Statistical Analysis of Data
Problem sheet #2

Please send by the announced due date to Glen Cowan, Physics Dept., Royal Holloway, University of London, Egham, Surrey, TW20 0EX, or e-mail to `g.cowan@rhul.ac.uk`.

Exercise 2.1: Suppose the independent random variables x_1 and x_2 have means $\mu_1 = \mu_2 = 10$ and variances $\sigma_1^2 = \sigma_2^2 = 1$. Use error propagation to find the variance of

$$y = \frac{x_1^2}{x_2} . \tag{1}$$

Comment on the validity of the procedure if one had $\mu_2 = 1$.

Exercise 2.2: Suppose the random variable x is uniformly distributed in the interval $[\alpha, \beta]$, with $\alpha, \beta > 0$. Find the expectation value of $1/x$, and compare the answer to $1/E[x]$ using $\alpha = 1$, $\beta = 2$.

Exercise 2.3: Suppose two independent random variables x and y are both uniformly distributed between zero and one, i.e. the p.d.f. $g(x)$ is given by

$$g(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

and similarly for the p.d.f. $h(y)$.

(a) Show that the p.d.f. $f(z)$ for $z = xy$ is

$$f(z) = \begin{cases} -\log z & 0 < z < 1 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

by defining an additional function, $u = x$. First, find the joint p.d.f. of z and u. Integrate this over u to find the p.d.f. for z.

(b) Show that the cumulative distribution of z is

$$F(z) = z(1 - \log z) . \tag{4}$$

Exercise 2.4: Consider the exponential p.d.f.,

$$f(x; \xi) = \frac{1}{\xi} e^{-x/\xi} , \quad x \geq 0 . \tag{5}$$

(a) Show that the corresponding cumulative distribution is given by

$$F(x) = 1 - e^{-x/\xi} , \quad x \geq 0 . \tag{6}$$

(b) Show that the conditional probability to find a value x between x_0 and $x_0 + x'$ given that $x > x_0$ is equal to the (unconditional) probability to find x less than x', i.e.

$$P(x \leq x_0 + x'|x \geq x_0) = P(x \leq x')\,. \tag{7}$$

(c) Cosmic ray muons produced in the upper atmosphere enter a detector at sea level, and some of them come to rest in the detector and decay. The time difference t between entry into the detector and decay follows an exponential distribution, and the mean value of t is the mean lifetime of the muon (approximately 2.2 μS). Explain why the time that the muon lived prior to entering the detector does not play a role in determining the mean lifetime.