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PH3930 Notes
Lecture # 5
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Review of particle physics

The purpose of this lecture is to review the basics of the Standard Model of particle physics which we will need in our discussions of the early universe. In addition, we'll take a look at Grand Unified Theories (GUTs), which are more speculative and are outside the Standard Model.

Units

Before we start it's useful to recall the system of units used in particle physics, which is summarised in Table 1. Making the indicated replacements amounts to working in units where $\hbar = c = k_{\rm B} = 1$. That is, whenever we write p for momentum, this really stands for the usual momentum (having units GeV/c) times c, so that the units of our 'particle physics' momentum are GeV.

Table 1: Units used in particle physics.

quantity	units	replacement
energy E	GeV	–
momentum p	GeV	$pc \to p$
mass m	GeV	$mc^2 \to m$
temperature T	GeV	$k_{\rm B}T \to T$
distance x	GeV^{-1}	$x/(\hbar c) \to x$
time t	GeV^{-1}	$t/\hbar \to t$

So for example, cross sections in standard units having dimensions of length squared will have units GeV^{-2} in the particle physics system. To get back to normal units, you have to multiply this by $(\hbar c)^2$. For converting between the two systems, it's sufficient to know c, $\hbar c$, and Boltzmann's constant $k_{\rm B}$:

$$\begin{aligned}
c &= 2.997 \times 10^8 \text{ m/s} = 2.997 \times 10^{-7} \text{ fm/s} \\
\hbar c &= 0.1973 \text{ GeV fm}, \\
k_{\rm B} &= 8.617 \times 10^{-14} \text{ GeV/K}.
\end{aligned}$$

In addition with we will usually give the electric charge of a particle as a multiple of the proton's charge e. In particle physics units, e is 0.303. It's easier, however, to remember the fine structure constant α, given by

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137} \ . \tag{1}$$

Review of fundamental particles

In the Standard Model, the fundamental particles of matter are spin-1/2 fermions, whose interactions are mediated by spin-1 gauge bosons. The fermions come in two types: quarks and leptons. Both types participate in electroweak interactions. In addition, quarks participate in the strong interactions, whereas leptons do not. There are six types or 'flavours' of quarks: up, down, strange, charm, bottom and top. The flavours are grouped into three 'families' or 'generations', each of which consists of an 'up-type' (u, c, and t, charge 2/3) and a corresponding 'down-type' (d, s, and b, charge $-1/3$):

charge +2/3:	u	c	t
charge $-1/3$:	d	s	b

The quarks are assigned a baryon number of 1/3. The corresponding antiquarks ($\overline{u}$, $\overline{d}$, $\overline{s}$, $\overline{c}$, $\overline{b}$, $\overline{t}$) have opposite electric charges and are assigned baryon number $-1/3$.

Leptons also come in three families, each of which contains a neutrino (charge 0) and a charged lepton (charge -1):

charge 0:	ν_{e}	ν_μ	ν_τ
charge -1:	e^-	μ^-	τ^-

These are assigned a lepton number of 1. The corresponding antileptons ($\overline{\nu}_{\mathrm{e}}$, $\overline{\nu}_\mu$, $\overline{\nu}_\tau$, e^+, μ^+, τ^+) have opposite electric charge and a lepton number of -1.

The fermions interact by exchange of spin-1 gauge bosons. Their properties are listed in Table 2.

Table 2: Gauge bosons and their properties.

particle	charge	mass (GeV)	interaction
photon (γ)	0	0	electromagnetic
$\mathrm{W}^\pm$	± 1	80.4	weak
Z	0	91.2	weak
gluon (g)	0	0	strong

The particles γ, $\mathrm{W}^\pm$ and Z can be seen as mediating a unified 'electroweak' interaction. Those involving the $\mathrm{W}^\pm$ and Z are weak in the sense that the masses of these particles lead to a suppression of the interaction probability when the typical energy scale is much less than the boson's mass.

In addition to the fundamental fermions and gauge bosons, the simplest version of the Standard Model has a neutral spin-0 particle known as the Higgs boson. This particle is related to the mechanism by which all particles of the Standard Model acquire mass.

Particle interactions

The Standard Model of particle physics is a quantum field theory, i.e. a quantum mechanical theory in which particles can be created and destroyed. The basic rule of quantum mechanics is that for every reaction such as a scattering or a decay, there exists an amplitude $\mathcal{M}$. The probability for the reaction to occur is proportional to the absolute square of the amplitude $|\mathcal{M}|^2$. These probabilities are expressed as scattering cross sections, usually denoted by σ, or as decay rates, Γ.

The amplitudes for a given reaction can be obtained using Feynman diagrams. These are space-time diagrams in which the type of line indicates the type of particle, as shown in Fig. 1. (In addition, arrows are usually drawn on fermion lines going forward in time for particles, and backwards in time for antiparticles. This won't be important in this course and we'll usually drop the arrows.)

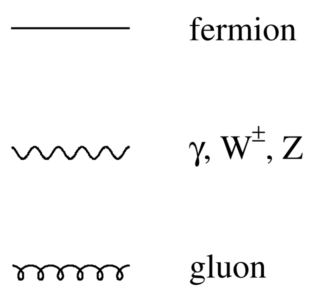

Figure 1: Lines used in Feynman diagrams to indicate different kinds of particles.

The diagrams are usually drawn such that the vertical direction corresponds to particle separation, and the horizontal direction denotes the flow of time. (The opposite convention is sometimes found as well.) Figure 2, for example, shows one of the diagrams for the reaction $e^-\gamma \to e^-\gamma$. The electron absorbs the photon and then emits it at a later time. (In fact, the opposite time ordering of emission and absorption is included in the same amplitude, but we can ignore this subtlety for now.)

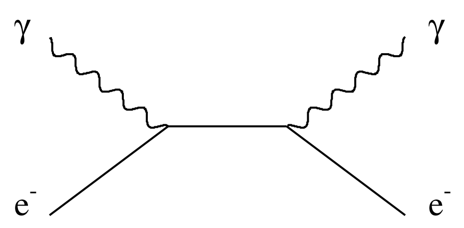

Figure 2: A Feynman diagram for the reaction $e^-\gamma \to e^-\gamma$ (Compton scattering).

The total amplitude for a reaction is obtained by summing all of the diagrams with the same initial and final states. There exists a set of 'Feynman rules', which tell how to write down the amplitudes directly from the diagrams. Deriving these rules is a complicated affair, beyond the scope of this course. We can, however, state a subset of the rules which will tell us some information about the amplitudes.

One of the rules is that every vertex in the diagram is associated with a 'coupling' or charge, which characterises the strength of the interaction. The amplitude contains a factor of the coupling for each vertex. For fermion–photon couplings, this is simply the fermion's electric charge, i.e. 0 for neutrinos, e for the charged leptons, and $2e/3$ or $e/3$ for for up- and down-type quarks. There are two electron–photon vertices in Fig. 2, so the corresponding amplitude is proportional to e^2. To get the cross section σ, we square the amplitude, so we have

$$\begin{aligned} \mathcal{M} &\propto e^2 , \\ \sigma &\propto |\mathcal{M}|^2 \propto e^4 \propto \alpha^2 . \end{aligned}$$

For every W–fermion vertex, the amplitude contains a factor of g, the weak coupling constant. This is the same coupling for quarks and leptons. Some possible vertices are shown in Fig. 3(a)–(b). Note that electric charge is conserved at each vertex, i.e. the charged electron is transformed into a neutrino by emission of a W^-, and the charge 2/3 up-quark is transformed into a charge $-1/3$ down quark.

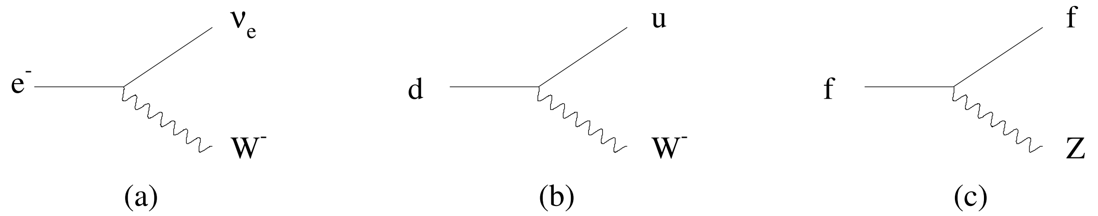

Figure 3: Possible couplings between W bosons and (a) leptons and (b) quarks (b). (c) The coupling between a fermion f and a Z, where f represents any of the fermion types.

The Z couples to all of the fermion types, as indicated in Fig. 3(c). The Z/fermion coupling depends on the fermion type and is a function of both e and g. The main rule to remember for Z–fermion vertices is that the fermion does not change its family. That is, $u \to uZ$ is allowed, but the direct coupling $u \to cZ$ is not present in the theory, even though it would conserve electric charge.

Quarks and gluons possess an additional type of charge known as colour. Quarks come in one of three colours, and a gluon is is characterised by a colour and an anticolour. The theory describing the interactions between quarks and gluons is called Quantum Chromodynamics (QCD).

The possible couplings between quarks and gluons are shown in Fig. 4. The quark–gluon coupling (a) and triple gluon vertex (b) both have the coupling g_{s}, whereas the four-gluon vertex in (c) has a coupling of g_{s}^2. In analogy with the fine structure constant, one often uses the strong coupling constant α_{s}, defined as

$$\alpha_{\mathrm{s}} = \frac{g_{\mathrm{s}}^2}{4\pi} \approx 0.12 \ , \tag{2}$$

where the value of 0.12 is valid for processes where the energy scale is of the order of the Z mass, $M_{\mathrm{Z}} = 91.2$ GeV. This gives $g_{\mathrm{s}} = 1.2$.

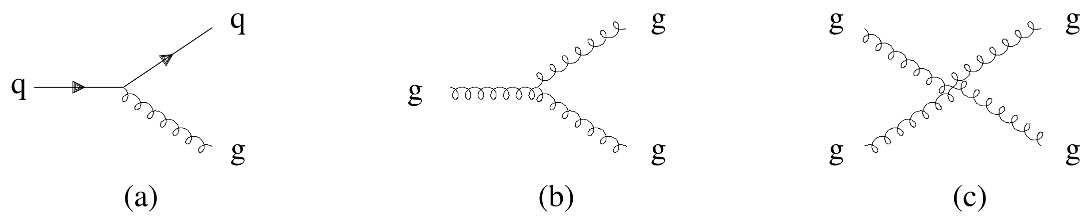

Figure 4: Possible couplings between quarks and gluons.

One of the rules for manipulating Feynman diagrams is that you can move any of the external lines between initial and final states by replacing particle with antiparticle and vice versa. For example, the incoming photon in the Compton scattering diagram of Fig. 2 can be pulled over to the right-hand side of the diagram, and the outgoing electron can be made an incoming positron. This results in the diagram shown in Fig. 5(a), corresponding to the reaction $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$. Similarly, the $\mathrm{e}^+$and e^-can both be pulled over to the right-hand side and the photons to the left, giving $\gamma\gamma \to \mathrm{e}^+\mathrm{e}^-$, as in Fig. 5(b).

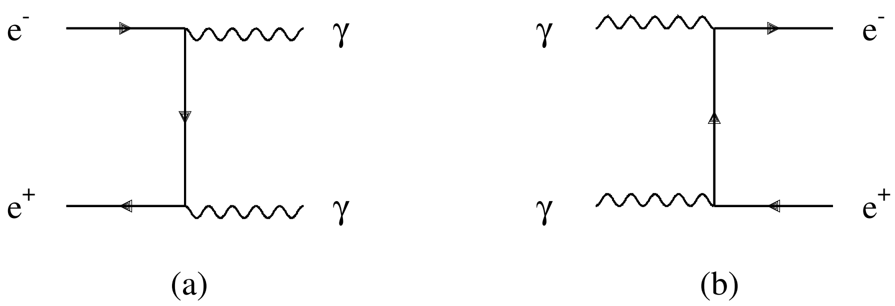

Figure 5: Feynman diagrams for (a) $\mathrm{e}^+\mathrm{e}^- \to \gamma\gamma$ and (b) $\gamma\gamma \to \mathrm{e}^+\mathrm{e}^-$.

From the rules encountered so far we can see that the cross sections for $\mathrm{e}^+\mathrm{e}^-$annihilation into two photons and $\mathrm{e}^+\mathrm{e}^-$pair production by the collision of two photons will both be proportional to e^4 (i.e. α^2). That is, a reaction and its inverse have the same couplings.

We can state another Feynman rule for diagrams in which a boson such as the W is exchanged. An example is shown in Fig. 6(a), in which a neutron (a bound state of udd quarks) decays into a proton (uud), an electron $\mathrm{e}^-$and an antineutrino $\overline{\nu}_{\mathrm{e}}$. The rule is that for every internal boson, the amplitude contains a factor

$$\frac{1}{q^2 - M^2} \ , \tag{3}$$

where M^2 is the boson's (rest) mass and q^2 is its four-momentum squared. In order to apply this to the physics of the early universe, all we need to know is that q^2 will be of the same order as the typical energy scale of the process (squared), as given, for example, by the temperature.

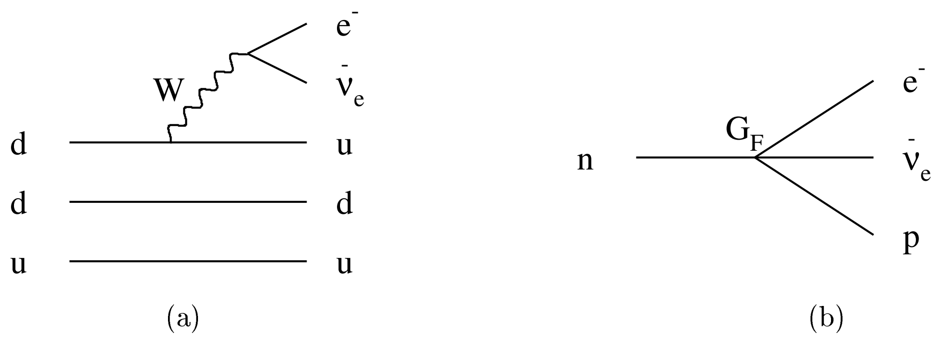

Figure 6: Feynman diagrams for beta decay of a neutron. (a) The process viewed as the decay of a d quark emitting a W boson. (b) The process viewed as a four-fermion coupling.

The decay shown in Fig. 6 is the beta decay of a neutron, something which will be of considerable importance in our discussion of the early universe. In Fig. 6(a), this is viewed as the decay of a d quark into a u quark by emission of a W^-, which then decays into an electron and antineutrino. Both vertices have the weak coupling g, and the amplitude will contain a propagator like (3). For this reaction, we will have $q^2 \ll M_{\mathrm{W}}^2$, since the momentum transfer to the electron–antineutrino pair is of the order of the neutron–proton mass difference (1.3 MeV), compared to $M_{\mathrm{W}} = 80.4$ GeV. The propagator can thus be approximated by $-1/M_{\mathrm{W}}^2$, the amplitude will be proportional to g^2/M_{W}^2, and thus the decay rate will be proportional to g^4/M_{W}^4.

Beta decay can also be described using an effective low-energy theory proposed by Fermi in 1934, long before the quark picture was developed. Here the neutron decays into a proton, electron and antineutrino by means of a four-fermion coupling, as shown in Fig. 6(b). The corresponding coupling is the Fermi constant G_{F}, which can be determined from measured rates of beta decay to be

$$G_{\mathrm{F}} = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}. \tag{4}$$

By equating the predictions of the Standard Model and of Fermi's theory, we can relate the weak coupling constant g to the Fermi constant. One obtains

$$\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g^2}{8M_{\mathrm{W}}^2}\,, \tag{5}$$

where the factors of $\sqrt{2}$ and 8 result from different conventions used in defining the couplings in the two theories. Solving (5) for g gives $g = 0.65$.

Running couplings

An important property of the Standard Model is that the couplings depend on the energy scale of the processes involved. The couplings are said to 'run' with energy. This is seen most dramatically in QCD, where the strong coupling α_{s} is found to decrease for increasing energy scale E as

$$\alpha_{\mathrm{s}}(E) = \frac{12\pi}{(33 - 2n_{\mathrm{f}})\log(E^2/\Lambda^2)} \ . \tag{6}$$

Here n_{f} is the number of quark flavours with masses less than E, and Λ is the QCD scale parameter, which has a value of approximately 200 MeV. From equation (6) we can see that if the energy is equal to Λ, then the coupling becomes infinite. In fact, (6) is itself based on a perturbative expansion in α_{s}, which for large α_{s} is no longer valid. The main message is, however, that for energies on the order of 1 GeV, the strong coupling becomes so large that quarks and gluons become bound into hadrons, i.e. states consisting of three quarks (baryons) or a quark and antiquark (mesons), which are colour neutral. This phenomenon is known as confinement. It is not well understood theoretically, since the coupling is too large to allow for reliable predictions using perturbation theory. For energies much larger than Λ, α_{s} becomes small and perturbation theory can be used. This decrease in the coupling is known as asymptotic freedom.

The electromagnetic and weak couplings e and g also depend on the energy scale of the reaction. The weak coupling g decreases slightly and the electric charge e increases with energy. The behaviour of e can be understood qualitatively by considering an electric charge in a vacuum, as shown in Fig. 7. If a virtual $\mathrm{e}^-\mathrm{e}^+$ pair is created out of the vacuum, this would appear to violate conservation of energy. This is allowed by the uncertainty principle, however, as long as the virtual particles only live for a very short time before recombining. The result is that a certain amount of positive and negative charge is constantly swimming around in the vacuum, and in the presence of the negative point charge in Fig. 7, the vacuum becomes polarised. That is, the positrons are attracted towards the charge and electrons are repelled. This has the effect of partially screening some of the original negative charge, just as if it were in a dielectric medium.

(a) (b)

Figure 7: (a) A point charge in a vacuum surrounded by virtual $\mathrm{e}^+\mathrm{e}^-$ pairs. (b) A high energy particle probes the point charge at short distances, penetrating past the screening of the $\mathrm{e}^+\mathrm{e}^-$ pairs.

If we consider the interaction of another charged particle with our original negative point charge, the effective coupling will be reduced because of the polarisation of the vacuum. If, however, the incoming particle has a higher energy, then it will probe the target at a smaller distance scale and the screening effect will be reduced. Hence, the effective electric charge increases for increasing energies.

This type of reasoning leads to the correct qualitative picture, but it does not tell us, for example, why the QCD coupling decreases while the electromagnetic and weak couplings increase. To see this more quantitatively, one must use perturbation theory to compute interaction cross sections. Here the couplings (e, g, g_{s}) appear as expansion parameters in a power series. Higher order terms correspond to Feynman diagrams with more virtual particles, and hence more powers of the coupling, as shown in Fig. 8.

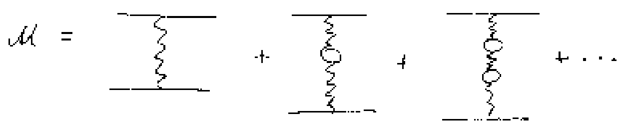

Figure 8: Some higher order corrections to a scattering amplitude.

It turns out that the diagrams with loops in the boson lines as shown in Fig. 8 lead to the dependence of couplings on the energy scale. Here the loops correspond to all of the different types of particles which can couple to the boson in question. If the exchanged particle in Fig. 8 is a photon, for example, then the loop could be be any particle–antiparticle pair with electric charge, such as $\mathrm{e}^+\mathrm{e}^-$, $\mu^+\mu^-$, $q\overline{q}$, etc. The contribution of the diagram with the loop depends on the mass of the particle in the loop. To a good approximation, one can say that only particles with masses less than the energy scale of the reaction contribute. This leads to the dependence of α_{s} on the number of quark flavours n_{f} with masses less than the energy E, as seen in equation (6).

The decrease in the strong coupling for increasing energy is related to the fact that the gluon couples to itself (cf. Fig. 4), whereas the photon does not. That is, an exchanged gluon can have loops with quark–antiquark pairs as shown in Fig. 9(a), and also with a gluon pair as in Fig. 9(b). It turns out that the gluon loops contribute to the energy dependence with the opposite sign, i.e. they result in the decrease in the coupling for increasing energy.

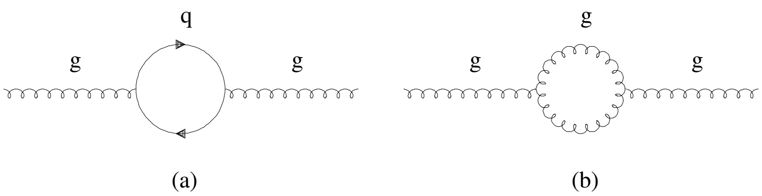

Figure 9: Higher order corrections to diagrams with exchanged gluons: (a) a quark–antiquark loop, (b) a gluon loop.

Grand unified theories (GUTs)

The energy dependence of the strong, electromagnetic and weak couplings predicted by the Standard Model is shown schematically in Fig. 10. The prediction is that they will meet at an energy of around 10^{15} GeV, known as the unification energy scale.

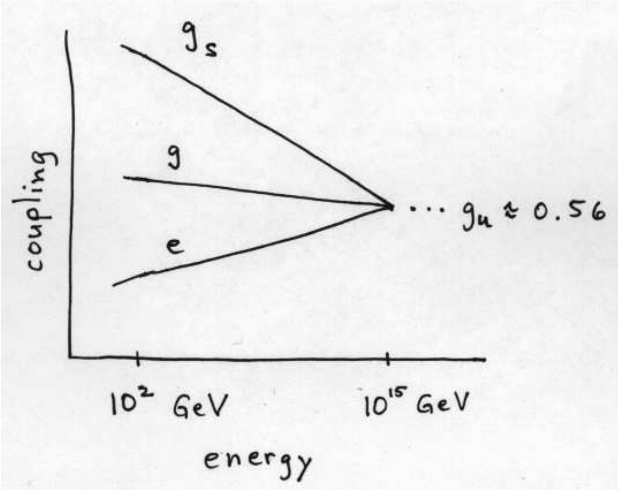

Figure 10: The strong, electromagnetic and weak couplings as a function of energy.

The idea between so-called Grand Unified Theories (GUTs) is that for energies higher than the unification scale, interactions are characterised by a single coupling $g_{\mathrm{U}} \approx 0.56$, or in analogy with the fine structure constant one defines

$$\alpha_{\mathrm{U}} = \frac{g_{\mathrm{U}}^2}{4\pi} \approx \frac{1}{40} \; . \tag{7}$$

GUTs are constructed following the same pattern as the electroweak theory and QCD. That is, one requires that the theory have a mathematical property known as gauge symmetry, related to invariance under a certain group of transformations. In order to have this symmetry, one has to introduce additional particles into the theory, namely gauge bosons. In one of the simplest GUTs based on the group SU(5), the gauge bosons are called $\mathrm{X}^{\pm}$, with charge $\pm 4/3$, and $\mathrm{Y}^{\pm}$, having charge $\pm 1/3$. Their masses are on the order of the GUT scale, i.e. 10^{15} GeV.

GUTs can explain a number of features of the Standard Model which would otherwise have to be put in by hand. For example, GUTs require that charge be quantized, and that the charge of the electron be equal and opposite to that of the proton. GUTs predict that all of the couplings come together at a single value for energies equal to the GUT scale, and to a certain approximation this appears to be the case. More accurate measurements of the couplings indicate, however, that the couplings don't meet at a point. The unification of the couplings can be restored, however, if one considers theories beyond the Standard Model, such as supersymmetry.

GUTs also predict that the proton should be unstable. One of the possible decay modes should be into a neutral pion and a positron, as shown in Fig. 11.

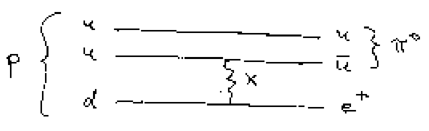

Figure 11: A Feynman diagram for the decay $p \to \pi^0 e^+$.

Note that baryon number B is violated in this decay, as is lepton number L, although $B - L$ is conserved. The simplest GUTs predict a mean proton lifetime of around 10^{30} years. This is in contradiction with recent experimental results, which place a lower limit on the proton's lifetime of around 10^{33} years. More complicated GUTs (including those with supersymmetry), however, can lead to sufficiently long proton lifetimes so as to not stand in disagreement with experiment. We will return to baryon number violation and GUTs in our discussion of the matter/antimatter asymmetry of the universe.