Statistical Methods for Particle Physics
Graduierten-Kolleg RWTH Aachen, February 2014
Problem sheet 1

Exercise 1: Show that

$$P(A \cup B) \;=\; P(A) + P(B) - P(A \cap B)\,.$$

(Hint: express $A \cup B$ as the union of disjoint sets and use the Kolmogorov axioms.)

Exercise 2: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{array}{lcl} P(0\,|\,\mathrm{e}) = 0.001 & \text{and} & P(0\,|\,\gamma) = 0.99899 \\ P(1\,|\,\mathrm{e}) = 0.01 & & P(1\,|\,\gamma) = 0.001 \\ P(2\,|\,\mathrm{e}) = 0.989 & & P(2\,|\,\gamma) = 10^{-5}\,. \end{array}$$

(a) What is the probability for the particle to be a photon given a detected signal in one layer only?

(b) What is the probability for a particle to be an electron given a detected signal in both layers?

Exercise 3: Consider two random variables x and y.

(a) Show that the variance of $\alpha x + y$ is given by

$$\begin{aligned} V[\alpha x + y] &= \alpha^2 V[x] + V[y] + 2\alpha \mathrm{cov}[x, y] \\ &= \alpha^2 V[x] + V[y] + 2\alpha\rho\sigma_x\sigma_y\,, \end{aligned} \tag{1}$$

where α is any constant value, $\sigma_x^2 = V[x]$, $\sigma_y^2 = V[y]$, and the correlation coefficient is $\rho = \mathrm{cov}[x, y]/\sigma_x\sigma_y$.

(b) Using the result of (a), show that the correlation coefficient always lies in the range $-1 \le \rho \le 1$. (Use the fact that the variance $V[\alpha x + y]$ is always greater than or equal to zero and consider the cases $\alpha = \pm\sigma_y/\sigma_x$.)

Exercise 4: Suppose the independent random variables x_1 and x_2 have means $\mu_1 = \mu_2 = 10$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Use error propagation to find the variance of

$$y = \frac{x_1^2}{x_2}\,. \tag{2}$$

Comment on the validity of the procedure if one had $\mu_2 = 1$.

Exercise 5: Suppose the independent random variables r_i are uniformly distributed between zero and one. Using the code from

`http://www.pp.rhul.ac.uk/~cowan/stat/root/mc/`

as a starting point, write a computer program to make histograms of

(a) $x = r_1 + r_2 - 1$

(b) $x = r_1 + r_2 + r_3 + r_4 - 2$

(c) $x = \sum_{i=1}^{12} r_i \quad - 6$

Calculate exactly (i.e., in closed form using the exact values $E[r_i] = 1/2$, $V[r_i] = 1/12$) the means and variances of the variables defined in (a)–(c) and compare these to the values you obtain from the histograms of generated numbers (the information is displayed when you plot the histograms using ROOT). Comment on the connection between your histograms and the central limit theorem. Remeber to adjust the minimum and maximum x values of the histogram so that you cover all of the generated values.

Exercise 6: Consider a sawtooth p.d.f.,

$$f(x) = \begin{cases} \frac{2x}{x_{\rm max}^2} & 0 < x < x_{\rm max} \ , \\ 0 & \text{otherwise} \ . \end{cases} \tag{3}$$

(a) Use the transformation method to find the function $x(r)$ to generate random numbers according $f(x)$. Implement the method in a short computer program and make a histogram of the results. (Use e.g. $x_{\rm max} = 1$.)

(b) Write a program to generate random numbers according to the sawtooth p.d.f. using the acceptance-rejection technique. Plot a histogram of the results.

G. Cowan
10 February, 2014