Statistical Methods for Particle Physics

Problem sheet: statistical test for discovery

https://www.lip.pt/events/2019/data-science/

School on Data Science
in (Astro)particle Physics
and Cosmology
Braga, 25-27 March, 2019

Glen Cowan
Physics Department
Royal Holloway, University of London
g.cowan@rhul.ac.uk
www.pp.rhul.ac.uk/~cowan

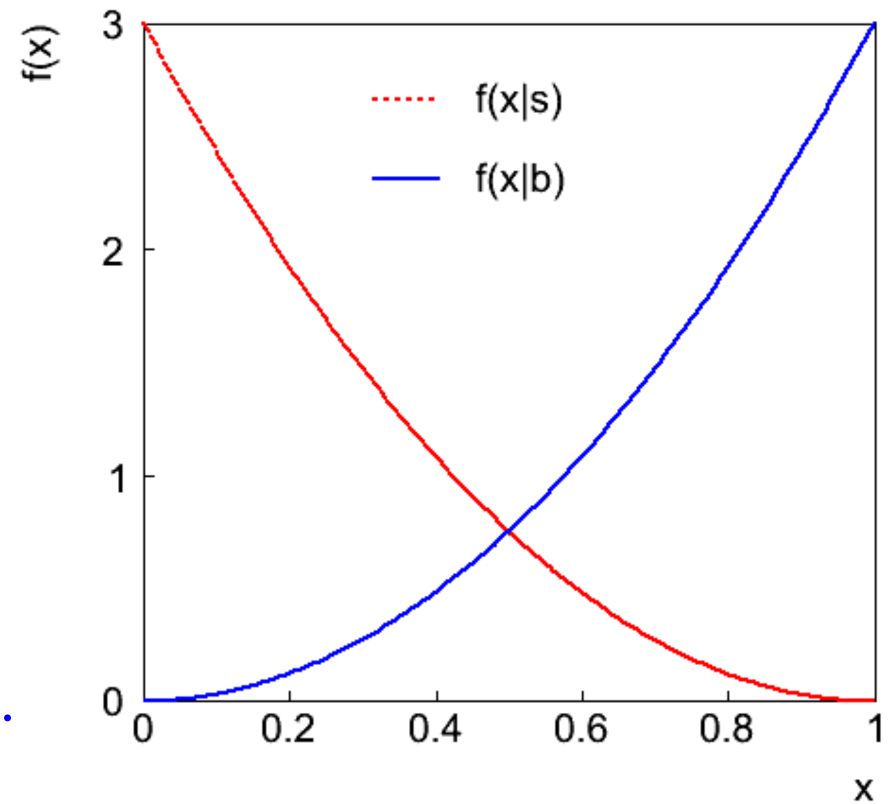
The exercise: discovering a small signal

Materials at **www.pp.rhul.ac.uk/~cowan/stat/braga19/**

Problem concerns searching for a signal such as Dark Matter by counting events. Suppose signal/background events are characterized by a variable x ($0 \leq x \leq 1$):

$$f(x|s) = 3(1-x)^2 \,,$$

$$f(x|b) = 3x^2 \,.$$



As a first step, test the background hypothesis for each event: if $x < x_{\text{cut}}$, reject background hypothesis.

Testing the outcome of the full experiment

In the full experiment we will find n events in the signal region ($x < x_{\rm cut}$), and we can model this with a Poisson distribution:

$$P(n|s,b) = \frac{(s+b)^n}{n!} e^{-(s+b)}$$

Suppose total expected events in $0 \le x \le 1$ are $b_{\rm tot} = 100$, $s_{\rm tot} = 10$; expected in $x < x_{\rm cut}$ are s, b.

Suppose for a given $x_{\rm cut}$, $b = 0.5$ and we observe $n_{\rm obs} = 3$ events. Find the p-value of the hypothesis that $s = 0$:

$$p = P(n \ge n_{\rm obs}|s=0,b) = \sum_{n=n_{\rm obs}}^{\infty} \frac{b^n}{n!} e^{-b} = 1 - \sum_{n=0}^{n_{\rm obs}-1} \frac{b^n}{n!} e^{-b}$$

and the corresponding significance: $Z = \Phi^{-1}(1-p)$

Experimental sensitivity

To characterize the experimental sensitivity we can give the median, assuming s and b both present, of the significance of a test of $s = 0$. For $s \ll b$ this can be approximated by

$$\mathrm{med}[Z_b|s+b] = s/\sqrt{b}$$

A better approximation is:

$$\mathrm{med}[Z_b|s+b] = \sqrt{2\left((s+b)\ln\left(1+\frac{s}{b}\right)-s\right)}$$

Try this for $x_{\mathrm{cut}} = 0.1$ and if you have time, write a small program to maximize the median Z with respect to x_{cut}.

We will discuss these formulae in later lectures, including methods for treating uncertainty in b.

Optional exercise: using the x values

Instead of just counting events with $x < x_{cut}$, we can define a statistic that takes into account all the values of x. I.e. the data are: $n, x_1, .., x_n$. Later we will discuss ways of doing this with the likelihood ratio L_{s+b}/L_b, which leads to the statistic

$$q = -2 \sum_{i=1}^{n} \left[1 + \frac{s_{\rm tot}}{b_{\rm tot}} \frac{f(x_i|s)}{f(x_i|b)} \right]$$

Using **`www.pp.rhul.ac.uk/~cowan/stat/invisibles/mc/invisibleMC.cc`** find the distribution of this statistic under the "b" and "$s+b$" hypotheses.

From these find the median, assuming the $s+b$ hypothesis, of the significance of the b (i.e., $s = 0$) hypothesis. Compare with result from the experiment based only on counting n events.