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Discussion Notes Week 4

Problem Sheet 1:

$\pi(e) = 0.01$, $\pi(\pi) = 0.99$ } prior probs.

data outcomes: A, B or C

$P(A|e) = 0.01$ $P(A|\pi) = 0.980$

$P(B|e) = 0.1$ $P(B|\pi) = 0.019$

$P(C|e) = 0.89$ $P(C|\pi) = 0.001$

a) $$P(\pi|A) = \frac{P(A|\pi)\,\pi(\pi)}{P(A|\pi)\,\pi(\pi) + P(A|e)\,\pi(e)}$$

$$= \frac{0.980 \times 0.99}{0.980 \times 0.99 + 0.01 \times 0.01}$$

$$= 0.9999$$

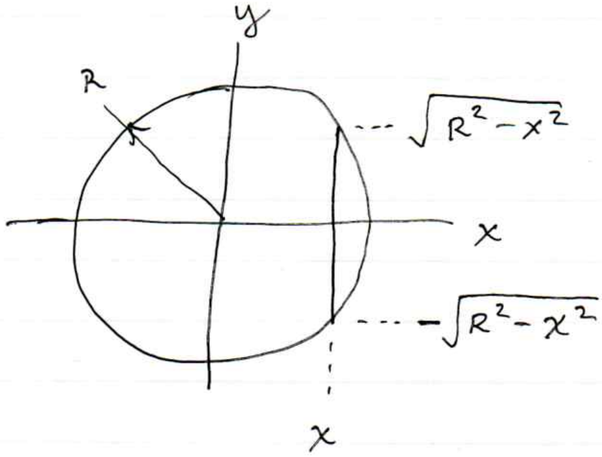
b) $$P(e|C) = \frac{P(C|e)\,\pi(e)}{P(C|e)\,\pi(e) + P(C|\pi)\,\pi(\pi)}$$

$$= \frac{0.89 \times 0.01}{0.89 \times 0.01 + 0.001 \times 0.99}$$

$$= 0.8999$$

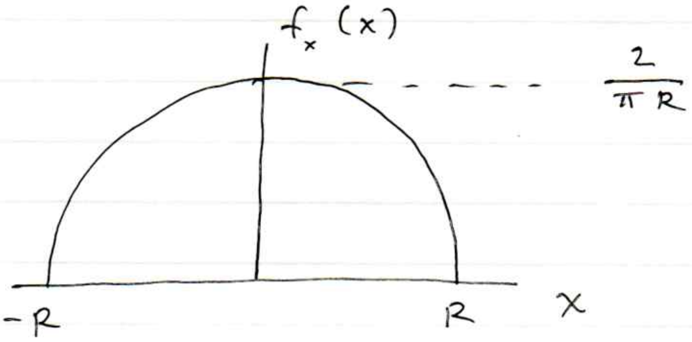
2) $$f(x,y) = \frac{1}{\pi R^2}, \quad x^2 + y^2 \leq R^2$$

a) $$f_x(x) = \int f(x,y)\, dy$$



$$= \int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}} \frac{1}{\pi R^2}\, dy$$

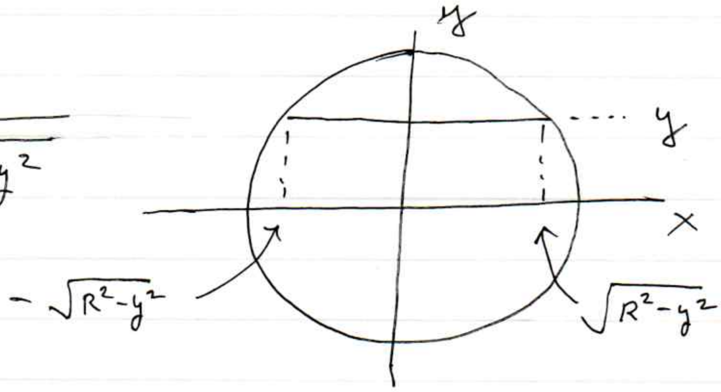
$$= \frac{2\sqrt{R^2-x^2}}{\pi R^2}, \quad -R \leq x \leq R$$



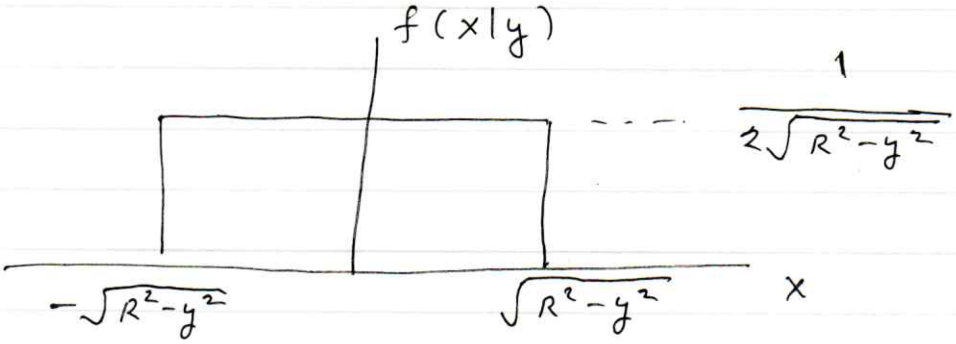
By symmetry, $f_y(y) = \dfrac{2\sqrt{R^2-y^2}}{\pi R^2}, \quad -R \leq y \leq R$

2b) Conditional pdf $f(x|y) = \dfrac{f(x,y)}{f_y(y)}$

$$\Rightarrow f(x|y) = \frac{\frac{1}{\pi R^2}}{\frac{2}{\pi R^2}\sqrt{R^2-y^2}}$$



$$= \frac{1}{2\sqrt{R^2-y^2}}, \quad -\sqrt{R^2-y^2} \le x \le \sqrt{R^2-y^2}$$



& by symmetry

$$f(y|x) = \frac{1}{2\sqrt{R^2-x^2}}, \quad -\sqrt{R^2-x^2} \le x \le \sqrt{R^2-x^2}$$

2c) By Bayes' thm, $f(x|y) = \dfrac{f(y|x)\, f_x(x)}{f_y(y)}$

$$\frac{1}{2\sqrt{R^2-y^2}} \stackrel{?}{=} \frac{\dfrac{1}{\cancel{2\sqrt{R^2-x^2}}} \times \cancel{\dfrac{2}{\pi R^2}\sqrt{R^2-x^2}}}{\cancel{\dfrac{2}{\pi R^2}}\sqrt{R^2-y^2}}$$

$$= \frac{1}{2\sqrt{R^2-y^2}} \quad \checkmark$$

2d) x & y <u>not</u> independent

because $f(x|y)$ depends on y

$f(y|x)$ " on x

From Week 4 extra slides

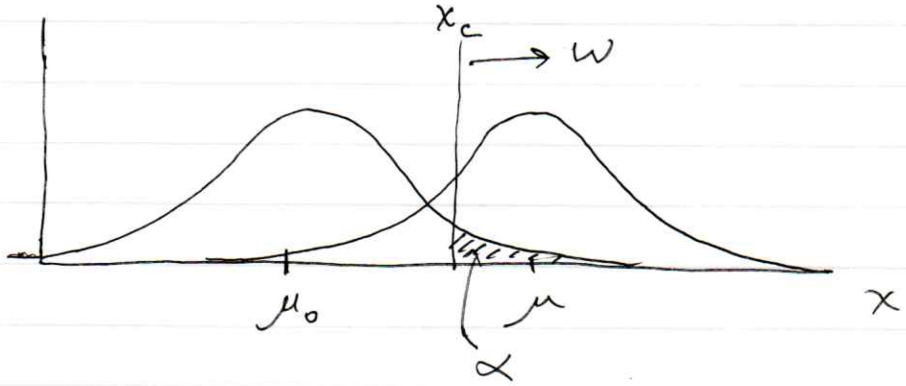
$$x \sim \text{Gauss}(\mu, \sigma)$$

want to test (μ)

known (σ).

$H_0 : \mu = \mu_0$ vs. $H_1 : \mu > \mu_0$

Take $w = \{ x : x \geq x_c \}$



$$\alpha = P(x \geq x_c \mid \mu_0)$$

$$= \int_{x_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu_0)^2/2\sigma^2} dx$$

let $y = \frac{x - \mu_0}{\sigma}$

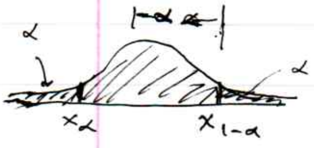
$$= \int_{\frac{x_c - \mu_0}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-y^2/2} \cdot \cancel{\sigma}\, dy$$

$$= 1 - \Phi\left(\frac{x_c - \mu_0}{\sigma}\right)$$

Standard Gauss. cumul. dist.

$$\Rightarrow x_c = \mu_0 + \sigma \Phi^{-1}(1 - \alpha)$$

Std. Gauss. quantile



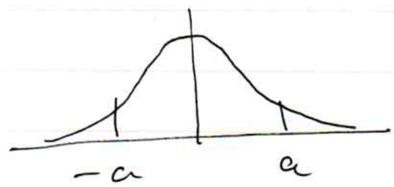
$$\Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$$

Power $M = P(x \geq x_c \mid \mu)$

$$= \int_{x_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \, dx$$

$$= 1 - \Phi\left(\frac{x_c - \mu}{\sigma}\right)$$

$$= \Phi\left(\frac{\mu - x_c}{\sigma}\right)$$



$\Phi(-a) = 1 - \Phi(a)$

$$= \Phi\left(\frac{\mu - \mu_0 - \sigma\,\Phi^{-1}(1-\alpha)}{\sigma}\right)$$

$$= \Phi\left(\frac{\mu - \mu_0}{\sigma} + \Phi^{-1}(\alpha)\right)$$

$\rightarrow$ see plots on week 4 extra slides.