Statistical Data Analysis
Problem sheet 1
Due Monday, 17 October 2022

Exercise 1: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{aligned} P(0\,|\,\mathrm{e}) &= 0.001 \qquad \text{and} \qquad & P(0\,|\,\gamma) &= 0.99899 \\ P(1\,|\,\mathrm{e}) &= 0.01 & P(1\,|\,\gamma) &= 0.001 \\ P(2\,|\,\mathrm{e}) &= 0.989 & P(2\,|\,\gamma) &= 10^{-5}\,. \end{aligned}$$

(a) [4 marks] What is the probability for the particle to be a photon given a detected signal in one layer only?

(b) [4 marks] What is the probability for a particle to be an electron given a detected signal in both layers?

Exercise 2: Consider the joint probability density for two continuous variables x and y given by

$$f(x,y) = \begin{cases} x+y & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

(a) [6 marks] Find the marginal pdfs $f_x(x)$ and $f_y(y)$ and indicate what they look like with a simple sketch. Are x and y independent? Explain.

(b) [6 marks] Find the conditional probabilities $f(x|y)$ and $f(y|x)$. State how these two densities are related by Bayes theorem, and demonstrate that the relation holds using the conditional pdfs you have found together with the marginal pdfs from (a).

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