GDC
PH4515

24.10.22

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# Discussion Notes – week 4

## Problem sheet 1

1) Use Bayes' theorem

$$P(A|B) = \frac{P(B|A)P(A)}{\sum_i P(B|A_i)P(A_i)}$$

a) $$P(\gamma|1) = \frac{P(1|\gamma)P(\gamma)}{P(1|\gamma)P(\gamma) + P(1|e)P(e)}$$

$$= \frac{0.001 \times 0.9999}{0.001 \times 0.999 + 0.01 \times 10^{-4}} \quad \leftarrow 1-10^{-4}$$

$$= \underline{0.999} \ (000999)$$

b) $$P(e|2) = \frac{P(2|e)P(e)}{P(2|e)P(e) + P(2|\gamma)P(\gamma)}$$

$$= \frac{0.989 \times 0.0001}{0.989 \times 0.0001 + 10^{-5} \cdot (1-0.0001)}$$

$$= 0.\underline{908}18$$

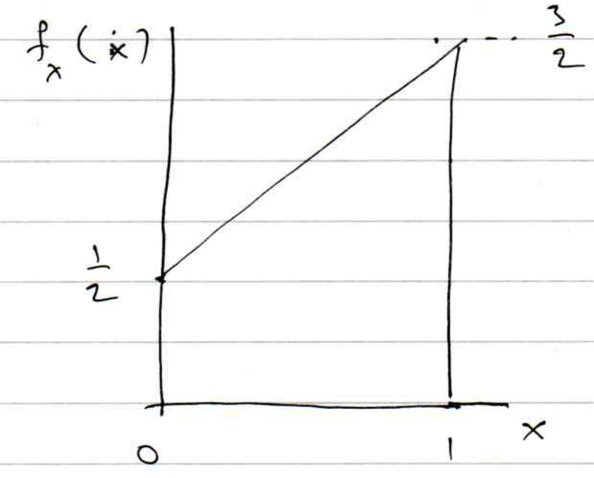
2) $$f(x,y) = \begin{cases} x + y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

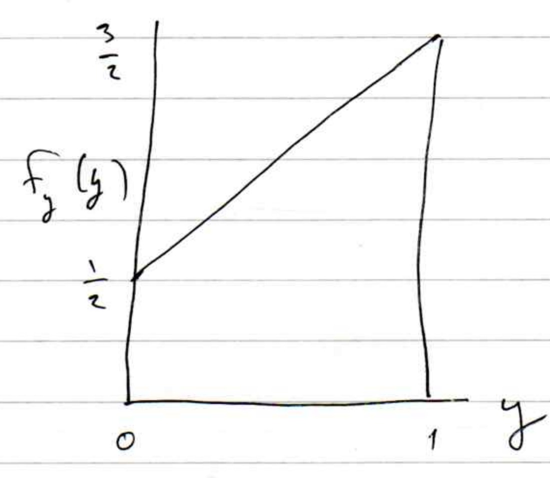
a) $$f_x(x) = \int f(x,y)\,dy$$

$$= \int_0^1 (x+y)\,dy$$

$$= \left(xy + \frac{y^2}{2}\right)\Big|_0^1 = \frac{1}{2} + x\,, \quad 0 \le x \le 1$$

By symmetry, $f_y(y) = \frac{1}{2} + y\,, \quad 0 \le y \le 1$




$$f(x,y) = x + y$$

$$\ne f_x(x)\,f_y(y) = \left(\frac{1}{2} + x\right)\left(\frac{1}{2} + y\right)$$

$\Rightarrow$ $x, y$ <u>not</u> independent.

2b) $$f(x \mid y) = \frac{f(x,y)}{f_y(y)}$$

$$= \frac{x+y}{\frac{1}{2}+y}, \quad 0 \le x \le 1$$

By symmetry,

$$f(y \mid x) = \frac{x+y}{\frac{1}{2}+x}, \quad 0 \le y \le 1$$

Bayes' thm. says

$$f(x \mid y) = \frac{f(y \mid x)\, f_x(x)}{f_y(y)}$$

Check:

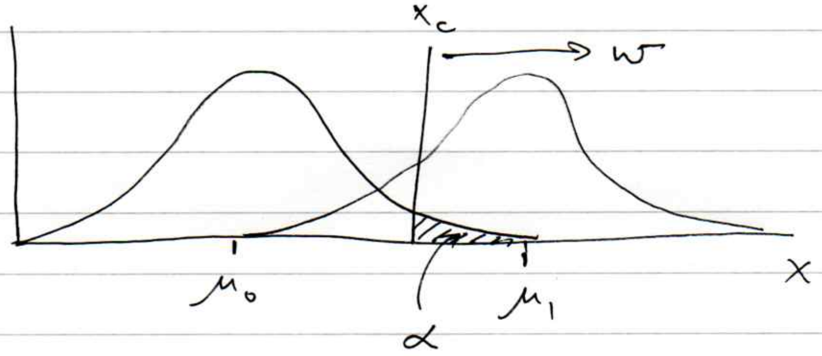
$$\frac{x+y}{\frac{1}{2}+y} \stackrel{?}{=} \frac{\frac{x+y}{\cancel{\frac{1}{2}+x}} \cdot \cancel{\left(\frac{1}{2}+x\right)}}{\frac{1}{2}+y} \quad \checkmark$$

From Week 4 <u>extra slides</u>

$$x \sim \text{Gauss}(\underset{\text{want to test}}{\mu}, \underset{\text{known}}{\sigma})$$

$$H_0 : \mu = \mu_0 \quad \text{vs.} \quad H_1 : \mu > \mu_0$$

Take $w = \{ x : x \geq x_c \}$




$$\alpha = P(x \geq x_c \mid \mu_0)$$

$$= \int_{x_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu_0)^2/2\sigma^2} \, dx \, , \qquad \text{let } y = \frac{x-\mu_0}{\sigma}$$

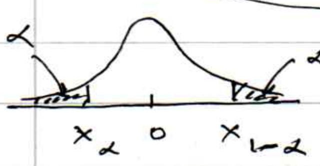
$$= \int_{\frac{x_c-\mu_0}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\,\cancel{\sigma}} e^{-y^2/2} \cdot \cancel{\sigma}\, dy$$

$$= 1 - \Phi\left(\frac{x_c - \mu_0}{\sigma}\right)$$

$\Phi$ ← Standard Gauss cumul. dist.

$$\Rightarrow x_c = \mu_0 - \sigma \Phi^{-1}(\alpha) = \mu_0 + \sigma \Phi^{-1}(1-\alpha)$$

$\Phi^{-1}$ ← std. Gauss quantile.



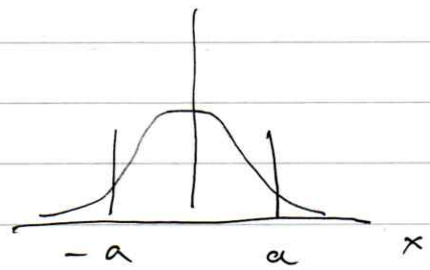

$$\Rightarrow \Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$$

Power $M(\mu) = P(x \ge x_c \mid \mu)$

$$= \int_{x_c}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2} \, dx$$

$$= 1 - \Phi\left(\frac{x_c - \mu}{\sigma}\right)$$

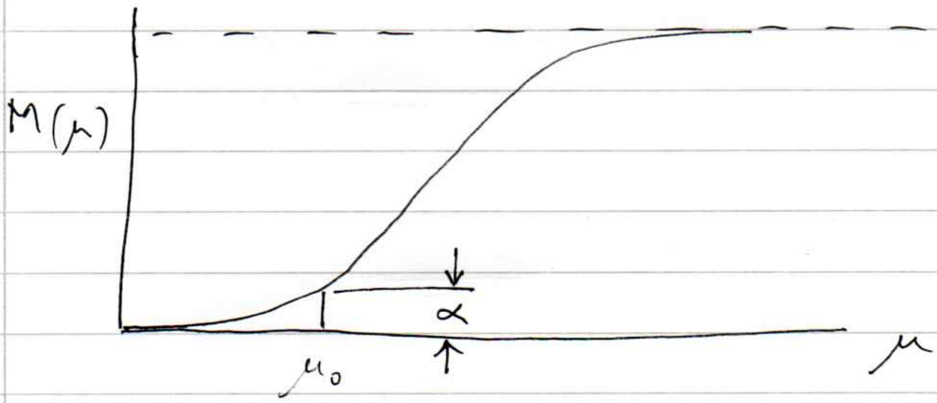
$$= \Phi\left(\frac{\mu - x_c}{\sigma}\right)$$




$$\Phi(-a) = 1 - \Phi(a)$$

$$= \Phi\left(\frac{\mu - \mu_0 - \sigma \Phi^{-1}(1-\alpha)}{\sigma}\right)$$

$$= \Phi\left(\frac{\mu - \mu_0}{\sigma} + \Phi^{-1}(\alpha)\right)$$



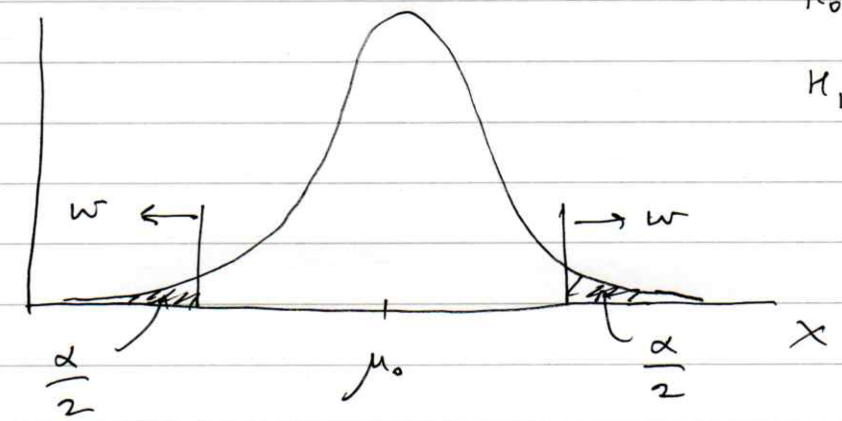
"Two-sided" test:

We might consider $\mu < \mu_0$ and $\mu > \mu_0$ as equally relevant alternatives.

$\Rightarrow$ choose two-sided critical region:

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$




Two-sided $w$ gives (some) power for both $\mu < \mu_0$ and $\mu > \mu_0$,

but less power for $\mu > \mu_0$ than the original one-sided test.