Statistical Analysis of Data
Problem sheet #1

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Exercise 1.1: Show that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) .$$

(Express $A \cup B$ as the union of three disjoint sets.)

Exercise 1.2: A beam of particles consists of a fraction 10^{-4} electrons and the rest photons. The particles pass through a double-layered detector which gives signals in either zero, one or both layers. The probabilities of these outcomes for electrons (e) and photons (γ) are

$$\begin{array}{lcl} P(0\,|\,\mathrm{e}) = 0.001 & \quad\text{and}\quad & P(0\,|\,\gamma) = 0.99899 \\ P(1\,|\,\mathrm{e}) = 0.01 & & P(1\,|\,\gamma) = 0.001 \\ P(2\,|\,\mathrm{e}) = 0.989 & & P(2\,|\,\gamma) = 10^{-5} . \end{array}$$

(a) What is the probability for a particle detected in one layer only to be a photon?

(b) What is the probability for a particle detected in both layers to be an electron?

Exercise 1.3: Consider a random variable x and constants α and β. Show that

$$\begin{aligned} E[\alpha x + \beta] &= \alpha E[x] + \beta , \\ V[\alpha x + \beta] &= \alpha^2 V[x] . \end{aligned} \tag{1}$$

Exercise 1.4: Consider two random variables x and y.

(a) Show that the variance of $\alpha x + y$ is given by

$$\begin{aligned} V[\alpha x + y] &= \alpha^2 V[x] + V[y] + 2\alpha \mathrm{cov}[x, y] \\ &= \alpha^2 V[x] + V[y] + 2\alpha\rho\sigma_x\sigma_y , \end{aligned} \tag{2}$$

where α is any constant value, $\sigma_x^2 = V[x]$, $\sigma_y^2 = V[y]$, and the correlation coefficient is $\rho = \mathrm{cov}[x, y]/\sigma_x\sigma_y$.

(b) Using the result of (a), show that the correlation coefficient always lies in the range $-1 \leq \rho \leq 1$. (Use the fact that the variance $V[\alpha x + y]$ is always greater than or equal to zero and consider the cases $\alpha = \pm\sigma_y/\sigma_x$.)