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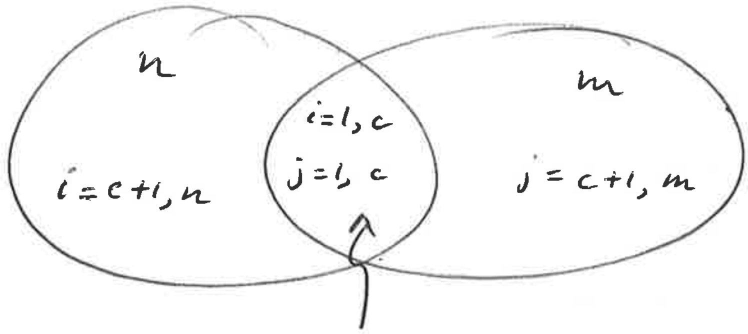
Covariance when using overlapping data



c events in common

$E[x_i] = \mu$,

$V[x_i] = \sigma^2$

all i

Suppose $y_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$, $y_2 = \frac{1}{m} \sum_{j=1}^{m} x_j$

$$\text{cov}[y_1, y_2] = \frac{1}{nm} \text{cov}\left[\sum_{i=1}^{n} x_i , \sum_{j=1}^{m} x_j \right]$$

$$= \frac{1}{nm} \text{cov}\left[\sum_{i=1}^{c} x_i + \sum_{i=c+1}^{n} x_i , \sum_{j=1}^{c} x_j + \sum_{j=c+1}^{m} x_j \right]$$

only this term $\neq 0$

$$= \frac{1}{nm} \sum_{i,j=1}^{c} \text{cov}[x_i, x_j] = \frac{1}{nm} \sum_{i,j=1}^{c} \delta_{ij} \sigma^2$$

$$= \frac{c\sigma^2}{nm}$$

$$\Rightarrow \rho = \frac{\text{cov}[y_1, y_2]}{\left(\frac{\sigma}{\sqrt{n}}\right)\left(\frac{\sigma}{\sqrt{m}}\right)} = \frac{c}{\sqrt{nm}}$$

note: $\sigma_1^2 = \frac{\sigma^2}{n}$, $\sigma_2^2 = \frac{\sigma^2}{m}$

$$\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{m}{n}}$$

$$\frac{\rho}{(\sigma_1/\sigma_2)} = \frac{c}{\sqrt{nm}} \cdot \sqrt{\frac{n}{m}}$$

$$= \frac{c}{m} \leq 1$$