$$\sigma_{\hat{\alpha}}$$

$$\sigma_{\hat{\beta}}$$

$$m = \sqrt{2E_1E_2(1-\cos\theta_{12})}$$

$$P(n|s,b) = \frac{(s+b)^n}{n!}e^{-(s+b)}$$

$$Q = -2\ln\frac{L_{s+b}}{L_b} = 2s - 2\sum_{i=1}^{n}\ln\left[1 + \frac{s}{b}\frac{f(\mathbf{x}_i|s)}{f(\mathbf{x}_i|b)}\right]$$

$$\lambda(\mu) = \frac{L(\mu)}{L(\hat{\mu})}$$

$$\hat{\mu} = \operatorname*{argmax}_{\mu} L(\mu)$$

$$p = 1 - \Phi\left(\sqrt{q_0}\right)$$

$$Z = \sqrt{q_0} = \hat{\mu}/\sigma_{\hat{\mu}}$$

$$L(\mu) = \frac{(\mu s+b)^n}{n!}e^{-(\mu s+b)}\prod_{i=1}^{n} f(x_i|\mu)$$

$$f(x|\mu) = \frac{\mu s}{\mu s+b}f(x|\mathrm{s}) + \frac{b}{\mu s+b}f(x|\mathrm{b})$$

$$x_1,\ldots,x_n$$

$$n \sim \mathrm{Poisson}(\mu s+b)$$

$$y(\mathbf{x}) = \frac{p(\mathbf{x}|\mathrm{s})}{p(\mathbf{x}|\mathrm{b})}$$

$$\frac{p(\mathbf{x}|\mathrm{s})}{p(\mathbf{x}|\mathrm{b})} > c$$

$$t_{\mathrm{p}}(x) = \frac{L_{\mathrm{p}}(x|s)}{L_{\mathrm{p}}(x|b)} = \frac{L(x|\hat{\hat{\nu}}(s),s)}{L(x|\hat{\hat{\nu}}(b),b)}$$

$$t_{\mathrm{m}}(x) = \frac{L_{\mathrm{m}}(x|s)}{L_{\mathrm{m}}(x|b)} = \frac{\int L(x|\nu,s)\pi_\nu(\nu)\,d\nu}{\int L(x|\nu,b)\pi_\nu(\nu)\,d\nu}$$

$$P(\theta < \theta_{\rm up}(x)|\theta) = ?$$

$$P(\theta < \theta_{\rm up}|x) = \int_{-\infty}^{\theta_{\rm up}} p(\theta|x)\, d\theta = 95\%$$

$$\pi_\nu(\nu) = \pi(\nu|y) \propto L(y|\nu)\pi_0(\nu)$$

$$p(\theta,\nu|x) \propto L(x|\theta,\nu)\pi_\theta(\theta)\pi_\nu(\nu)$$

$$p(\theta,\nu|x,y) \propto L(x|\theta,\nu)L(y|\nu)\pi_\theta(\theta)\pi_0(\nu)$$

$$p(\theta|x) = \int p(\theta,\nu|x)\, d\nu \propto \int L(x|\theta,\nu)\pi_\nu(\nu)\pi_\theta(\theta)\, d\nu = L_{\rm m}(x|\theta)\pi_\theta(\theta)$$

$$L_{\rm m}(x|\theta) = \int L(x|\theta,\nu)\,\pi_\nu(\nu)\, d\nu$$

$$p(\theta|x) = \int p(\theta,\nu|x)\, d\nu$$

$$p(\theta,\nu|x) \propto L(x|\theta,\nu)\pi(\theta,\nu)$$

$$\pi_\theta(\theta)$$

$$\pi_\nu(\nu)$$

$$\pi(\theta,\nu) = \pi_\theta(\theta)\pi_\nu(\nu)$$

$$\pi(\theta,\nu)$$

$$p_\theta = \int_{t_{\theta,\rm obs}}^{\infty} f(t_\theta|\theta,\nu)\, dt_\theta$$

$$t_\theta = -2\ln\frac{L(\theta,\hat{\hat{\nu}}(\theta))}{L(\hat{\theta},\hat{\nu})}$$

$$L_{\rm p}(\theta) = L(\theta,\hat{\hat{\nu}}(\theta))$$

$$\hat{\hat{\nu}}(\theta) = \operatorname*{argmax}_\nu L(\theta,\nu)$$

$$L(\theta)$$

$$L(x|\theta)$$

$$L_{\mathrm{p}}(\mu) = L(\mu, \hat{\hat{\theta}}(\mu))$$

$$L_{\mathrm{m}}(\mu) = \int L(\mu, \theta)\pi_\theta(\theta)\, d\theta$$

$$L(L_1, L_2, T, \tilde{\tau}_0, \tilde{\alpha}_1, \tilde{\alpha}_2|\lambda, \tau, \tau_0, \alpha_1, \alpha_2) =$$

$$\frac{1}{\sqrt{2\pi}\sigma_T} e^{-(T-\tau)^2/2\sigma_T^2} \prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(L_i-\lambda+\alpha_i(\tau-\tau_0))^2/2\sigma_i^2}$$

$$\times \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\tau}_0}} e^{-(\tilde{\tau}_0-\tau_0)^2/2\sigma_{\tilde{\tau}_0}^2}$$

$$\prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_{\tilde{\alpha}_i}} e^{-(\tilde{\alpha}_i-\alpha_i)^2/2\sigma_{\tilde{\alpha}_i}^2}$$

$$\mathrm{cov}[y_1, y_2] = \alpha_1\alpha_2\sigma_T^2$$

$$\mathrm{cov}[y_i, T] = \alpha_i\sigma_T^2$$

$$y_i = L_i + \alpha_i(T - \tau_0)$$

$$L(T, L_1, L_2|\lambda, \tau) = \frac{1}{\sqrt{2\pi}\sigma_T} e^{-(T-\tau)^2/2\sigma_T^2} \prod_{i=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(L_i-\lambda+\alpha_i(\tau-\tau_0))^2/2\sigma_i^2}$$

$$L_i \sim \mathrm{Gauss}(\lambda - \alpha_i(\tau - \tau_0), \sigma_i)$$

$$T \sim \mathrm{Gauss}(\tau, \sigma_T)$$

$$E[L_i] = \lambda - \alpha_i(T - \tau_0), \qquad i = 1, 2$$

$$\mathrm{cov}[\theta_i, \theta_j] = \int \theta_i\theta_j p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta} - \int \theta_i p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta} \int \theta_j p(\boldsymbol{\theta}|x)\, d\boldsymbol{\theta}$$

$$\rho_{xy} = \frac{\mathrm{cov}[x, y]}{\sigma_x\sigma_y}$$

$$\mathrm{cov}[x,y] = E[xy] - E[x]E[y]$$

$$t_\theta = -2\ln\frac{L(\theta)}{L(\hat{\theta})}$$

$$\widehat{V}^{-1}_{ij} \approx -\left.\frac{\partial^2 \ln L}{\partial\theta_i\,\partial\theta_j}\right|_{\theta=\hat{\theta}}$$

$$b = E[\hat{\theta}] - \theta$$

$$V_{ij} = \mathrm{cov}[\hat{\theta}_i, \hat{\theta}_j]$$

$$\hat{\theta} = \operatorname*{argmax}_\theta L(x|\theta)$$

$$p(\theta|x) = \frac{L(x|\theta)\pi(\theta)}{\int L(x|\theta)\pi(\theta)\,d\theta}$$

$$Z = \sqrt{2\left((s+b)\ln\left(1+\frac{s}{b}\right) - s\right)}$$

$$t_{\text{float}} = -2\ln\frac{L(0)}{L(\hat{\mu},\hat{m})} \equiv -2\ln\lambda(0)$$

$$\chi^2(\boldsymbol{\mu}) + \tau\sum_i \left[(\mu_{i+1}-\mu_i) - (\mu_i - \mu_{i-1})^2\right]$$

$$\chi^2 = \sum_{i=1}^{N}\frac{(\nu_{0i}-n_i)^2}{\nu_{0i}}$$

$$\nu_{0i} = \sum_{j=1}^{M} R_{ij}\mu_{0j} + \beta_i$$

$$\Phi(\boldsymbol{\mu}) = \alpha\left(\ln L(\boldsymbol{\mu}) + \ln L_\theta(\theta)\right) + S(\boldsymbol{\mu})$$

$$C_i = 0.1$$

$$n_i = 100$$

$$\beta_i = 0$$

$$\hat{\mu}_i = C_i n_i = 10$$

$$\sigma_{\hat{\mu}_i} = C_i \sqrt{n_i} = 1.0$$

$$E[\hat{\boldsymbol{\mu}}] = R^{-1}(E[\mathbf{n}] - \boldsymbol{\beta}) = \boldsymbol{\mu}$$

$$U_{ij} = \mathrm{cov}[\hat{\mu}_i, \hat{\mu}_j] = \sum_{k,l=1}^{N} (R^{-1})_{ik} (R^{-1})_{jl} \, \mathrm{cov}[n_k, n_l]$$

$$= \sum_{k=1}^{N} (R^{-1})_{ik} (R^{-1})_{jk} \, \nu_k$$

$$\frac{\partial^2 \ln L}{\partial \mu^2} = -\frac{n}{\mu^2}$$

$$\ln L(\boldsymbol{\mu}) = \sum_{i=1}^{N} (n_i \ln \nu_i - \nu_i)$$

$$\hat{\boldsymbol{\nu}} = \mathbf{n}$$

$$\hat{\boldsymbol{\mu}} = R^{-1}(\mathbf{n} - \boldsymbol{\beta})$$

$$P(n_i; \nu_i) = \frac{\nu_i^{n_i}}{n_i!} e^{-\nu_i}$$

$$\boldsymbol{\mu} = R^{-1}(\boldsymbol{\nu} - \boldsymbol{\beta})$$

$$\boldsymbol{\mu} = (\mu_1, \ldots, \mu_M), \quad \mu_{\mathrm{tot}} = \sum_{j=1}^{M} \mu_j$$

$$\mathbf{p} = (p_1, \ldots, p_M) = \boldsymbol{\mu} / \mu_{\mathrm{tot}}$$

$$\boldsymbol{\nu} = (\nu_1, \ldots, \nu_N)$$

$$\varepsilon_j = \sum_{i=1}^{N} R_{ij}$$

$$\boldsymbol{\beta} = (\beta_1, \ldots, \beta_N)$$

$$E[\mathbf{n}] = \boldsymbol{\nu} = R\boldsymbol{\mu} + \boldsymbol{\beta}$$

$$\sum_{i=1}^{N} R_{ij} = \sum_{i=1}^{N} P(\text{observed in bin } i \,|\, \text{true value in bin } j)$$

$$= P(\text{observed anywhere} \,|\, \text{true value in bin } j)$$

$$= \varepsilon_j$$

$$\nu_i = \sum_{j=1}^{M} R_{ij}\mu_j + \beta_i$$

$$\mathbf{n} = (n_1, \ldots, n_N)$$

$$\nu_i = E[n_i] = \sum_{j=1}^{M} R_{ij}\mu_j \,, \quad i = 1, \ldots, N$$

$$R_{ij} = P(\text{observed in bin } i \,|\, \text{true in bin } j)$$

$$f_{\text{meas}}(x) = \int R(x|y) f_{\text{true}}(y) \, dy$$

$$p_j = \int_{\text{bin } j} f(y) \, dy$$

$$\mu_j = \mu_{\text{tot}} p_j$$

$$\pi(b) = \frac{1}{\sqrt{2\pi}\sigma_b} e^{-(b-b_{\text{meas}})^2/2\sigma_b^2}$$

$$p_{\text{float}} \approx p_{\text{fix}} + \langle N(c) \rangle$$

$$c = t_{\text{fix}} = Z_{\text{fix}}^2$$

$$Z_{\text{fix}} = \Phi^{-1}(1 - p_{\text{fix}})$$

$$\mathcal{N} = \frac{\langle N(c) \rangle}{1 - F_{\chi_2^2}(c)}$$

$$\langle N(c) \rangle \approx \langle N(c_0) \rangle e^{-(c-c_0)/2}$$

$$F_{\text{trials}} \equiv \frac{p_{\text{float}}}{p_{\text{fix}}} \approx \sqrt{\frac{\pi}{2}} \frac{\langle N(t_{\text{float,obs}}) \rangle}{1 - F_{\chi_2^2}(t_{\text{float,obs}})} Z_{\text{fix}}$$

$$F_{\text{trials}} \equiv \frac{p_{\text{float}}}{p_{\text{fix}}} \approx \sqrt{\frac{\pi}{2}} \mathcal{N} Z_{\text{fix}}$$

$$P(t_{\text{float}} > t_{\text{float,obs}} | H_0) \leq 1 - F_{\chi^2_1}(t_{\text{float,obs}}) + \langle N(t_{\text{float,obs}}) \rangle$$

$$-2 \ln \lambda(\mu) = -2 \ln(L(\mu)/L(\hat{\mu})) < 1 \text{ i.e., } \ln L(\mu) > \ln L(\hat{\mu}) - \frac{1}{2}$$

$$\frac{P(\mathbf{x}|H_1)}{P(\mathbf{x}|H_0)} \geq k$$

$$x \sim \text{Gauss}(\mu, \sigma)$$

$$\alpha = 1 - \Phi\left(\frac{x_{\text{c}} - \mu_0}{\sigma}\right)$$

$$\text{power} = 1 - \beta = P(x > x_{\text{c}}|\mu) = 1 - \Phi\left(\frac{\mu_0 - \mu}{\sigma} + \Phi^{-1}(1 - \alpha)\right)$$

$$x_{\text{c}} = \mu_0 + \sigma \Phi^{-1}(1 - \alpha)$$

$$L(x, y|\theta, \nu) = L(x|\theta, \nu) L(y|\nu)$$

$$\pi(\nu|y) \propto L(y|\nu) \pi_0(\nu)$$

$$L_{\text{m}}(x|\theta) = \int L(x|\theta, \nu) \pi(\nu) \, d\nu$$

$$\hat{\hat{\nu}}(\theta)$$

$$p_\theta = \int_{q_{\theta,\text{obs}}}^{\infty} f(q_\theta|\theta, \nu) \, dq_\theta$$

$$L(x|\theta) \to L(x|\theta, \nu)$$

$$L(x|\theta) = \theta x$$

$$L(x|\theta) = \theta x + \alpha x^2 + \beta x^3 + \cdots$$

$$p_\theta = \int_{q_{\theta,\text{obs}}}^{\infty} f(q_\theta|\theta) \, dq_\theta$$

$$p(s|n) = \int p(s,b|n)\,db$$

$$p(\mathbf{x}|H_0), p(\mathbf{x}|H_1)$$

$$s_{\rm up} = \tfrac{1}{2} F^{-1}_{\chi^2}(1-\alpha; 2(n+1)) - b$$

α

$s_{\rm up} = -\ln\alpha \approx 3.00$

$$\begin{aligned} P(\mathrm{D}) &= 0.001 \\ P(\text{no D}) &= 0.999 \end{aligned}$$

$$\begin{aligned} P(+|\mathrm{D}) &= 0.98 \\ P(-|\mathrm{D}) &= 0.02 \end{aligned}$$

$$\begin{aligned} P(+|\text{no D}) &= 0.03 \\ P(-|\text{no D}) &= 0.97 \end{aligned}$$

$$\begin{aligned} p(\mathrm{D}|+) &= \frac{P(+|\mathrm{D})P(\mathrm{D})}{P(+|\mathrm{D})P(\mathrm{D}) + P(+|\text{no D})P(\text{no D})} \\ &= \frac{0.98 \times 0.001}{0.98 \times 0.001 + 0.03 \times 0.999} \\ &= 0.032 \end{aligned}$$