Statistical Data Analysis
Problem sheet 1
Due Monday, 19 October 2020

Exercise 1 [7 marks]: Using the Kolmogorov axioms, show that

$$P(A \cup B) \;=\; P(A) + P(B) - P(A \cap B)\,.$$

(Hint: express $A \cup B$ as the union of disjoint sets and use Kolmogorov's third axiom.)

Exercise 2 [5 marks]: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{array}{lcl} P(0\,|\,\mathrm{e}) = 0.001 & \quad\text{and}\quad & P(0\,|\,\gamma) = 0.99899 \\ P(1\,|\,\mathrm{e}) = 0.01 & & P(1\,|\,\gamma) = 0.001 \\ P(2\,|\,\mathrm{e}) = 0.989 & & P(2\,|\,\gamma) = 10^{-5}\,. \end{array}$$

(a) What is the probability for the particle to be a photon given a detected signal in one layer only?

(b) What is the probability for a particle to be an electron given a detected signal in both layers?

Exercise 3 [8 marks]: Consider the joint probability density for two continuous variables x and y given by

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the marginal pdfs $f_x(x)$ and $f_y(y)$ and indicate what they look like with a simple sketch. Are x and y independent? Explain.

(b) Find the conditional probabilities $f(x|y)$ and $f(y|x)$. State how these two densities are related by Bayes theorem, and demonstrate that the relation holds using the conditional pdfs you have found together with the marginal pdfs from (a).

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