PH3930 (Particle Astrophysics)
Problem sheet 3
To be turned in by Friday 7 February, 2003, 17:00.

1: Consider the universe at a point when the temperature was $T = 30$ GeV.

(a) Find the effective number of degrees of freedom g_*.

(b) What is the total energy density in GeV^4?

(c) At what time was this temperature reached?

(d) What was the number density of u quarks?

(e) What was the number density of Z bosons at this temperature? (The chemical potential for Z bosons is zero. Optional question: Explain why.)

2: You are kidnapped by space aliens who place you in suspended animation. When you awake, you are in a spaceship whose instruments indicate a CMBR temperature of 1.73 K. How long has it been since you were abducted? Take for the time of the abduction $t_\mathrm{a} = 1.5 \times 10^{10}$ years after the Big Bang and assume that the energy density is and continues to be dominated by nonrelativistic matter.

3(a): Suppose the universe starts with equal amounts of baryons and antibaryons, i.e.,

$$n_\mathrm{b} = n_{\overline{\mathrm{b}}} \, .$$

Assume further that there are no baryon number violating processes to allow this to change. Consider temperatures below around $T \approx 0.05$ GeV, so that the only relativisitic particles in thermal equilibrium are $\mathrm{e}^\pm$, neutrinos and photons.

(a) What is the effective number of degrees of freedom g_* at this temperature? (Treat muons as non-relativistic.)

(b) At this temperature, quarks and gluons have become bound into neutrons and protons, which are non-relativistic. Riotto gives for the baryon–antibaryon annihilation cross section times speed at this energy

$$\langle \sigma v \rangle \approx \frac{1}{m_\pi^2} \, ,$$

where $m_\pi = 0.14$ GeV is the pion mass. Plot the baryon–antibaryon annihilation rate $\Gamma = n_b \langle \sigma v \rangle$ versus temperature in the range up to $T = 50$ MeV. Also plot the expansion rate H as a function of temperature in this range. (Use mathematica or whatever you like.)

(c) At what temperature do you find $\Gamma = H$? At this temperature, baryon–antibaryon annihilation 'freezes out', i.e., the baryons and antibaryons are too spread out to annihilate and their number density will scale as $n_\mathrm{b} \propto R^3 \propto 1/T^3$, just like photons.

(d) Compute at this temperature the baryon density n_b (equal to the antibaryon density in this scenario), the photon density n_γ and their ratio. You should find a number that is much smaller than the actual measured value of $n_\mathrm{b}/n_\gamma \approx 5 \times 10^{-10}$.

G. Cowan
30 January, 2003