Week 6 Discussion Session Notes

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Problem Sheet 3

1 a) $f(x;\xi) = \frac{1}{\xi} e^{-x/\xi}$ $(x \geq 0)$

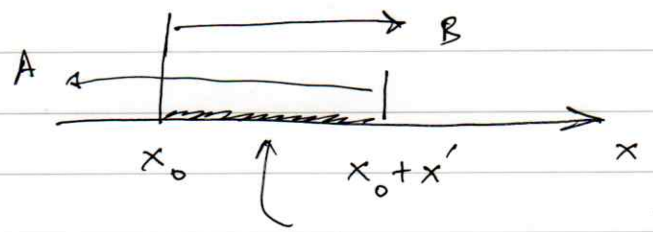
$$F(x) = \int_0^x \frac{1}{\xi} e^{-x'/\xi}\, dx'$$

$$= -e^{-x'/\xi}\Big|_0^x = 1 - e^{-x/\xi}$$

1 b) Find $P(x < x_0 + x' \mid x > x_0)$ and show equals $P(x < x')$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

For $P(\underbrace{x < x_0 + x'}_{A} \mid \underbrace{x > x_0}_{B})$



$A \cap B = x_0 < x < x_0 + x'$

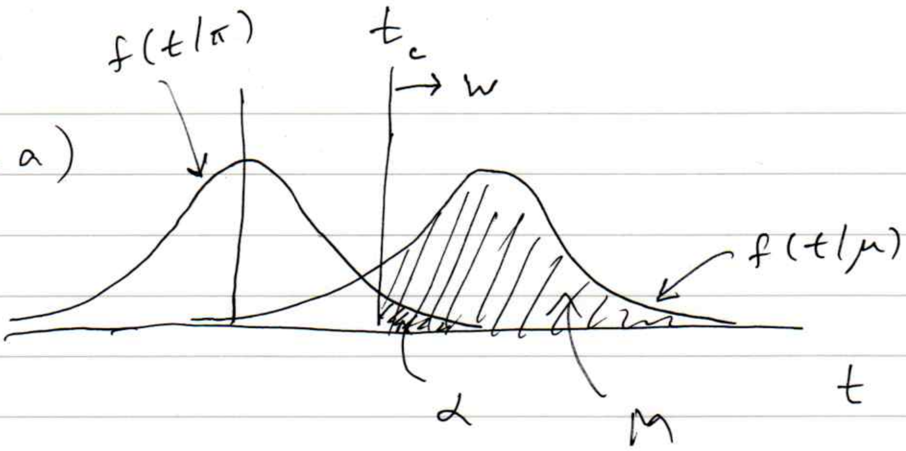
$$\Rightarrow P(x < x_0 + x' \mid x > x_0) = \frac{P(x_0 < x < x_0 + x')}{P(x > x_0)}$$

$$= \frac{\int_{x_0}^{x_0+x'} \frac{1}{\xi} e^{-x/\xi}\, dx}{\int_{x_0}^{\infty} \frac{1}{\xi} e^{-x/\xi}\, dx} = \frac{F(x_0 + x') - F(x_0)}{F(\infty) - F(x_0)}$$

$$= \frac{1 - e^{-(x_0 + x')/\xi} - \left[1 - e^{-x_0/\xi}\right]}{1 - (1 - e^{-x_0/\xi})}$$

$$= 1 - e^{-x'/\xi} = F(x') = P(x \leq x')$$

2 a)



want $\alpha = 0.05$ for test of $H_0 : \pi$

using critical region $t > t_c$

$$\Rightarrow \alpha = P(t > t_c | \pi) = \int_{t_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_\pi} e^{-(t-\mu_\pi)^2/2\sigma_\pi^2} dt$$

let $x = \frac{t - \mu_\pi}{\sigma_\pi}$ $\quad dx = \frac{dt}{\sigma_\pi}$

$$= \int_{(t_c - \mu_\pi)/\sigma_\pi}^{\infty} \frac{1}{\sqrt{2\pi}\,\cancel{\sigma_\pi}} e^{-x^2/2} \cancel{\sigma_\pi}\, dx$$

$$= 1 - \Phi\left(\frac{t_c - \mu_\pi}{\sigma_\pi}\right)$$

$$\Rightarrow t_c = \mu_\pi + \sigma_\pi \Phi^{-1}(1 - \alpha)$$

TMath::NormQuantile

or scipy.stats.norm.ppf

using $\mu_\pi = 0$, $\sigma_\pi = 1$, $\alpha = 0.05$

$$\Rightarrow \underline{t_c = 1.64}$$

2b) Power $M = \int_{t_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_\mu} e^{-(t-\mu_\mu)^2/2\sigma_\mu^2}\, dt$

$= 1 - \Phi\left(\frac{t_c - \mu_\mu}{\sigma_\mu}\right)$ or as in (a)

$= 0.639$ TMath::Freq for Φ

scipy.stats.norm.cdf or sf

2c) Purity $= P(\mu \mid t > t_c)$

$$= \frac{P(t > t_c \mid \mu)\,\pi_\mu}{P(t>t_c \mid \mu)\,\pi_\mu + P(t > t_c \mid \pi)\,\pi_\pi}$$

Prior probabilities are $\pi_\pi = 0.99$, $\pi_\mu = 0.01$

From (a), (b) we have

$P(t > t_c \mid \pi) = \alpha = 0.05$

$P(t > t_c \mid \mu) = M = 0.639$

$\Rightarrow$ Purity $= P(\mu \mid t > t_c) = \frac{0.639 \times 0.01}{0.639 \cdot 0.01 + 0.05 \times 0.99}$

$= 0.114$