PH3930 (Particle Astrophysics)
Problem sheet 1
To be turned in by 5:00 p.m., Friday, 24 January, 2003.

1: This is an exercise in units; don't worry about the formula, which is something we will see later. The universe is bathed in primordial neutrinos, which have been passing through you unnoticed your entire life. Using units where $\hbar = c = k_{\rm B} = 1$, the number of neutrinos (plus antineutrinos) per unit volume for a given family is

$$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2} 2T^3 \ , \tag{1}$$

where T is the temperature, ζ is the Riemann zeta function and $\zeta(3) \approx 1.20206$. The neutrinos have a temperature of $T = 1.95$ K (note that this is lower than the temperature of the cosmic microwave background radiation). Find the number of neutrinos and antineutrinos for a single family per cm^3. (Hint: look at the example in Section 1.2 of the Lecture Notes.)

2: Using the vertices seen in the lecture from the SU(5) Grand Unified Theory, draw two possible Feynman diagrams for the decay $\mathrm{n} \to \pi^- \mathrm{e}^+$. Check whether B, L and $B - L$ are conserved.

3: (a) You are not decaying at a noticeable rate (radioactively, that is). From this, make an order of magnitude estimate of the mean lifetime of the proton.

You could, for example, agrue that if proton decays in your body were to release more than 1 J/kg in one year, then this dose (10^3 times a typical yearly dose from natural radiation sources) would lead to a measurable level of radiation sickness, which we hopefully do not observe. Your are welcome to think up a different measure of the decay rate.

Use the fact that the number of decays $n_{\rm dec}$ in 1 kg of matter is related to the time t, the mean proton lifetime $\tau_{\rm p}$ and the total number of protons $n_{\rm tot}$ by

$$n_{\rm dec} = n_{\rm tot} \frac{t}{\tau_{\rm p}} \ .$$

Assume that if a proton in your body decays, some significant fraction of its rest mass energy is deposited in your body.

(b) Using the limit on the proton lifetime from (a), place a lower limit on the masses of the X and Y bosons. (Assume they have equal mass, take $g_{\rm U} = 0.56$ and use the forumla from the lecture.)

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16 January, 2003