

1: Starting from the Friedmann equation (here $c = 1$),

$$\frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi}{3}G\rho$$

and the fluid equation

$$\dot{\rho} + 3\frac{\dot{R}}{R}(\rho + P) = 0 ,$$

derive the acceleration equation,

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G(\rho + 3P) .$$

2: The current value of the Hubble constant is close to $H_0 = 70$ km/s/Mpc. Find $1/H_0$ in years. (You can find the necessary conversion factors near the beginning of the PDG Particle Physics Booklet.)

3: The critical density ρ_c is defined as

$$\rho_c = \frac{3H^2}{8\pi G} . \tag{1}$$

Find the current value of ρ_c in GeV^4 and in GeV per m^3 . Use the current value for the Hubble parameter of $H_0 = 70$ km/s/Mpc. You may want to use G in particle physics units: $G = 6.7 \times 10^{-39} \text{ GeV}^{-2}$.

4: Consider an empty room $5 \times 5 \times 2 \text{ m}^3$ with walls at a temperature $T = 300$ K. How many photons from blackbody radiation are in the room?

Would it make sense to apply the corresponding formula for the number density of neutrinos in the room? Justify your answer.