PH3520 (Particle Physics)
Problem sheet 8
To be turned in by 10:00 a.m., Thursday, 24 November, 2011.

1: Consider production of a resonance of mass M in a reaction with centre-of-mass energy $E_{\rm cm}$. We saw in the lecture that the cross section will follow a Breig–Wigner form,

$$\sigma(E_{\rm cm}) = \frac{M^2\Gamma^2\sigma_0}{(E_{\rm cm}^2 - M^2)^2 + M^2\Gamma^2} ,$$

where σ_0 is the cross section at $E_{\rm cm} = M$. Show that for $E_{\rm cm}$ close to M, this reduces to the other form of the curve that we have used, namely,

$$\sigma(E_{\rm cm}) = \frac{\Gamma^2}{4} \frac{\sigma_0}{(E_{\rm cm} - M)^2 + \Gamma^2/4} .$$

(Hint: factorize $E_{\rm cm}^2 - M^2$ and use $E_{\rm cm} \approx M$.)

2: Consider elastic electron–proton scattering where the incoming electron has a four-momentum $p = (E, \vec{p})$ and the scattered electron has $p' = (E', \vec{p}')$. If we ignore the proton's magnetic moment, the differential cross section can be expressed as

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm point-like} |G_{\rm E}(Q^2)|^2 \qquad (1)$$

where the electric form factor $G_{\rm E}(Q^2)$ is the Fourier transform of the proton's charge distribution $\rho(\vec{x})$ and $Q^2 \equiv -q^2$, where $q^2 = (p - p')^2$ is the four-momentum transfer squared. Assuming a distribution of the form $\rho(\vec{x}) = \rho_0 e^{-\mu r}$, $G_{\rm E}(Q^2)$ can be expressed as

$$G_{\rm E}(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mu^2}\right)^2} . \qquad (2)$$

(a) Look at the electron–proton cross section measured by Hofstadter and McAllister with an electron-beam energy of 188 MeV (Fig. 8.3 in the lecture notes) and carefully estimate the ratio of measured to predicted point-like cross sections at $\theta = 130°$ (note that the vertical scale is logarithmic).

(b) What is the value of Q^2 (in units of MeV^2) at this angle?

(c) Using the equations given above, what value of μ (in MeV) is necessary to explain the observed suppression of the cross section relative to the expectation for a point-like proton?

(d) What is the rms radius of the proton (in fm) that corresponds to the value of μ that you obtain? (Use $\hbar c = 197.3$ MeV-fm to get back to 'normal' distance units.)

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