Statistical Data Analysis Problem sheet 9 Due Monday, 4 December 2023

**Exercise 1:** Suppose the outcome of a measurement consists of a single value t modeled as following an exponential pdf

$$f(t|\tau) = \frac{1}{\tau}e^{-t/\tau}$$
,  $(t \ge 0)$ .

(a) [3 marks] Write down the log-likelihood function and find the Maximum-Likelihood estimator for the parameter  $\tau$ .

(b) We observe a single value t and want to test hypothetical values of  $\tau$ .

(i) [3 marks] Take the critical region of the test to be  $t > t_{cut}$ . Find the value of  $t_{cut}$  needed to have a test of size  $\alpha$ .

(ii) [2 marks] For a given observed value t, find the corresponding p-value of a hypothesized value of  $\tau$ , taking larger values of t to constitute increasing incompatibility with  $\tau$ .

(iii) [2 marks] Suppose t = 1 s. Find the lower limit on  $\tau$  at a confidence level of CL = 95%. Evaluate numerically.

(c) [4 marks] Consider the Bayesian approach to inference about  $\tau$ . Using the second derivative of the log-likelihood, show that the Jeffreys prior for  $\tau$  is

$$\pi(\tau) \propto \frac{1}{\tau} \qquad (\tau > 0).$$

(d) [3 marks] Using the Jeffreys prior, show that the posterior pdf is

$$p(\tau|t) = \frac{t}{\tau^2} e^{-t/\tau} .$$

(e) [3 marks] Find the mode of the posterior pdf above and comment on why it is less than the Maximum Likelihood estimator.