PH3930 (Particle Astrophysics)
Problem sheet 7
Due 17:00, Friday 14 March, 2003

1(a) Suppose the number density of magnetic monopoles is given by

$$n_{\rm m} = \frac{1}{(2t_{\rm GUT})^3} ,$$

where $t_{\rm GUT}$ is the time where the temperature reaches $T_{\rm GUT} \approx 10^{16}$ GeV. Assuming the mass of magnetic monopoles is $m_{\rm m} \approx M_{\rm X}/\alpha_{\rm U} \approx 10^{17}$ GeV, find the energy density of monopoles in GeV^4. (Hints: Take $g_* \approx 10^2$ to relate $t_{\rm GUT}$ to $T_{\rm GUT}$; keep time in units of GeV^{-1}.)

(b) Compare the answer from (a) to the energy density of photons at $T_{\rm GUT}$.

(c) Assuming that the scale factor follows $R \propto t^{1/2}$, find the time in seconds when the energy density of monopoles and photons becomes equal. (Hint: Use the fact that $\rho \propto 1/R^4$ for photons but as $1/R^3$ for the nonrelativistic monopoles.)

2(a) Sketch a possible potential $V(\phi)$ for an inflaton field ϕ and describe qualitatively how the state of the field evolves through the period of inflation.

(b) Describe qualitatively how inflation leads to density fluctuations in the early universe.

(c) Describe qualitatively how density fluctuations in the early universe lead to temperature variations in the CMBR measured from different directions. Does a lower CMBR temperature correspond to a region of higher or lower density? (Explain.)

3 Consider a neutralino with mass $m_\chi = 100$ GeV. Suppose that at a temperature of $T = 5$ GeV, the annihilation rate Γ drops below the expansion rate H.

(a) What is the number density n_χ of neutralinos in cm^{-3} at this point (i.e., at $T = 5$ GeV)?

(b) Consider a comoving volume R^3. Make a sketch (with mathematica or by hand) of the number of neutralinos N_χ in this volume as a function of temperature. Use a logarithmic scale for both axes and consider temperatures from 1000 GeV down to 0.1 GeV.

(c) What would be n_χ today in cm^{-3}?

(d) What would be $\Omega_\chi = \rho_\chi/\rho_{\rm c}$ today?

G. Cowan
6 March 2003