Statistical Data Analysis
Problem sheet 1
Due Monday, 18 October 2021

Submit your solutions as a single pdf file, named `YourName_stat_prob_sheet_1.pdf`.
Remember to put in the pdf file: full name, college, degree programme (PhD, MSci, MSc,...).

Exercise 1 [6 marks]: A sample of particles produced in high-energy proton-proton collisions
are known to be either electrons (e) or pions (π). The prior probabilities for the particles to be
of these types are $\pi(e) = 0.01$ and $\pi(\pi) = 0.99$.

The signals produced by the particles as they interact with a detector result in one of three
possible outcomes labeled A, B and C. The probabilities for these outcomes given that the
particle is either e or π are:

$$P(A \,|\, e) = 0.01 \qquad \text{and} \qquad P(A \,|\, \pi) = 0.980$$
$$P(B \,|\, e) = 0.1 \qquad\qquad\qquad\; P(B \,|\, \pi) = 0.019$$
$$P(C \,|\, e) = 0.89 \qquad\qquad\qquad P(C \,|\, \pi) = 0.001\,.$$

(a) [3] A single particle is observed resulting in data outcome A. What is the probability that
the particle is a pion?

(b) [3] What is the probability for a particle to be an electron given data outcome C?

Exercise 2 [14 marks]: Consider the joint pdf for the continuous random variables x and y

$$f(x, y) = \begin{cases} \frac{1}{\pi R^2} & x^2 + y^2 \le R^2\,, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [6] Find the marginal pdf $f_x(x)$ and by symmetry also $f_y(y)$ (remember to give the relevant
ranges for the variables). Make a rough sketch of the result for $f_x(x)$.

(b) [6] Find the conditional pdfs $f(x|y)$ and by symmetry $f(y|x)$ (remember to give the relevant
ranges for the variables). Make a rough sketch of the result for $f(x|y)$.

(c) [1] Show explicitly that the conditional and marginal pdfs satisfy Bayes' theorem.

(d) [1] Are x and y independent? Justify your answer.

G. Cowan