Statistical Data Analysis
Problem sheet 1
Due Monday, 14 October 2024

Exercise 1: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$
\begin{array}{lll}
P(0\,|\,\text{e}) = 0.001 & \text{and} & P(0\,|\,\gamma) = 0.99899 \\
P(1\,|\,\text{e}) = 0.01 & & P(1\,|\,\gamma) = 0.001 \\
P(2\,|\,\text{e}) = 0.989 & & P(2\,|\,\gamma) = 10^{-5}\,.
\end{array}
$$

(a) [4 marks] What is the probability for the particle to be a photon given a detected signal in one layer only?

(b) [4 marks] What is the probability for a particle to be an electron given a detected signal in both layers?

Exercise 2: Consider the joint probability density for two continuous variables x and y given by

$$
f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1,\ 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}
$$

(a) [6 marks] Find the marginal pdfs $f_x(x)$ and $f_y(y)$ and indicate what they look like with a simple sketch. Are x and y independent? Explain.

(b) [6 marks] Find the conditional probabilities $f(x|y)$ and $f(y|x)$. State how these two densities are related by Bayes theorem, and demonstrate that the relation holds using the conditional pdfs you have found together with the marginal pdfs from (a).

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