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## Note on Event Weights

Suppose an event consists of $N$ objects, which could be identified as belonging to a certain object type $t$ : jets (b or non-b), photons, electrons, etc. We are given for each event the set of reconstruction probabilities $P_{i j}$, where $i=1, \ldots, N$ labels the object and $j=1, \ldots, N_{\mathrm{t}}$ identifies the various possible types.

The goal is to estimate the expected number of events corresponding to a given signal configuration in bins of the kinematic variables that characterize the final state. This can be done by assigining a set of weights to each event corresponding to each possible mapping of objects onto object types, and then summing the weights of all events found in a given kinematic bin.

Out of the event's objects, $M$ are used to construct a particular signal configuration. For a given choice of these $M$ objects, there exist possible mappings $\mathbf{m}=\left(m_{1}, \ldots, m_{M}\right)$ relating objects to the possible types. In the example of interest, $N \geq 4, M=4$, and the mapping that corresponds to the signal configuration is when 2 of the 4 objects are photons and the other 2 are b-jets. For each choice of 4 out of the $N$ objects there are thus 6 distinct mappings that correspond to the signal configuration:

$$
\begin{aligned}
& \{b, b, \gamma, \gamma\} \\
& \{\gamma, \gamma, b, b\} \\
& \{b, \gamma, b, \gamma\} \\
& \{b, \gamma, \gamma, b\} \\
& \{\gamma, b, b, \gamma\} \\
& \{\gamma, b, \gamma, b\}
\end{aligned}
$$

For each mapping $\mathbf{m}=\left(m_{1}, \ldots, m_{M}\right)$, we can therefore calculate the probability $\pi(\mathbf{m})$ that the given objects will correspond to a signal configuration such as one of those listed above, i.e.,

$$
\pi(\mathbf{m})=\prod_{i=1}^{M} P_{i, m(i)}
$$

where $P_{i j}$ is the probability for object $i$ to be reconstructed as type $j$ and $m(i)$ is the type for the $i$ th object assigned by the mapping $\mathbf{m}=\left(m_{1}, \ldots, m_{M}\right)$.

Once an event is observed, each reconstructed object is assigned a specific type. Therefore out of the 6 signal configurations shown above, at most one can be realzed for a given set of 4 objects. If the event contains more than 4 objects, however, we may find more than one signal configuration for a given event. For example, an event may contain 2 b-jets and 3
photons, so one may choose any of the 3 possible photon pairs from the 3 photons to form a possible signal configuration.

To estimate the event rate, we need to specify how an event such as the one described above would be treated in real data. For now, suppose that one rejects the event if more than one signal configuration is possible. That is, one selects events in which there exist 2 b-jets, 2 photons, and zero photons or b-jets amongst the remaining objects.

In this case, the probability to select the event as corresponding to a particular mapping $\mathbf{m}$ is

$$
\begin{equation*}
P(\text { accept event with } \mathbf{m})=\pi(\mathbf{m}) \prod_{\mathbf{m}^{\prime} \neq \mathbf{m}}\left(1-\pi\left(\mathbf{m}^{\prime}\right)\right) . \tag{1}
\end{equation*}
$$

One thus obtains a value for each possible mapping within an event, and each mapping corresonds to a given bin in the space of kinematic variables. Therefore for each of the

$$
\begin{equation*}
\frac{N!}{M!(N-M)!} \tag{2}
\end{equation*}
$$

subsets of $M(=4)$ objects one calculates the probability from Eq. (1) for each of the (6) possible mappings and enters this weight into the corresponding kinematic bin.

One can imagine different rules for the game, e.g., if an event has more than one possible signal configuration, then each real event may be used more than once. In this case the event weight would simply be

$$
\begin{equation*}
P(\text { accept event with } \mathbf{m})=\pi(\mathbf{m}) \tag{3}
\end{equation*}
$$

Alternative one might choose to use only the "best" configuration according to some criterion. In this case, the event weight would be given by

$$
\begin{equation*}
P(\text { accept event with } \mathbf{m})=\pi(\mathbf{m}) P(\mathbf{m} \text { best }) . \tag{4}
\end{equation*}
$$

