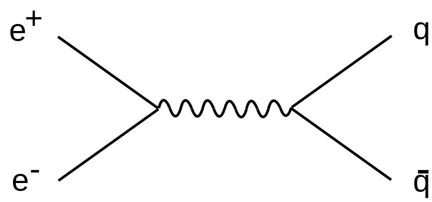
(*iii*) iterate until all $y_{ij} > y_{\rm cut}$

remaining pseudoparticles → jets

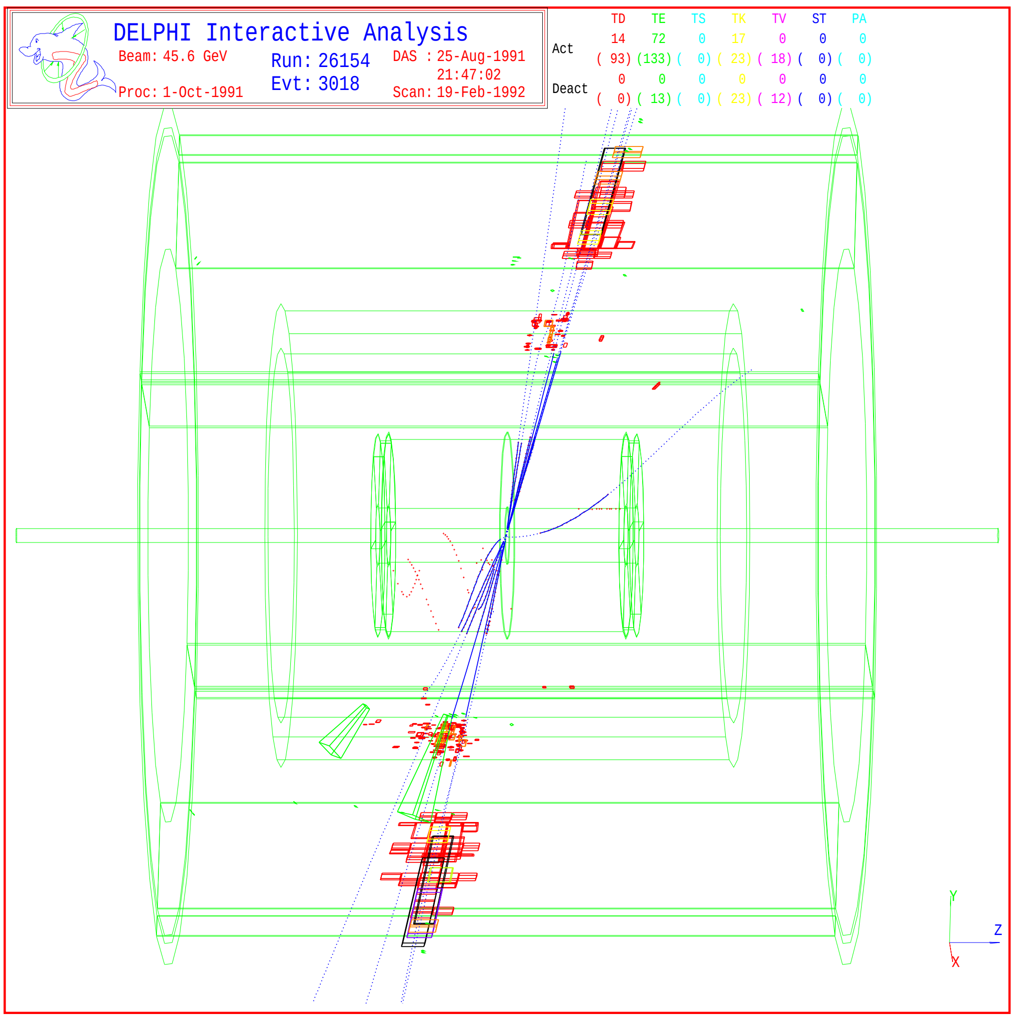
Other jet definitions also used, e.g., cone algorithm.

# Two-jet events

$e^+e^- \rightarrow q\bar{q}$ leads to two back-to-back jets of hadrons

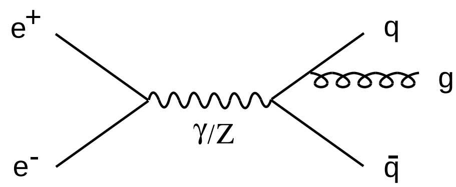



$\rightarrow$ angular distribution of jets depends on quark spin

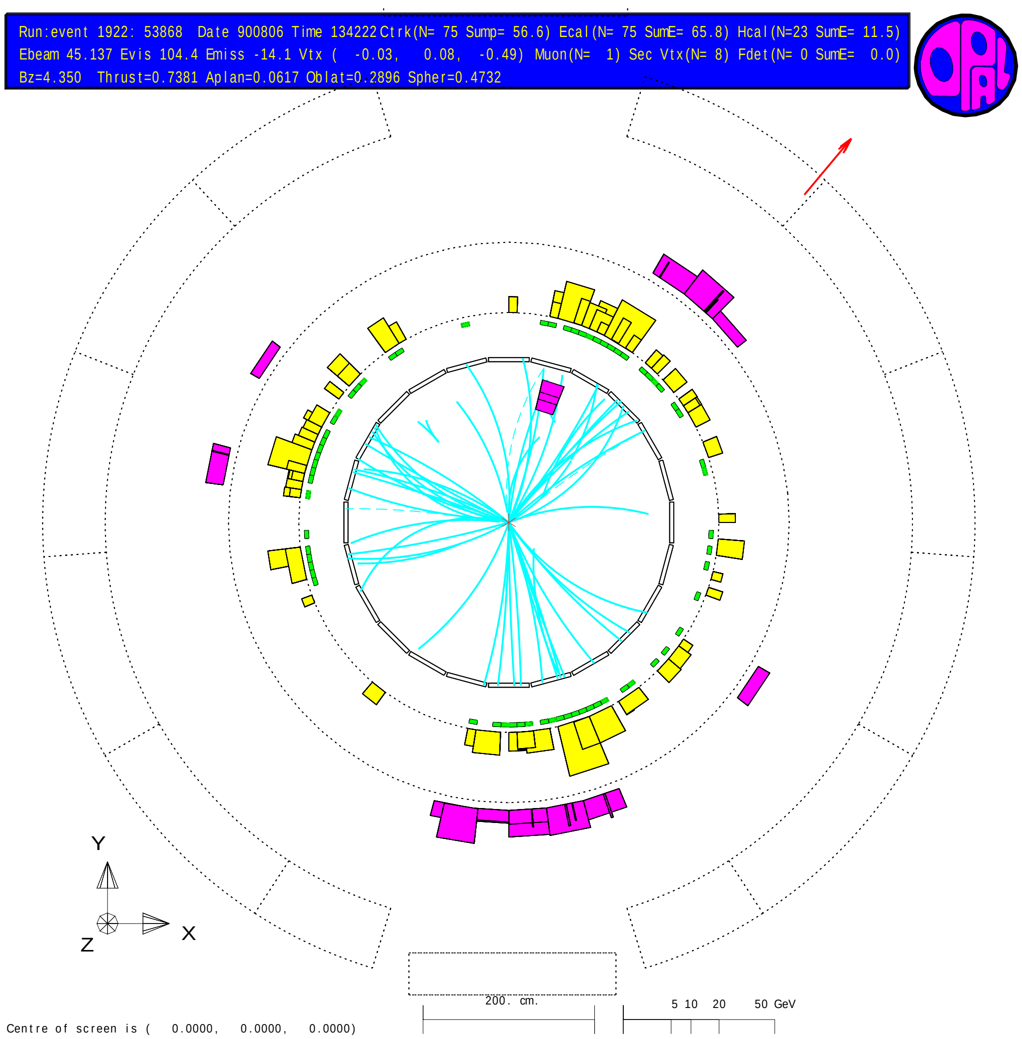


# Three-jet events

Bremsstrahlung-like gluon radiation (cf. $e^+e^- \to \mu^+\mu^-\gamma$)



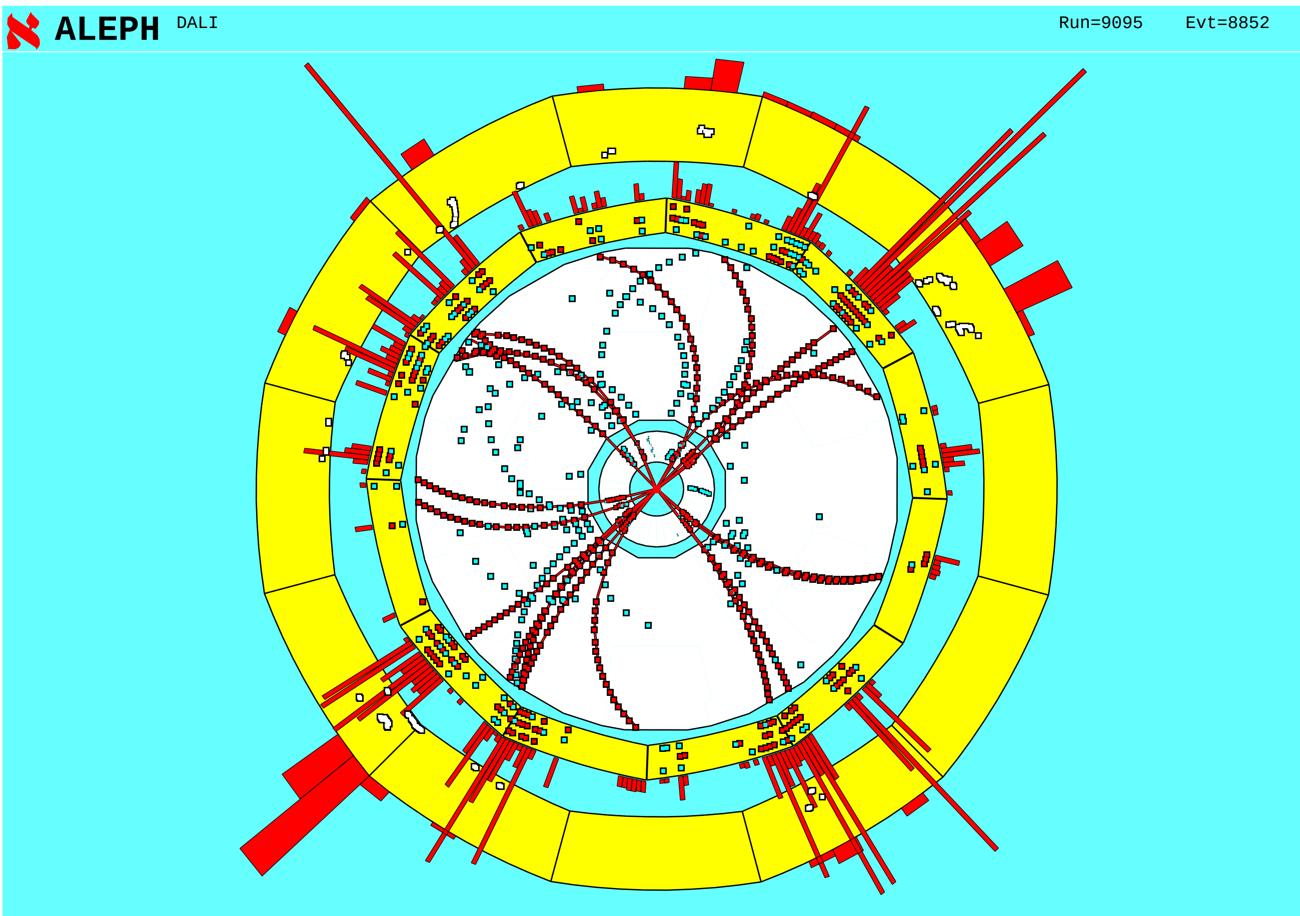

Additional jets $\to$ rate sensitive to strong coupling $\alpha_s$



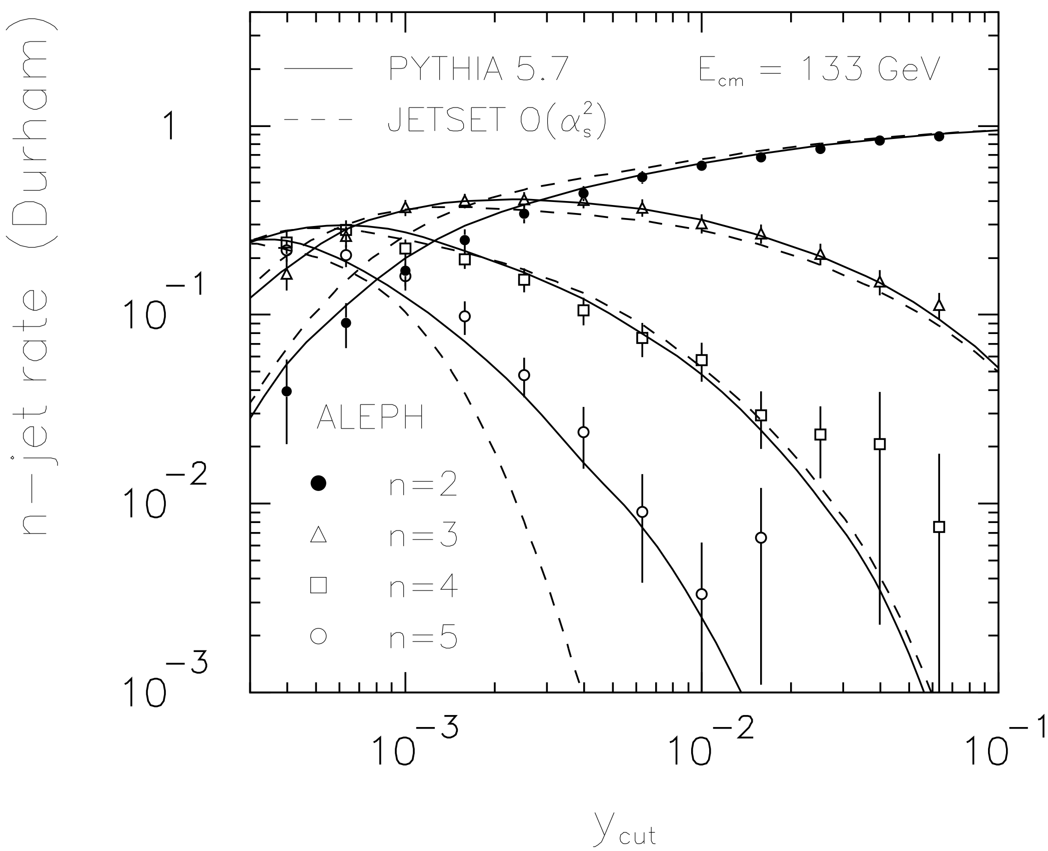
# Multijet events

More gluon radiation leads to multijet events


→ sensitive to triple-gluon vertex, non-abelian character of QCD



Relative rate of finding $n$ jets for $n = 2, 3, 4, 5$

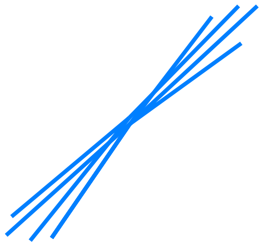



Parton shower based model (PYTHIA) gives good description of multijet rates

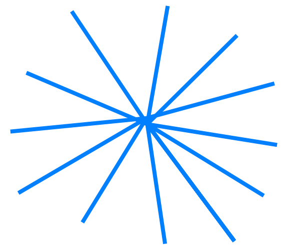
$\mathcal{O}(\alpha_s^2)$ based model has at most 4 partons in final state, falls short for rates of $n \geq 5$ jets

## Event-shape variables

Thrust:
$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



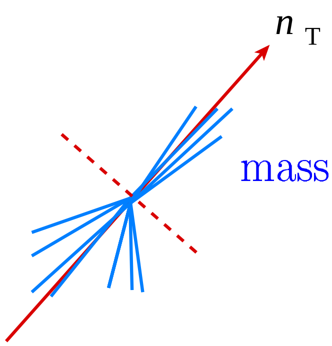
$T \approx 1$



$T \approx 1/2$

Heavy jet mass: divide event into hemispheres with thrust axis

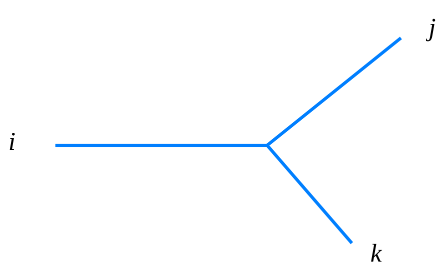
mass $M_h$




mass $M_l$

Often use $\rho = M_h^2/s$

$y_3$: cluster event to three jets

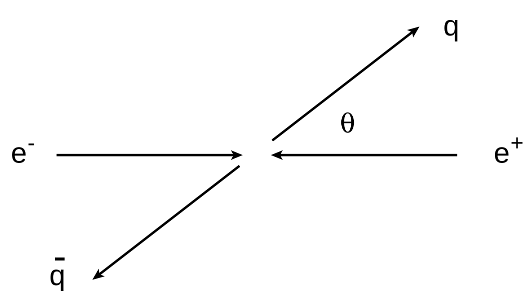


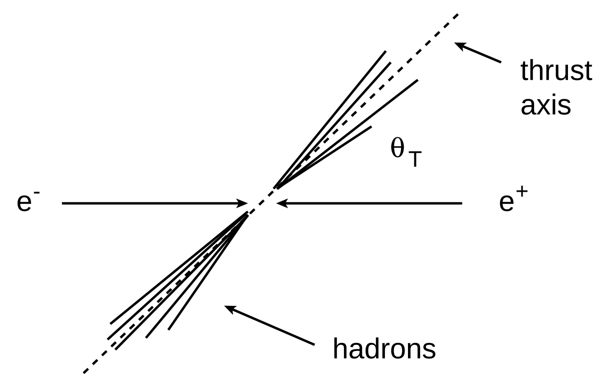

$y_3 = \min(y_{ij})$

# Spin of quarks

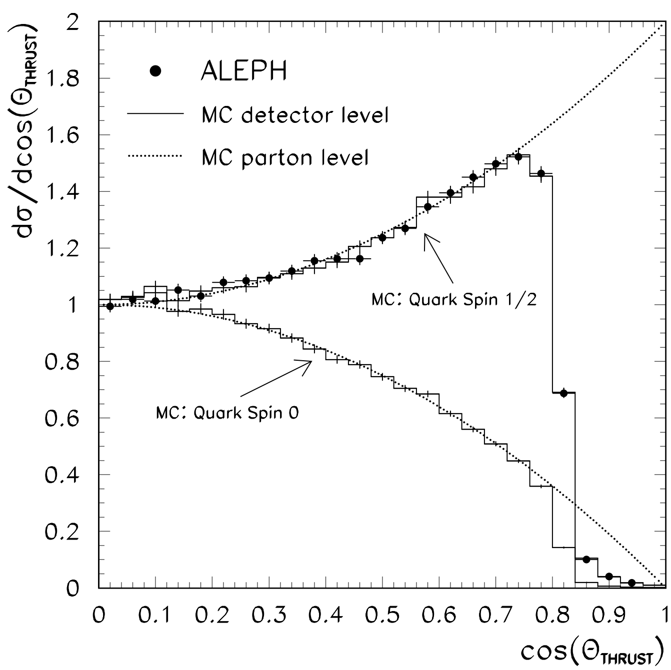
distribution of outgoing quark's angle relative to incoming $e^-$

$$\frac{d\sigma}{d\cos\theta} \sim \begin{cases} 1+\cos^2\theta & \text{spin-}\frac{1}{2}\text{ quarks} \\ 1-\cos^2\theta & \text{spin-0 quarks} \end{cases}$$




estimate $\theta$ with angle of thrust axis (doesn't distinguish $q$ direction)




$\rightarrow$ quarks have spin $\frac{1}{2}$

## QCD predictions to $\mathcal{O}(\alpha_s^2)$

Consider variable $y = y_3,\ \rho,\ 1-T,\ \ldots$

$$\frac{1}{\sigma_0}\frac{d\sigma}{dy} = A(y)\frac{\alpha_s(\mu)}{2\pi} + \left[B(y) + 2\pi b_0 A(y)\ln\left(\frac{\mu^2}{s}\right)\right]\left(\frac{\alpha_s(\mu)}{2\pi}\right)^2$$

$A(y),\ B(y)$ = computable functions

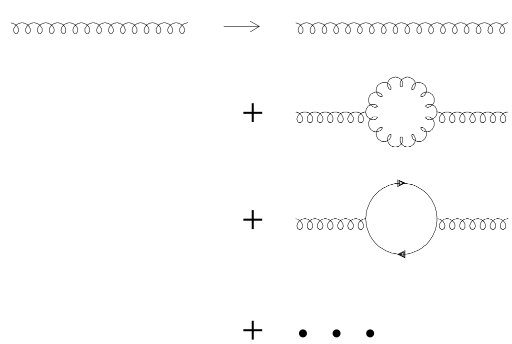
$\mu$ = renormalization scale

$d\sigma/dy$ should be independent of $\mu$ (to $\mathcal{O}(\alpha_s^2)$)

→ this determines the $\mu$ dependence of $\alpha_s$ (RGE)

$$\frac{1}{\alpha_s(\mu)} = \frac{1}{\alpha_s(Q)} + b_0\ln\left(\frac{\mu^2}{Q^2}\right), \qquad b_0 = \frac{33-2n_f}{12\pi}$$

Equivalent to



## This $\mu$ business

- $\mu$ reflects an ambiguity of perturbation theory

  not a QCD parameter

- Suppose we measure $\alpha_\mathrm{s}(\mu)$ with some $\mu$,

  Use RGE: $\alpha_\mathrm{s}(\mu) \to \alpha_\mathrm{s}(M_\mathrm{Z})$

  resulting $\alpha_\mathrm{s}(M_\mathrm{Z})$ still depends on chosen $\mu$

  since $\mu$ dependence only cancels to $\mathcal{O}(\alpha_\mathrm{s}^2)$

- Higher order coefficients will contain $\sim \left[\ln\left(\frac{\mu^2}{s}\right)\right]^n$

  $\to$ $\mu^2 \approx s$ gives some hope that series is converging.

- But ... at $\mathcal{O}(\alpha_\mathrm{s}^2)$ , data best described with $\mu^2 \approx 0.002s$ (!?!)

  $\to$ need higher order terms

## Resumming large logs

Consider cumulative distribution $R(y) = \int_0^y \frac{1}{\sigma}\frac{d\sigma}{dy'}dy'$

$$
\begin{aligned}
\ln R(y) \;=\; & \alpha_s(G_{12}\ln^2 y \;+\; G_{11}\ln y \;+\; \ldots) \\
&+ \alpha_s^2(G_{23}\ln^3 y \;+\; G_{22}\ln^2 y \;+\; \ldots) \\
&+ \alpha_s^3(G_{34}\ln^4 y \;+\; G_{33}\ln^3 y \;+\; \ldots) \\
&+ \alpha_s^4(G_{45}\ln^5 y \;+\; G_{44}\ln^4 y \;+\; \ldots) \\
&+ \ldots
\end{aligned}
$$

↑ leading logs ↑ next-to-leading logs

Large logs dominate for $y \to 0$ (two-jet region)

LL and NLL summed to all orders for several variables (including $1-T$, $y_3$, $M_h^2/s$)

Matching $\mathcal{O}(\alpha_s^2)$ and (N)LL parts:

subtract double-counted part using $R$ or $\ln R$? (difference $\mathcal{O}(\alpha_s^3)$)

Data no longer prefer small $\mu$

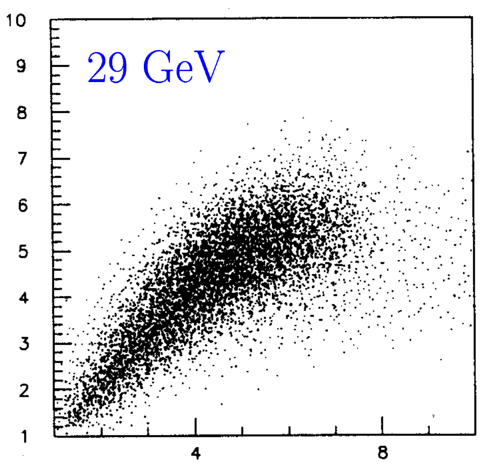
estimate (roughly) magnitude of missing higher orders by varying $\mu$ , e.g., $-1 < \ln(\mu^2/s) < 1$

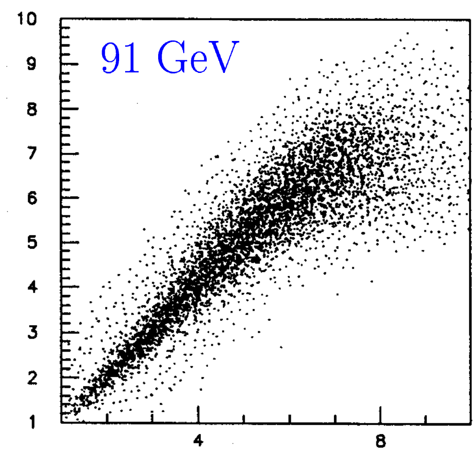
## Hadronization corrections

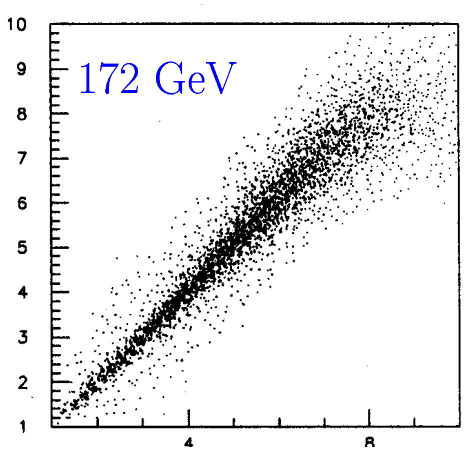
$$\left(\frac{d\sigma}{dy}\right)_{\text{had}} (\text{bin } i; \alpha_s) = \sum_j \left(\frac{d\sigma}{dy}\right)_{\text{QCD}} (\text{bin } j; \alpha_s) \cdot P_{ij}$$

$$P_{ij} = P\left(\begin{array}{c|c} \text{hadron level} & \text{parton level} \\ \text{in bin } i & \text{in bin } j \end{array}\right)$$ ← from MC model

e.g. for $-\ln y_3$ hadron vs. parton level (JETSET):



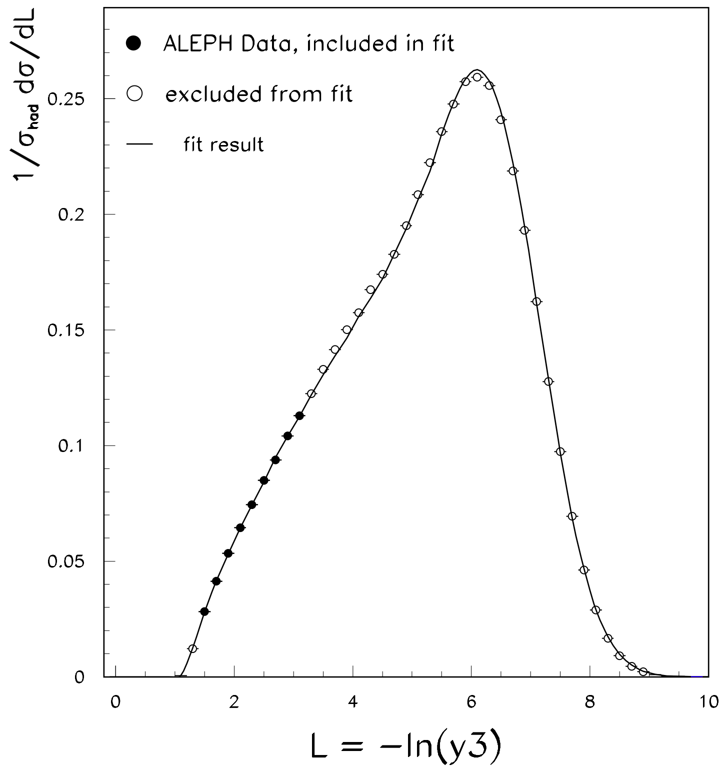




Partons follow hadrons better with increasing $E_{\text{cm}}$

Example with $y_3$:

restrict fit range
to three-jet region;
data also well described
outside fit range




$$\alpha_s(M_Z) = 0.1195 \pm 0.0002 \text{ (stat.)} \pm 0.0038 \text{ (sys.)}$$

Systematic error usually dominated by theory (as here)

hadronization corrections: try different models.

missing higher orders: try varying $\mu$ in ‘reasonable range’,

vary matching scheme to combine NLLA and $\mathcal{O}(\alpha_s^2)$ parts.

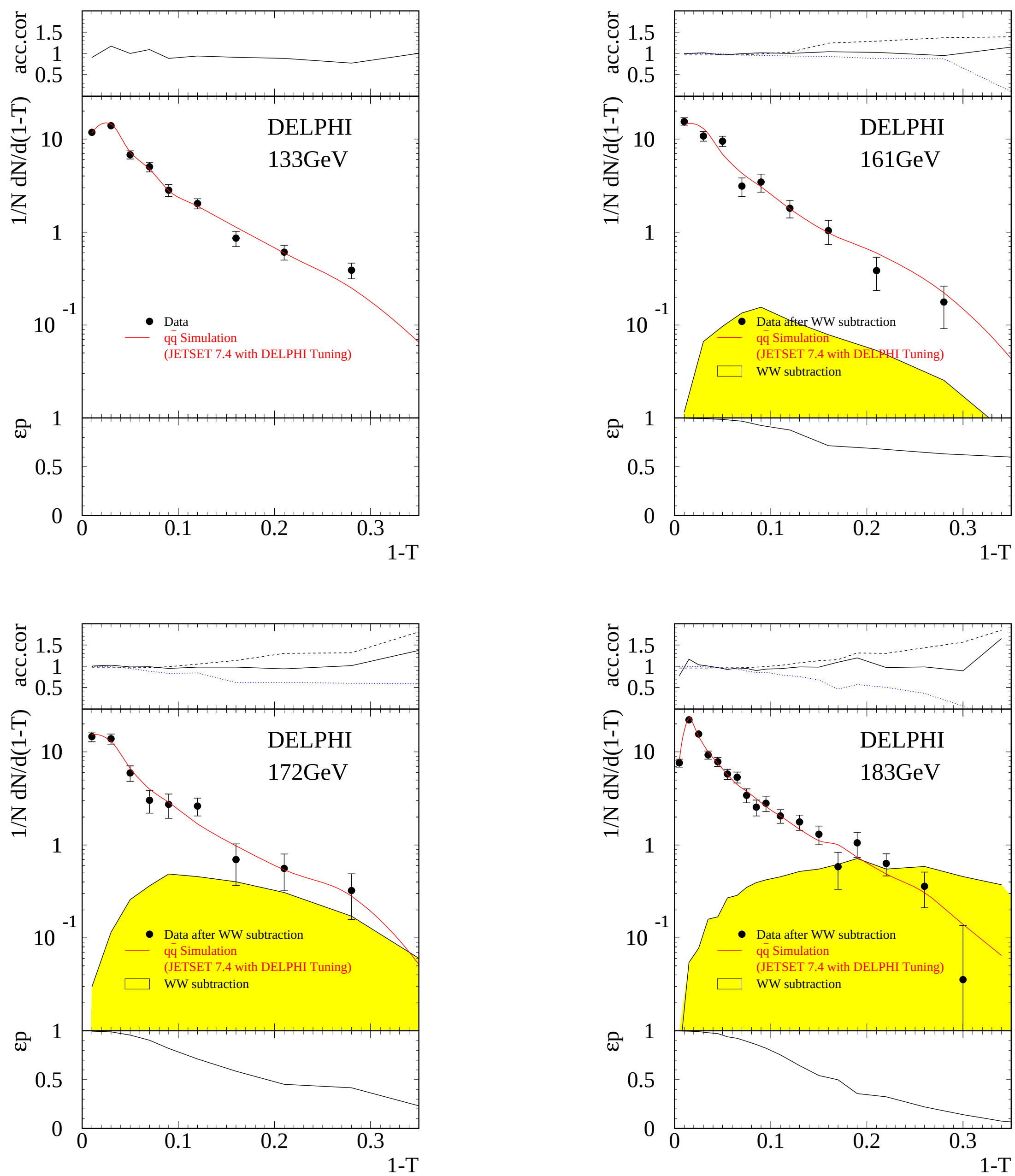
# Event-shape distributions at different $E_{\rm cm}$

Many systematic uncertainties in $\alpha_s$ common to all $E_{\rm cm}$

$\rightarrow$ not a problem for studying running of $\alpha_s$,

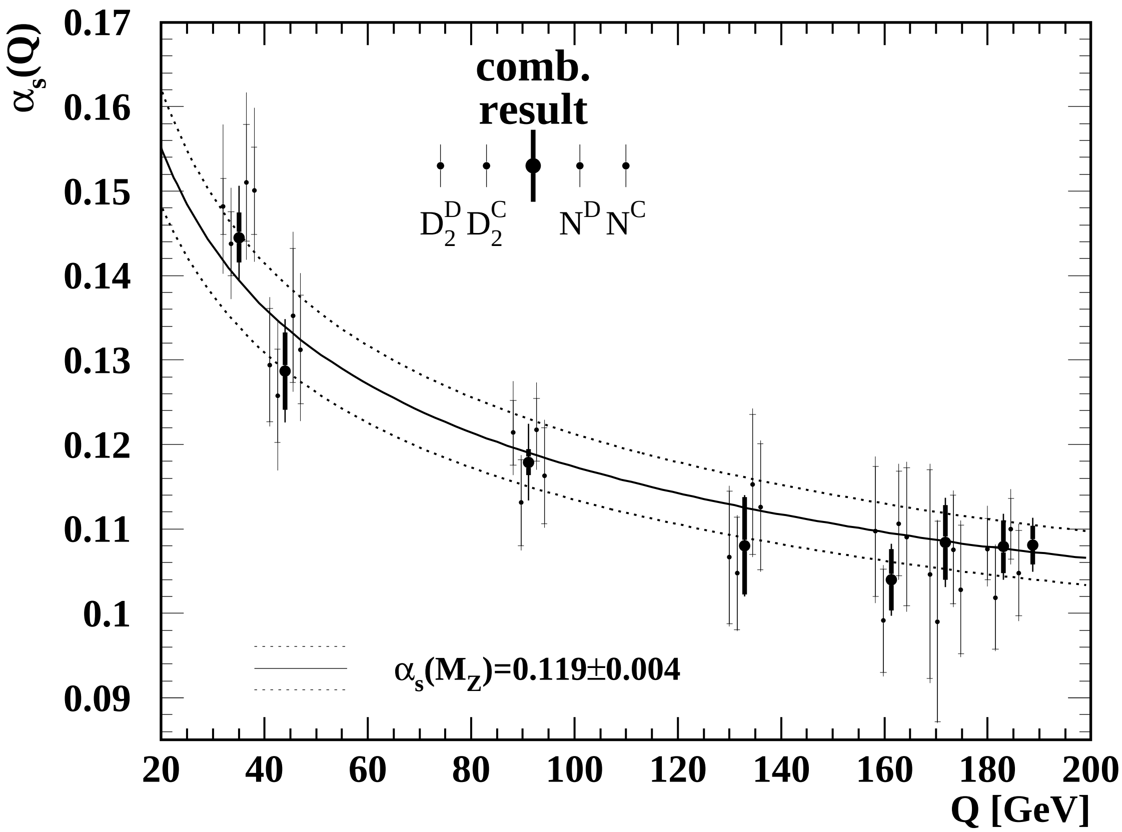
but ... LEP II has hadronic events from $e^+e^- \rightarrow W^+W^-$

initial state photon radiation, low statistics ...



# Event-shape distributions at different $E_{cm}$

Common $\alpha_s$ measurements by JADE and OPAL experiments




Inner error bars – uncorrelated errors (e.g. stat.)

Outer error bars – total uncertainty

→ good agreement with predicted running.

# Flavour independence of $\alpha_s$

Does $q\bar{q}g$ coupling depend on quark flavour?

QCD: no. (Still, good to test.)

Select event samples enriched in $b\bar{b}$, $c\bar{c}$ and $uds$

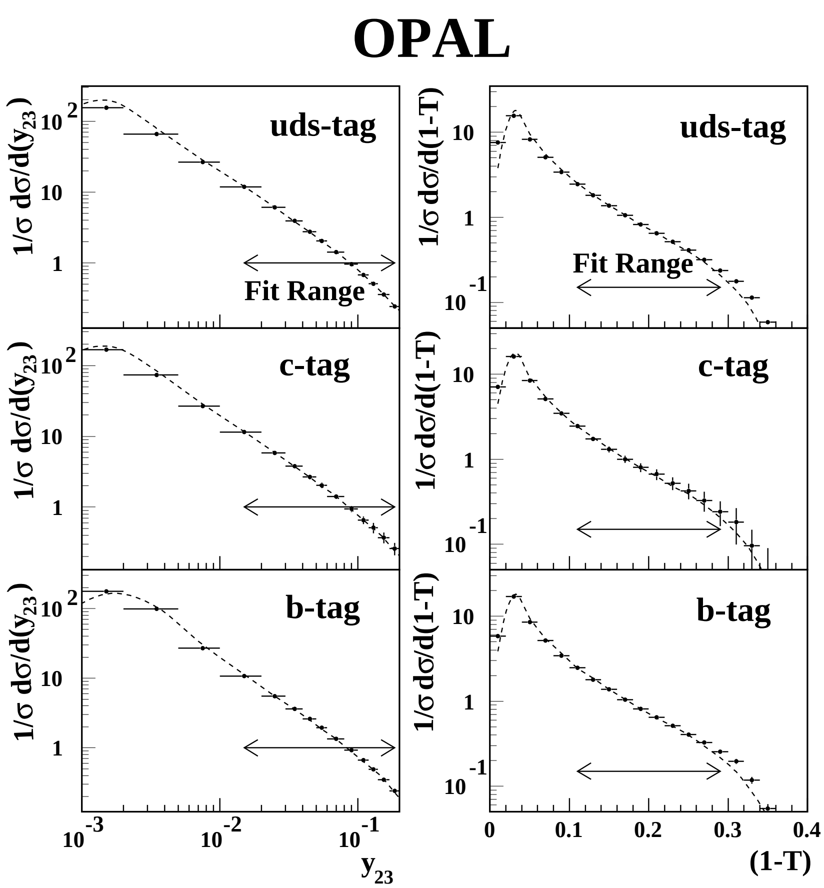
lifetime tag $\rightarrow b$

lifetime antitag $\rightarrow uds$

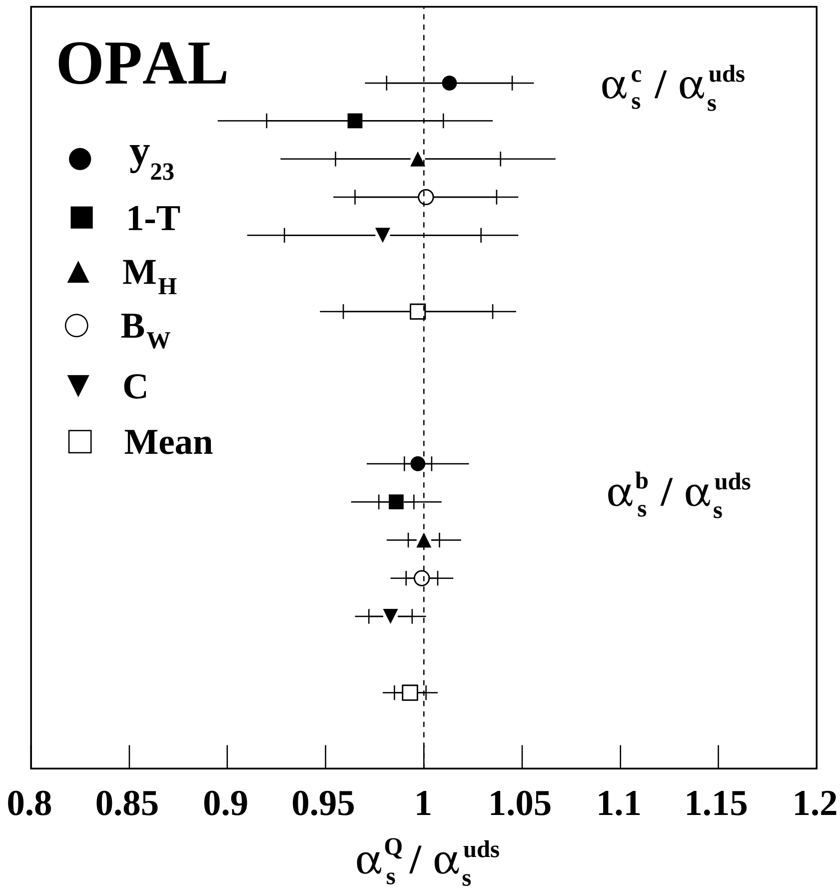
fast D mesons in jets $\rightarrow c$

Fit $\alpha_s^{uds}$, $\alpha_s^c/\alpha_s^{uds}$, $\alpha_s^b/\alpha_s^{uds}$ as separate parameters,

use $\mathcal{O}(\alpha_s^2)$ including with $b$, $c$ mass effects.



# Flavour independence of $\alpha_{\rm s}$: results




Averages:

$$\alpha_{\rm s}^c/\alpha_{\rm s}^{uds} = 0.997 \pm 0.038 \text{ (stat.)} \quad \pm 0.030 \text{ (sys.)} \quad \pm 0.012 \text{ (theo)}$$

$$\alpha_{\rm s}^b/\alpha_{\rm s}^{uds} = 0.993 \pm 0.008 \text{ (stat.)} \quad \pm 0.006 \text{ (sys.)} \quad \pm 0.011 \text{ (theo)}$$

$\rightarrow$ $\alpha_{\rm s}$ flavour independence tested to $\sim$ percent level.

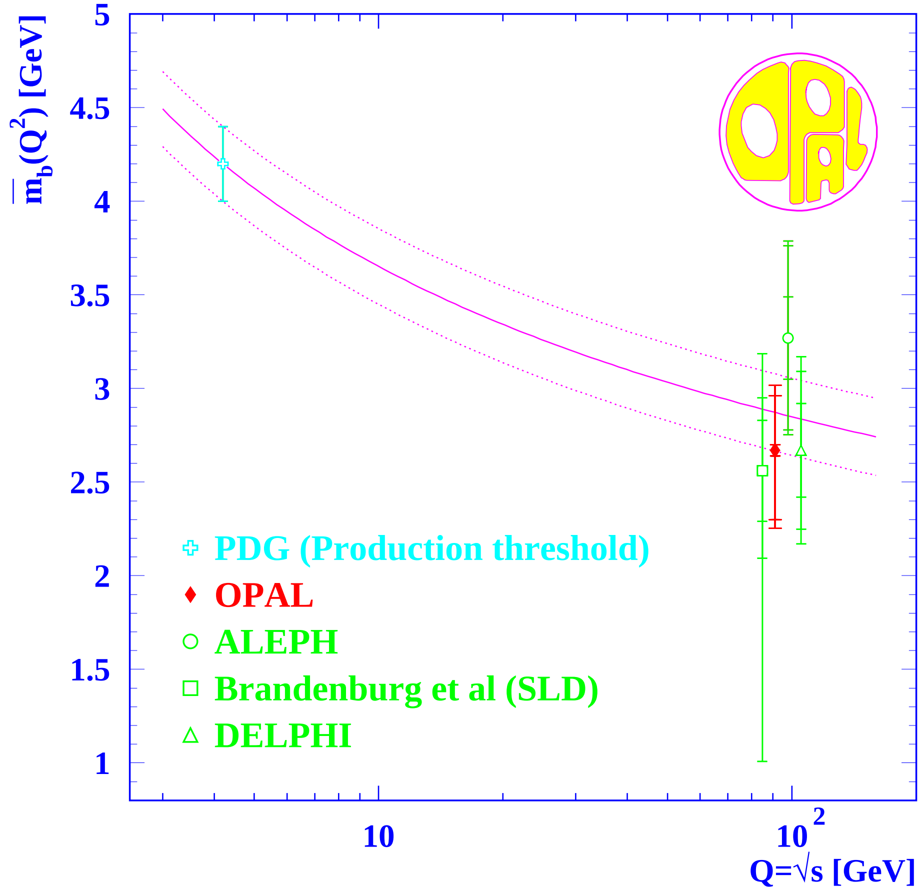
## $m_b$

High $b$-mass suppresses gluon radiation,

same reason less Bremsstrahlung for muons

$\rightarrow$ event-shapes, jet-rates differ for $b\bar{b}$, $uds$ events

Here *assume* $\alpha_s$ flavour independent, fit $m_b$



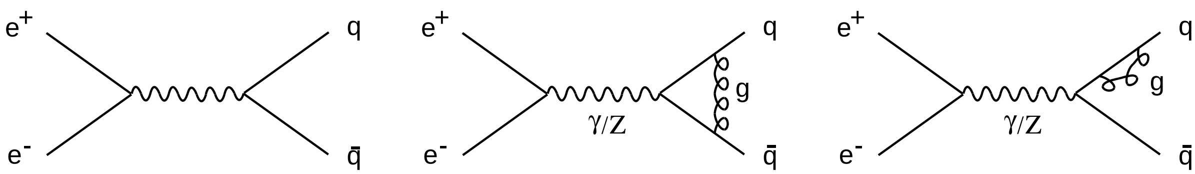

Running of mass observed, e.g., OPAL finds

$$m_b(M_Z) = 2.67 \pm 0.03 \text{ (stat) } \pm^{0.29}_{0.37} \text{ (sys) } \pm .19 \text{ (theo)}$$

## $\alpha_s$ from the total hadronic cross section

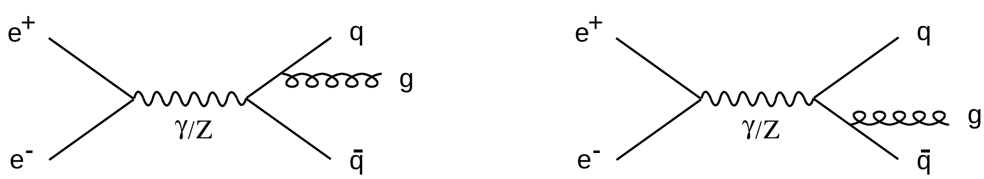
$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) + \ldots$$

### $e^+e^- \to q\bar{q}$




$\rightarrow$ ultraviolet divergences

### $e^+e^- \to q\bar{q}g$




$\rightarrow$ infrared, collinear divergences

The divergences (almost) cancel, leaving a finite correction

$$\sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) = \sigma_0 \left(1 + \frac{\alpha_s}{\pi} + \ldots\right)$$

works at every order in perturbation theory

## $\alpha_s$ from $R_l$ and $R_\tau$

$$R_l = \frac{\Gamma(\mathrm{Z} \to \text{ hadrons})}{\Gamma(\mathrm{Z} \to l\bar{l})} \to \text{(almost) same as } \sigma_{\text{had}}:$$

$$= 19.934 \left[1 + 1.045\frac{\alpha_s}{\pi} + 0.44\left(\frac{\alpha_s}{\pi}\right)^2 - 15\left(\frac{\alpha_s}{\pi}\right)^3\right]$$

LEP measures $R_l = 20.768 \pm 0.024$

$$\to \alpha_s(M_\mathrm{Z}) = 0.124 \pm 0.004 \text{ (stat) } \pm^{0.003}_{0.002} \text{ (theo)}$$

Uncertainties from missing higher orders, Higgs mass

Similarly, decay of virtual W from $\tau$ sensitive to QCD corrections

$$R_\tau = \frac{\mathcal{B}(\tau^- \to \nu_\tau \text{ hadrons})}{\mathcal{B}(\tau^- \to \nu_\tau \mathrm{e}^- \overline{\nu}_\mathrm{e})}$$

ALEPH, OPAL average result (Bethke hep-ex/0004021):

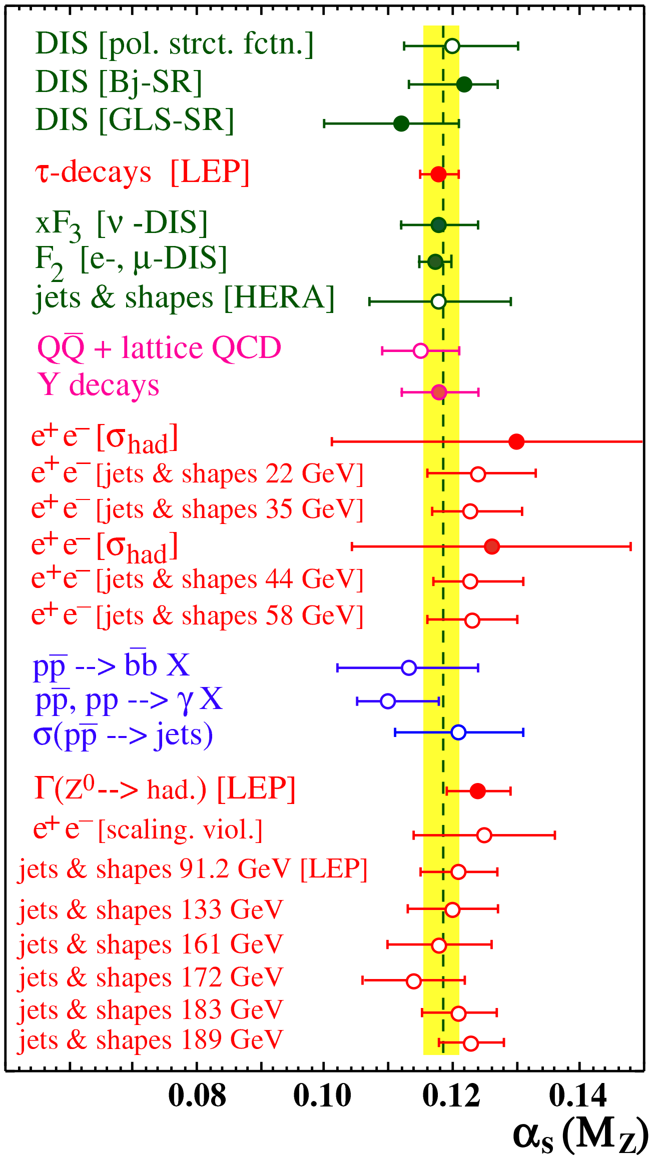
$$\alpha_s(m_\tau) = 0.323 \pm 0.005 \text{ (exp) } \pm 0.030 \text{ (theo)}$$

Evolve to $M_\mathrm{Z}$,

$$\alpha_s(M_\mathrm{Z}) = 0.118 \pm 0.001 \text{ (exp) } \pm 0.003 \text{ (theo)}$$

# Summary of $\alpha_s(M_Z)$

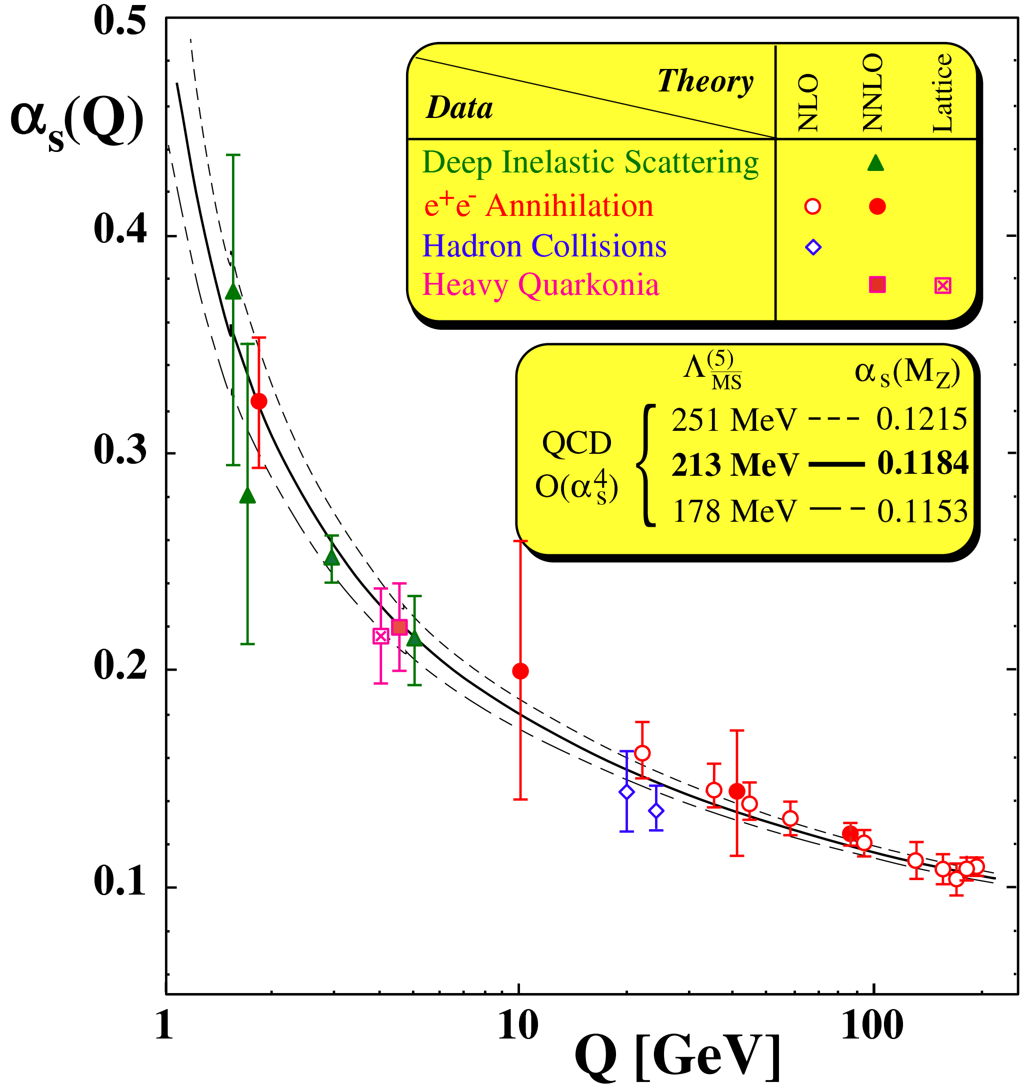
S. Bethke, hep-ex/0004021




Bethke's average (hep-ex/0004021):

$$\alpha_s(M_Z) = 0.1184 \pm 0.0031$$

S. Bethke, hep-ex/0004021



## Last word?

Many QCD predictions are well verified in $e^+e^-$ collilsions,

consistent $\alpha_s(M_Z)$ values measured each to several percent from:

event-shapes and jet rates, total cross section, $\tau$ decays

What we missed

$\gamma\gamma$ physics ($e^+e^- \to e^+e^- +$ hadrons)

QCD with $e^+e^- \to W^+W^- \to$ hadrons

scaling violations of fragmentation functions

four-jet properties ($\to$ Bill Gary)

⋮

Next step:

use our knowledge of QCD to discover new physics!

Many thanks for help in preparing this talk to:

Günther Dissertori

Michael Schmelling