PH2900 Complex Problem #2

If you are unable to obtain observatory time for Practical #2, you should follow the same data analysis procedure but using the data provided on the course website. There is at least one practice set of data from 24 October 2006. We will try to add more data in the course of the coming weeks.

If you did not take the data yourself, you should do the 'extended' error analysis described below. If you are using your own data you can still do this if you like, but it is not required for full marks. To do the error analysis you can use whatever computing tools you like, e.g., pocket calculator or Excel.

You should find the angular distances for all of the images available for a given target and use the variation in the values that you find in order to estimate the measurement uncertainty. Here is a quick reminder of how to do this. Suppose we have n measurements, $x_1, \ldots, x_n$ of some quantity whose true (and unknown) value is μ. In order to *estimate* μ we can use the arithmetic average:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i \, . \tag{1}$$

Here the hat refers to the fact that $\hat{\mu}$ is an estimator for μ, and will differ in general from the true value because of random measurement errors.

We can estimate the standard deviation of the x values using the formula

$$\hat{\sigma}_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \hat{\mu})^2}{n-1}} \, , \tag{2}$$

where here again the hat refers to the fact that this value will not be equal to the true standard deviation σ_x but rather is an estimate of it. Recall that the standard deviation of x is a measure of how widely spread the values of x will be if we repeat the measurement many times.

Now if we only used a single observation of x to estimate μ, e.g., if we had $n = 1$ and we took $\hat{\mu} = x_1$, then the appropriate measurement uncertainty would be σ_x. Normally we would estimate by $\hat{\sigma}_x$ using equation (2), but to do this we need more than one observation of x.

If, however, we have n measurements and we average the values of x by using equation (1), then we achieve a much smaller measurement error for our estimate of the true value μ. We can show under rather general conditions that the appropriate standard deviation of the estimator for the mean is

$$\sigma_{\hat{\mu}} = \frac{\sigma_x}{\sqrt{n}} \, . \tag{3}$$

Therefore when you estimate the angular separation between the stars you should use equation (1) to determine their average and then use equations (2) and (3) to estimate its standard deviation. Then you report the final result as

$$\text{final result} = \hat{\mu} \pm \hat{\sigma}_{\hat{\mu}} \, . \tag{4}$$

Note that the reported measurement uncertainty finally has two hats: $\hat{\sigma}_{\hat{\mu}}$. It is the *estimate* of the standard deviation of the *estimate* of μ.

The procedure described above is strictly speaking only correct in the case where each of the individual measurements has the measurement uncertainty σ_x. You should look at all of the images and see if this is true. You may want to discard some of the images that have very large measurement errors before applying the procedure described above. Comment on why you think there is variation from image to image for a given target.

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