Computing and Statistical Data Analysis
Problem sheet #5

1: Suppose the random variable x is uniformly distributed in the interval $[\alpha, \beta]$, with $\alpha, \beta > 0$. Find the expectation value of $1/x$, and compare the answer to $1/E[x]$ using $\alpha = 1$, $\beta = 2$.

Exercise 2: Consider a random variable x and constants α and β. Show that

$$\begin{aligned} E[\alpha x + \beta] &= \alpha E[x] + \beta\,, \\ V[\alpha x + \beta] &= \alpha^2 V[x]\,. \end{aligned} \tag{1}$$

Exercise 3: Consider two random variables x and y.

(a) Show that the variance of $\alpha x + y$ is given by

$$\begin{aligned} V[\alpha x + y] &= \alpha^2 V[x] + V[y] + 2\alpha \mathrm{cov}[x, y] \\ &= \alpha^2 V[x] + V[y] + 2\alpha\rho\sigma_x\sigma_y\,, \end{aligned} \tag{2}$$

where α is any constant value, $\sigma_x^2 = V[x]$, $\sigma_y^2 = V[y]$, and the correlation coefficient is $\rho = \mathrm{cov}[x, y]/\sigma_x\sigma_y$.

(b) Using the result of (a), show that the correlation coefficient always lies in the range $-1 \leq \rho \leq 1$. (Use the fact that the variance $V[\alpha x + y]$ is always greater than or equal to zero and consider the cases $\alpha = \pm\sigma_y/\sigma_x$.)

4: Suppose the independent random variables x_1 and x_2 have means $\mu_1 = \mu_2 = 10$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Use error propagation to find the variance of

$$y = \frac{x_1^2}{x_2}\,. \tag{3}$$

Comment on the validity of the procedure if one had $\mu_2 = 1$.