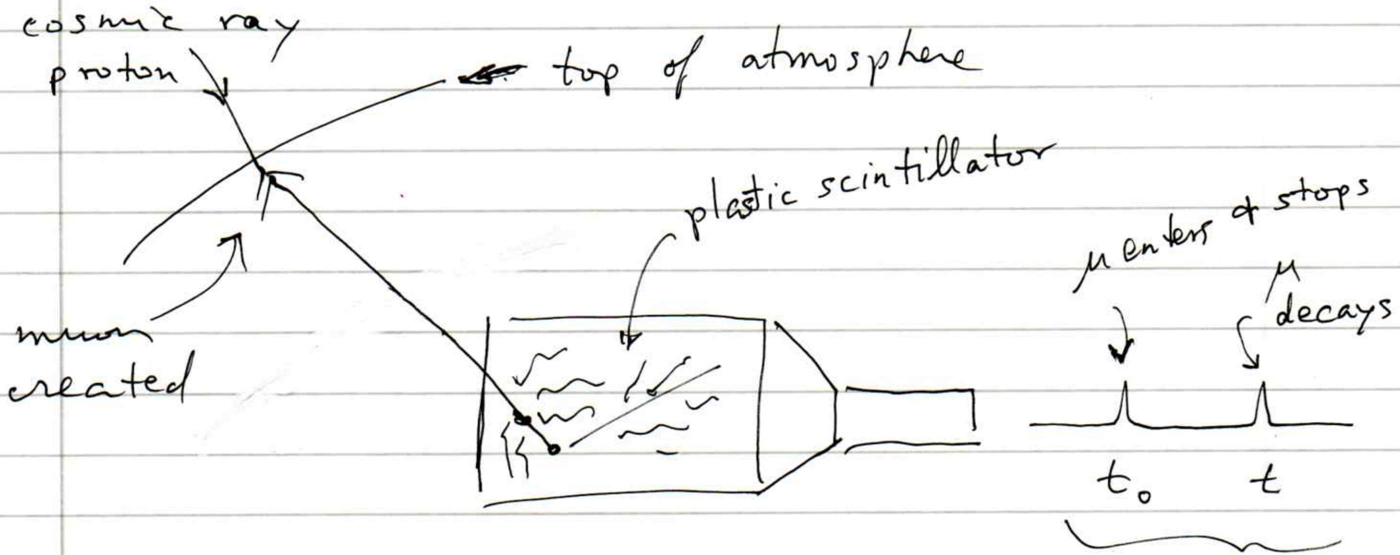
GLEN COWAN
RHUL PHYSICS

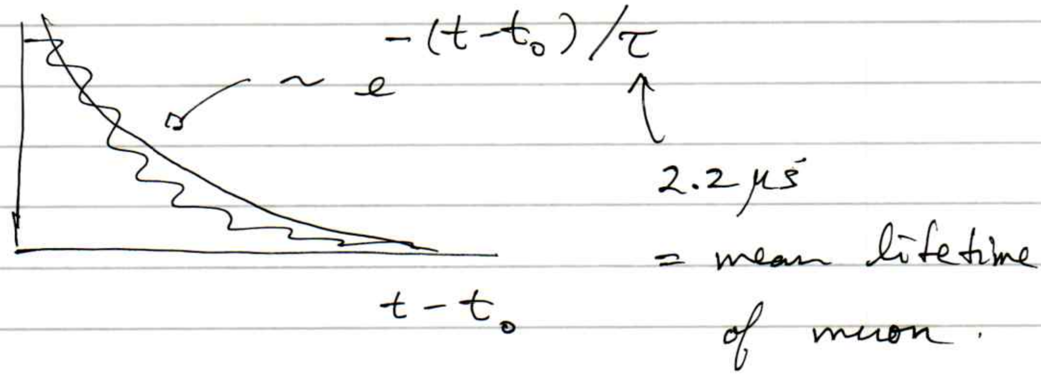
PH4515 Statistical Data Analysis

Notes from online session #3 (19 Oct 20)

Memorylessness of exponential:

$$f(t - t_0 \mid t > t_0) = f(t)$$





What about the time the muon lived before t_0?

Nothing to do with relativity. Because of memorylessness of exponential, muon is as "young and healthy" at t_0 as it was when it was born: $f(t-t_0 \mid t>t_0) = f(t) = \frac{1}{\tau} e^{-t/\tau}$.

Consider $x, y \sim f(x,y)$ & consider a function $a(x,y)$

$$E\left[a(x,y)\right] = \iint a(x,y) \underbrace{f(x,y)}_{f(x|y)\, f_y(y)} \, dx \, dy$$

$$= \int dy \, f_y(y) \underbrace{\int dx \, a(x,y) \, f(x|y)}_{E_x\left[a(x,y) \mid y\right]}$$

$$= E_y\left[E_x\left[a(x,y) \mid y\right]\right]$$

or similar proof for discrete var.

Example: $a = \sum_{i=1}^{n} x_i = a(n, \vec{x})$

$n \sim \text{Poisson}(\nu)$

$x_i \sim \text{Gauss}(\mu, \sigma)$ for all i

$$E[a] = \sum_{n=0}^{\infty} \underbrace{P(n|\nu)}_{\frac{\nu^n e^{-\nu}}{n!}} \int f(\vec{x}) \sum_{i=1}^{n} x_i \, d\vec{x}$$

$$= E_n\left[E_x\left[\sum_{i=1}^{n} x_i \mid n \right]\right]$$

$$= E_n\left[\sum_{i=1}^{n} \underbrace{E[x_i]}_{\mu} \right] = E_n\left[n\mu\right] = \nu\mu$$

Log-normal & variable transformation

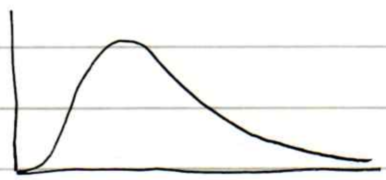
Gauss : $f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$

Let $y = e^x$. ~~so~~ find pdf of y

$x = \ln y \qquad \frac{dx}{dy} = \frac{1}{y}$

$$g(y) = f(x(y))\left|\frac{dx}{dy}\right| = \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma y}\exp\left[-\frac{(\ln y-\mu)^2}{2\sigma^2}\right]}_{\text{"log-normal pdf}}$$

(i.e. $\ln y \sim$ Gauss)



$y \rightarrow (y > 0)$

μ, σ^2 are mean, var. of Gaussian x, not the log-normal y. Can find

$E[y] = \exp\left[\mu + \frac{\sigma^2}{2}\right]$, - - -

$x = \sum_{i=1}^{\text{many}} u_i \xrightarrow{\text{CLT}} x \sim$ Gauss $\qquad$ CLT → Central Limit Theorem

$y = e^x = \exp\left[\sum_i u_i\right] = \prod_i e^{u_i} \xrightarrow{\text{CLT}}$ log-normal

Sum of many terms $\xrightarrow{\text{CLT}}$ Gauss

Product of many factors $\xrightarrow{\text{CLT}}$ log-normal

Example of MC transformation method

pdf. $f(x) = \frac{1}{\pi} \frac{1}{1+x^2}$ Cauchy

cumulative dist. $F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+x'^2)} dx'$

$$\Rightarrow F(x) = \frac{1}{\pi} \tan^{-1} x' \Big|_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left(\tan^{-1} x + \frac{\pi}{2}\right)$$

$\overset{\text{set}}{=} r$ & solve for x

$$x(r) = \tan\left[\pi\left(r - \tfrac{1}{2}\right)\right]$$

$\Rightarrow$ if $r_1, r_2, \ldots$ indep. & $U[0,1]$

then $x_i = x(r_i)$ indep. & $\sim f(x) = \frac{1}{\pi}\frac{1}{1+x^2}$

Code: cauchyMC.py

" .ipynb.