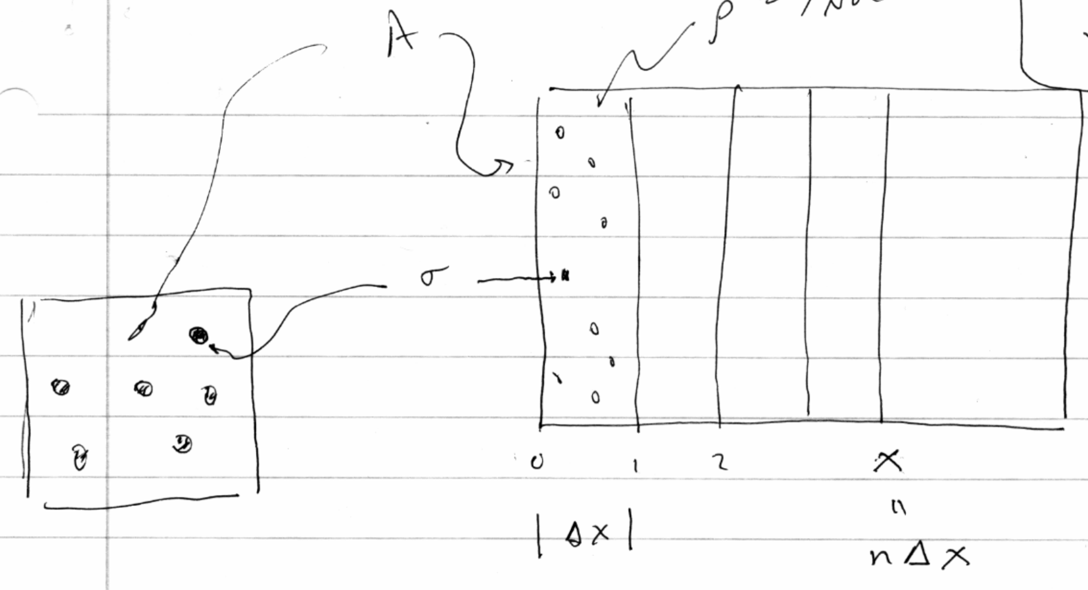
1 GDC

x-ray MC



P(no interaction before x)

$$= \prod_{i=1}^{n}\left(1 - \rho A \Delta x \cdot \frac{\sigma}{A}\right) = \prod_{i=1}^{n}\left(1 - \frac{\rho\sigma x}{n}\right)$$

$$= \left(1 - \frac{\rho\sigma x}{n}\right)^{n}$$

$$\to \exp\left(-\rho\sigma x\right)$$

$$\Rightarrow P(\text{interaction before } x) = 1 - e^{-\rho\sigma x} = \int_{-\infty}^{x} p(x')\,dx'$$

Probability density of interaction position

$$p(x) = \frac{\partial}{\partial x}\left(1 - e^{-\rho\sigma x}\right)$$

$$= \rho\sigma e^{-\rho\sigma x}$$

$$= \frac{1}{\lambda} e^{-x/\lambda} \qquad \lambda \equiv \frac{1}{\rho\sigma}$$

= mean free path.

For more than one interaction process.

$$p(x) = \frac{1}{\lambda} e^{-x/\lambda}$$

$$\frac{1}{\lambda} = \sum_i \sigma_i \rho_i = \sum_i \frac{1}{\lambda_i}$$

For photoelectric, get λ_{ph} from website.

For Compton,

$$\sigma_c = 2\pi r_0^2 \left\{ \frac{1+\gamma}{\gamma^2} \left[\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma} \ln(1+2\gamma) \right] + \frac{1}{2\gamma} \ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2} \right\}$$

$$\gamma = \frac{E_\gamma}{m_e c^2}$$

r_0 = classical e^- radius.

$$= \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2} = 2.8179 \times 10^{-15} \text{ m}$$

To generate x, use

$$p(x) = \frac{1}{\lambda} e^{-x/\lambda} \qquad \left(\Rightarrow x = -\lambda \ln r \text{ where } r \sim \mathrm{Uni}[0,1] \right)$$

where $\frac{1}{\lambda} = \frac{1}{\lambda_{Ph}(E)} + \frac{1}{\lambda_c(E)} + \cdots$

from table

$$1/\lambda_c(E) = \sigma_c \cdot \rho_e$$

$\#\ e^-$ per unit volume

$$= Z \times \frac{\#\ \text{atoms}}{\text{volume}}.$$

If $x >$ Length of absorber, γ escapes.

if $x < L$, decide which process takes place,

$$P_i = \frac{\lambda_i^{-1}}{\sum_i \lambda_i^{-1}} = \frac{\sigma_i \rho_i}{\sum_i \sigma_i \rho_i}.$$

If selected process is photo effect, γ is absorbed.

If process is Compton scattering, generate ~~new~~ scattering angle, (Klein-Nishina formula)

$$\frac{d\sigma_c}{d\cos\theta} = \pi r_0^2 \frac{\gamma'^2}{\gamma^2}\left(\frac{\gamma}{\gamma'} + \frac{\gamma'}{\gamma} + \cos^2\theta - 1\right)$$

$$\gamma = E_\gamma / m_e c^2 \qquad r_0 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_e c^2}$$

$$p(\cos\theta) = \frac{1}{\sigma_c}\frac{d\sigma_c}{d\cos\theta}$$

[use accep.-reject].

Get new energy from

$$E' = \frac{E}{1 + \frac{E}{m_e c^2}(1-\cos\theta)}$$