

Exercise 1: Show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(Express $A \cup B$ as the union of disjoint sets and use the Kolmogorov axioms.)

Exercise 2: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{aligned} P(0 | e) &= 0.001 & \text{and} & & P(0 | \gamma) &= 0.99899 \\ P(1 | e) &= 0.01 & & & P(1 | \gamma) &= 0.001 \\ P(2 | e) &= 0.989 & & & P(2 | \gamma) &= 10^{-5}. \end{aligned}$$

(a) What is the probability for the particle to be a photon given a detected signal in one layer only?

(b) What is the probability for a particle to be an electron given a detected signal in both layers?

Exercise 3: Suppose the joint pdf for the independent random variables x and y is given by

$$f(x, y) = \begin{cases} 1 & 0 < x < 1 \text{ and } 0 < y < 1 \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

(a) Show that the p.d.f. $g(z)$ for $z = xy$ is

$$g(z) = \begin{cases} -\log z & 0 < z < 1 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

by defining an additional function u , which you can choose to be $u = x$. First, find the joint p.d.f. of z and u by using the Jacobian determinant as shown in the lecture notes. Integrate this over u to find the p.d.f. for z .

(b) Show that you obtain the same result for $g(z)$ using the formula for the Mellin convolution from the notes. In both (a) and (b) you will need to be careful about limits of integration.

(c) Show that the cumulative distribution of z is

$$G(z) = z(1 - \ln z). \quad (3)$$