Statistical Data Analysis
Problem sheet 9
Due Monday, 12 December 2022

Problem 1: The binomial distribution is given by

$$f(n;N,\theta) = \frac{N!}{n!(N-n)!}\theta^n(1-\theta)^{N-n} ,$$

where n is the number of 'successes' in N independent trials, with a success probability of θ for each trial. Recall that the expectation value and variance of n are $E[n] = N\theta$ and $V[n] = N\theta(1-\theta)$, respectively. Suppose we have a single observation of n and using this we want to estimate the parameter θ.

1(a) [**4 marks**] Find the maximum likelihood estimator $\hat{\theta}$.

1(b) [**4 marks**] Show that $\hat{\theta}$ has zero bias and find its variance.

1(c) [**4 marks**] Suppose we observe $n = 0$ for $N = 10$ trials. Find the upper limit for θ at a confidence level of CL = 95% and evaluate numerically.

1(d) [**4 marks**] Suppose we treat the problem with the Bayesian approach using the Jeffreys prior, $\pi(\theta) \propto \sqrt{I(\theta)}$, where

$$I(\theta) = -E\left[\frac{\partial^2 \ln L}{\partial\theta^2}\right]$$

is the expected Fisher information. Find the Jeffreys prior $\pi(\theta)$ and the posterior pdf $p(\theta|n)$ as proportionalities.

1(e) [**4 marks**] Explain how in the Bayesian approach how one would determine an upper limit on θ using the result from (d). (You do not actually have to calculate the upper limit.)

Explain briefly the differences in the interpretation between frequentist and Bayesian upper limits.