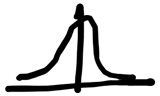


$$x \sim f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

$$y \sim f_y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}$$

$$f_x(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

$$f(x, y) = f_x(x)\, f_y(y) \longleftarrow \text{independent}$$

$$\underline{Z} = \frac{x}{y}, \quad u = x \quad \Rightarrow \quad x = u, \quad y = \frac{u}{z}$$

$$g(u,z) = |J|\, f(x(u,z), y(u,z))$$

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial z} \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ \frac{1}{z} & -\frac{u}{z^2} \end{vmatrix} = -\frac{u}{z^2}$$

$$= \frac{|u|}{z^2} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} \cdot \frac{1}{\sqrt{2\pi}} e^{-u^2/2z^2}$$

$$= \frac{|u|}{2\pi z^2} \exp\left[-\frac{u^2}{2}\left(\frac{z^2+1}{z^2}\right)\right] \qquad \begin{array}{l} -\infty < u < \infty \\ -\infty < z < \infty \end{array}$$

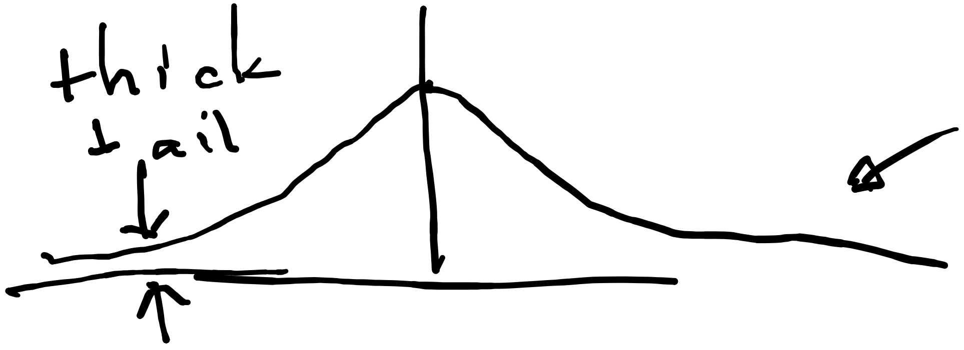
$\Rightarrow$ does not factorize $\rightarrow$ not indep.

$$g_z(z) = \int_{-\infty}^{\infty} g(u,z)\,du$$ —— complicated.

$$= 2\int_0^{\infty} \text{——}\ du$$

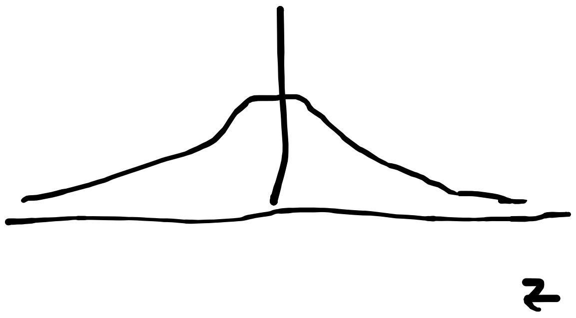
$$= \frac{1}{\pi}\,\frac{1}{1+z^2}$$

Cauchy, Lorentzian, Breit-Wigner.



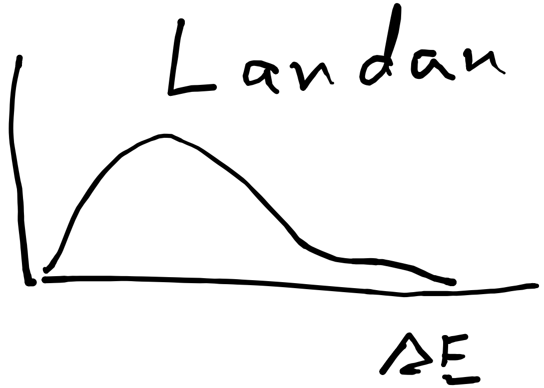
$$V[z] = E[z^2] - \cancel{(E[z])^2}$$

$$\underline{E[z]} = \int_{-\infty}^{\infty} z \, \frac{1}{\pi} \frac{1}{1+z^2} \, dz = 0$$



$$= \lim_{a,b \to \infty} \int_{-a}^{0} \sim \, dz + \int_{0}^{b} \sim \, dz \longrightarrow 0$$

$$E[z^2] = \int_{-\infty}^{\infty} z^2 \, \frac{1}{\pi} \frac{1}{1+z^2} \, dz \longrightarrow \infty$$



$$g(u \mid \underline{z}) = \frac{g(u,z)}{g_z(z)}$$

$$= \frac{|u|}{2\pi z^2} \exp\left[- \frac{u^2(1+z^2)}{z^2} \right] \times \pi(1+z^2)$$

$\Rightarrow$ depends on z $\Rightarrow$ u, z not indep.

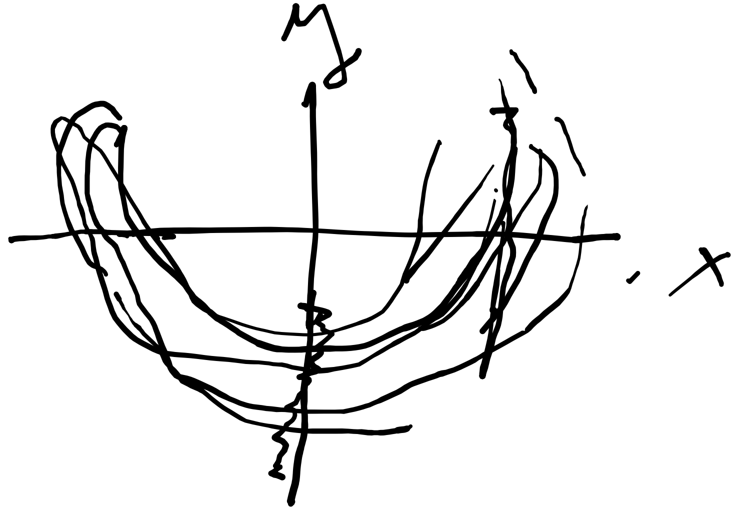
$$g(z \mid u) = \frac{g(u,z)}{g_u(u)}$$

$$= \frac{g(u \mid z)\, g_z(z)}{g_u(u)}$$

$$g_u(u) = \int g(u,z)\, dz$$

$$\frac{1}{\sqrt{2\pi}} e^{-u^2/2}$$

$$u = x$$

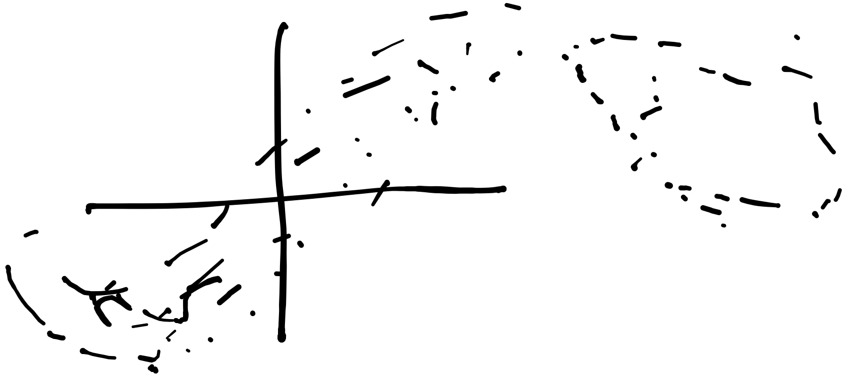


Are x, y corr. ?

(no tilt)

No

$$\boxed{E[xy] = E[x]E[y]}$$



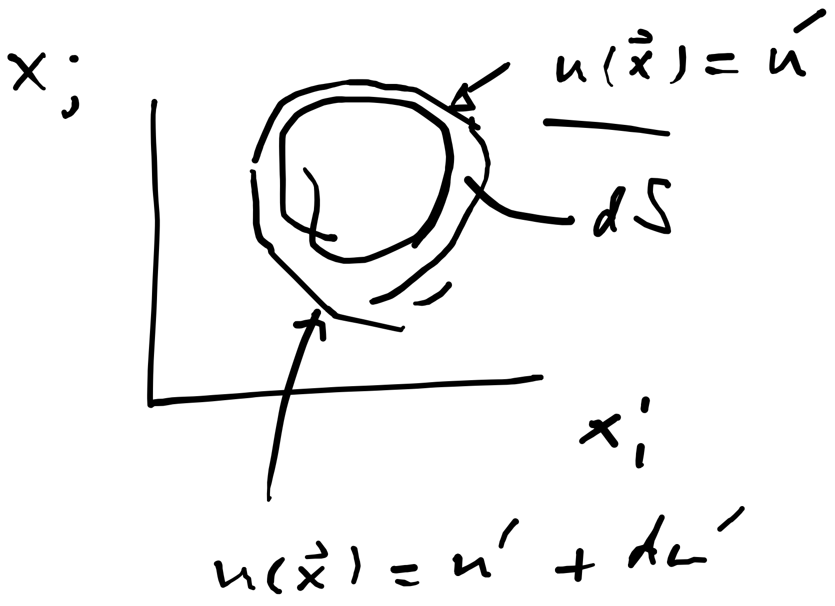
$\Rightarrow$ $f(y|x)$ depends on x

$\Rightarrow$ not indep.

$E[x^\alpha y^\beta \ldots]$

$u(\vec{x})$

$$g(u')\,du' = \iint_{dS} f(\vec{x})\, d\vec{x}$$



$$g(u') = \int \cdots \int f(\vec{x})\, \delta(u' - u(\vec{x}))\, d\vec{x}$$