GDC 13.9.18

Full $\ln L$ & $\ln L'$

$$L(\vec{\mu}, \vec{\theta}, \vec{\sigma}_u^2) = P(\vec{y}|\vec{\mu},\vec{\theta}) \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi\,\sigma_{u_i}^2}} e^{-(u_i-\theta_i)^2/2\sigma_{u_i}^2}$$

$$\times \frac{\beta_i^{\alpha_i}}{\Gamma(\alpha_i)} \nu_i^{\alpha_i - 1} e^{-\beta_i \nu_i} , \qquad \alpha_i = \frac{1}{4r_i^2}$$

$$\beta_i = \frac{\alpha_i}{\sigma_{u_i}^2} = \frac{1}{4r_i^2\sigma_{u_i}^2}$$

$$\Rightarrow \ln L(\vec{\mu},\vec{\theta},\vec{\sigma}_u^2) = \ln P(\vec{y}|\vec{\mu},\vec{\theta})$$

$$- \frac{1}{2}\sum_{i=1}^{N}\left[\ln \sigma_{u_i}^2 + \frac{(u_i-\theta_i)^2}{\sigma_{u_i}^2}\right]$$

$$+ \sum_{i=1}^{N}\left[\alpha_i \ln\beta_i - \ln\Gamma(\alpha_i) + (\alpha_i - 1)\ln\nu_i - \beta_i\nu_i - \frac{1}{2}\ln(2\pi)\right]$$

Use $\hat{\sigma}_{u_i}^2 = \dfrac{\nu_i + 2r_i^2(u_i-\theta_i)^2}{1+2r_i^2}$

$$\ln L'(\vec{\mu},\vec{\theta}) = \ln P(\vec{y}|\vec{\mu},\vec{\theta}) - \frac{1}{2}\sum_{i=1}^{N}\left[\ln\left(\frac{\nu_i + 2r_i^2(u_i-\theta_i)^2}{1+2r_i^2}\right)\right.$$

$$\left. + \frac{(1+2r_i^2)(u_i-\theta_i)^2}{\nu_i + 2r_i^2(u_i-\theta_i)^2}\right]$$

$$+ \sum_{i=1}^{N}\left[\frac{1}{4r_i^2}\ln\frac{1+2r_i^2}{4r_i^2(\nu_i + 2r_i^2(u_i-\theta_i)^2)} - \ln\Gamma\left(\frac{1}{4r_i^2}\right)\right.$$

$$\left. + \left(\frac{1}{4r_i^2} - 1\right)\ln\nu_i - \frac{\nu_i(1+2r_i^2)}{4r_i^2(\nu_i + 2r_i^2(u_i-\theta_i)^2)} - \frac{1}{2}\ln(2\pi)\right]$$

$$\ln L'(\vec{\mu}, \vec{\theta}) = \ln P(\vec{y} \mid \vec{\mu}, \vec{\theta})$$

$$- \frac{1}{2} \sum_{i=1}^{N_1} \left[\frac{(1+2r_i^2)(u_i - \theta_i)^2 + \nu_i (1+2r_i^2)/2r_i^2}{\nu_i + 2r_i^2 (u_i - \theta_i)^2} \right.$$

$$+ \ln \left(\frac{\nu_i + 2r_i^2 (u_i - \theta_i)^2}{1+2r_i^2} \right)$$

$$+ \frac{1}{2r_i^2} \left(\ln \frac{\nu_i + 2r_i^2 (u_i - \theta_i)^2}{1+2r_i^2} + \ln 4r_i^2 \right)$$

$$\left. + 2 \ln \Gamma\left(\frac{1}{4r_i^2}\right) - 2\left(\frac{1}{4r_i^2} - 1\right) \ln \nu_i + \ln 2\pi \right]$$

$$= \ln P(\vec{y} \mid \vec{\mu}, \vec{\theta})$$

$$- \frac{1}{2} \sum_{i=1}^{N_1} \left[\frac{1+2r_i^2}{2r_i^2} \cdot \cancel{\frac{\nu_i + 2r_i^2 (u_i - \theta_i)^2}{\nu_i + 2r_i^2 (u_i - \theta_i)^2}} \right.$$

$$+ \left(1 + \frac{1}{2r_i^2}\right) \ln \left(\frac{\nu_i + 2r_i^2 (u_i - \theta_i)^2}{1+2r_i^2} \times \frac{\nu_i}{\nu_i} \right)$$

$$\left. + \frac{1}{2r_i^2} \ln 4r_i^2 + 2 \ln \Gamma\left(\frac{1}{4r_i^2}\right) - \frac{\ln \nu_i}{2r_i^2} + 2 \ln \nu_i + \ln 2\pi \right]$$

$$= \ln P(\vec{y} \mid \vec{\mu}, \vec{\theta}) - \frac{1}{2} \sum_{i=1}^{N_1} \left[\left(1 + \frac{1}{2r_i^2}\right) \ln \left(1 + 2r_i^2 \frac{(u_i - \theta_i)^2}{\nu_i} \right) \right.$$

$$+ \left(1 + \frac{1}{2r_i^2}\right) \left(1 + \ln \frac{\nu_i}{1+2r_i^2} \right) + \frac{1}{2r_i^2} \ln 4r_i^2 + 2 \ln \Gamma\left(\frac{1}{4r_i^2}\right) - \frac{\ln \nu_i}{2r_i^2}$$

$$\left. + 2 \ln \nu_i + \ln 2\pi \right]$$

$$\ln L'(\vec{\mu}, \vec{\theta}) = \ln P(\vec{y} \,|\, \vec{\mu}, \vec{\theta})$$

$$= -\frac{1}{2} \sum_{i=1}^{N'} \left(1 + \frac{1}{2r_i^2}\right) \ln\left(1 + 2r_i^2 \frac{(\mu_i - \theta_i)^2}{\nu_i}\right)$$

$$+ \; C$$

where

$$C = -\frac{1}{2} \sum_{i=1}^{N'} \Bigg[\left(1 + \frac{1}{2r_i^2}\right)\left(1 + \ln \frac{\nu_i}{1+2r_i^2}\right)$$

$$\underbrace{+ \frac{1}{2r_i^2} \ln 4r_i^2 + 2 \ln \Gamma\left(\frac{1}{4r_i^2}\right) - \frac{\ln \nu_i}{2r_i^2}}_{\text{must retain if } r_i \text{ fitted}} + \underbrace{2 \ln \nu_i + \ln 2\pi}_{\text{can drop}} \Bigg]$$

$$C = \sum_{i=1}^{N'} \Bigg[-\frac{1}{2}\left(1 + \frac{1}{2r_i^2}\right)\left(1 + \ln \frac{\nu_i}{1+2r_i^2}\right)$$

$$+ \frac{1}{4r_i^2} \ln \frac{1}{4r_i^2} - \ln \Gamma\left(\frac{1}{4r_i^2}\right) + \frac{1}{4r_i^2} \ln \nu_i$$

$$- \ln \nu_i - \frac{1}{2} \ln 2\pi \Bigg]$$

checked numerically