PH3930 (Particle Astrophysics)
Problem sheet 6
To be turned in by Friday 7 March 2003, 17:00.

1(a) Show that the Friedmann equation,

$$H^2 + \frac{k}{R^2} = \frac{8\pi G}{3}\rho \,,$$

can be written

$$\Omega - 1 = \frac{k}{\dot{R}^2} \,,$$

where $\Omega = \rho/\rho_c$ and the critical density is $\rho_c = 3H^2/8\pi G$.

(b) Consider the universe when its energy density is dominated by nonrelativistic matter, so that the equation of state is $P = 0$. Recall from a previous problem sheet (or rederive) how the scale factor depends on time and use this to show

$$R\dot{R}^2 = \text{const} \,.$$

(c) Combine the results from (a) and (b) to show

$$\Omega - 1 \propto kR \,.$$

(d) There has been strong observational evidence for a number of years that the current total energy density of the universe is constrained to lie in the range $0.2 \leq \Omega \leq 2$. Using the results above, what range must Ω lie in at the time of photon decoupling ($T = 0.3$ eV) in order for this to be true?

2 For this question, go to the website of the WMAP experiment (via course website) and browse through the section called 'Universe'. Find the link on 'Parameters of Cosmology' (currently `map.gsfc.nasa.gov/m_mm/sg_parameters.html`).

(a) Sketch the power spectrum as a function of the multipole number l for $\Omega = 1$. (On the MAP website, this is called T_l; it corresponds to $l(l+1)C_l$ that we saw in the lecture.) At what angle is the position of the first acoustic peak at and what value of l does this correspond to?

(b) Sketch how the power spectrum would change if $\Omega = 0.5$. (Get the position of the 1st acoustic peak in the right place ... Recall $l_{\text{peak}} \approx 200/\sqrt{\Omega}$.) Suppose MAP is able to measure the position of this peak to an accuracy of 2%. What will be the resulting accuracy for Ω?

G. Cowan
27 February, 2003