

Please send by the announced due date to Glen Cowan, Physics Dept., Royal Holloway, University of London, Egham, Surrey, TW20 0EX, or e-mail to g.cowan@rhbnc.ac.uk.

Exercise 1.1: Show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

(Express $A \cup B$ as the union of three disjoint sets.)

Exercise 1.2: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{aligned} P(0 | e) &= 0.001 & \text{and} & & P(0 | \gamma) &= 0.99899 \\ P(1 | e) &= 0.01 & & & P(1 | \gamma) &= 0.001 \\ P(2 | e) &= 0.989 & & & P(2 | \gamma) &= 10^{-5}. \end{aligned}$$

- (a) What is the probability for a particle detected in one layer only to be a photon?
(b) What is the probability for a particle detected in both layers to be an electron?

Exercise 1.3: Consider a random variable x and constants α and β . Show that

$$\begin{aligned} E[\alpha x + \beta] &= \alpha E[x] + \beta, \\ V[\alpha x + \beta] &= \alpha^2 V[x]. \end{aligned} \tag{1}$$

Exercise 1.4: Consider two random variables x and y .

- (a) Show that the variance of $\alpha x + y$ is given by

$$\begin{aligned} V[\alpha x + y] &= \alpha^2 V[x] + V[y] + 2\alpha \text{cov}[x, y] \\ &= \alpha^2 V[x] + V[y] + 2\alpha \rho \sigma_x \sigma_y, \end{aligned} \tag{2}$$

where α is any constant value, $\sigma_x^2 = V[x]$, $\sigma_y^2 = V[y]$, and the correlation coefficient is $\rho = \text{cov}[x, y] / \sigma_x \sigma_y$.

- (b) Using the result of (a), show that the correlation coefficient always lies in the range $-1 \leq \rho \leq 1$. (Use the fact that the variance $V[\alpha x + y]$ is always greater than or equal to zero and consider the cases $\alpha = \pm \sigma_y / \sigma_x$.)