Statistical Data Analysis Problem sheet #3 Due Monday, 27 October 2025

**Exercise 1 [6 marks]:** Consider two continuous random variables x and y that follow the joint pdf f(x, y) and define u = x + y. Show that the pdf of u can be written

$$g(u) = \int_{-\infty}^{\infty} f(x, u - x) dx.$$

Use a method analogous to what was shown in the lectures for the product of two random variables (see p. 9 of the week 2 slides).

Exercise 2 [7 marks]: Suppose x and y are independent and exponentially distributed each with mean values  $\theta$  and define u = x + y. By using the result from Ex. 1, find the pdf of u. (In fixing the limits of integration, remember that the pdf is nonzero only for  $x \ge 0$  and  $y \ge 0$ .)

**Exercise 3 [7 marks]:** Consider a continuous random variable x that follows the pdf f(x) with cumulative distribution F(x), and suppose r follows a uniform distribution on [0,1]. Prove (as was claimed in the lectures) that if we set F(x) = r and solve for x, that x(r) follows the pdf f(x). To do this, use the method discussed in the lectures for finding the pdf of a function, and use the inverse function theorem, which says that

$$\frac{d}{dr}F^{-1}(r) = \frac{1}{\frac{dF}{dx}(x(r))}.$$

Exercise 4 (computing warm up [0 marks]): There is nothing to turn in for this exercise – it is just a warm-up exercise to ensure that you have your computing environment set up.

Starting with simpleMC.py, simpleMC.ipynb or simpleMC.cc from the course website, generate 10000 random values uniformly distributed between 0 and 1 and display the result as a histogram with 100 bins. (This is what simpleMC already does; you just need to ensure that you can run it.)