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BUSSTEPP lectures on Supersymmetry

Stephen M. West

Royal Holloway, University of London Egham, Surrey, TW20 0EX

E-mail: Stephen.West@rhul.ac.uk

Abstract: These lecture notes accompany the Supersymmetry lectures delivered at BUSSTEPP in 2014. They cover a basic introduction to supersymmetry with a detailed description of how to construct a supersymmetric action.

Contents

1 Books and references

Much of these lecture notes are derived from books, reviews and other lectures. A partial list accompanied by other useful references are

- S. P. Martin, A Supersymmetry primer; hep-ph/9709356
- I. J. R. Aitchison, Supersymmetry and the MSSM: An Elementary Introduction, hep-ph/0505105v1
- J. Wess, J. Bagger, Supersymmetry and Supergravity, Princeton University Press, (1992).
- H. Baer, X. Tata; Weak Scale Supersymmetry; CUP (2006)
- P. Binétruy, Supersymmetry, OUP (2008)
- J. D. Lykken, "Introduction to supersymmetry," hep-th/9612114.
- F. Quevedo, S. Krippendorf and O. SchlottererCambridge Lectures on Supersymmetry and Extra Dimensions, Cambridge Lectures on Supersymmetry and Extra Dimensions, arXiv:1011.1491 [hep-th].

  Further reading...
- M. A. Luty, 2004 TASI lectures on supersymmetry breaking, hep-th/0509029.
- J. Terning, TASI 2002 lectures: Non-perturbative supersymmetry, hep-th/0306119.

Warning: Although I have made every effort to make these notes consistent and correct, minus signs and factors of 2 are notoriously difficult to get right in supersymmetry. I urge the reader to check results presented in these notes carefully and I apologies in advance for the inevitable inclusion of typos and stupid errors.

2 Conventions

Before we start, it is worth establishing some notation. In these notes I will use the mostly negative metric, $\eta^{\mu\nu} = \text{diag}(+,-,-,-)$. In addition we use the following representation of the gamma matrices

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.1}$$

where

$$\sigma^0 = \overline{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \sigma^1 = -\overline{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\sigma^2 = -\overline{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = -\overline{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.2}$$

3 The Gauge Hierarchy Problem and the motivation for Supersymmetry.

One of the earliest and strongest motivations for Supersymmetry is the apparent need to solve the so called "gauge hierarchy problem". Although of late with the ever more constraining limits coming from the LHC it has become increasingly difficult to maintain this motivation with Supersymmetry losing its ability to solve this problem in the manner in which it was designed. Never the less, we start this introduction to Supersymmetry with the motivation of solving the gauge hierarchy problem.

The gauge hierarchy problem is concerned with the sensitivity of (fundamental) scalar states (in the Standard Model it is the Higgs) to potentially large mass scales in the UV. This manifests itself in terms of quantum corrections to the scalar masses of order the heaviest mass scale in the theory.

To be concrete let us consider the Standard Model Higgs and suppose there exists an additional heavy complex scalar particle, ϕ, with mass m_ϕ that has a coupling to the Higgs via the Lagrangian term $\mathcal{L} = \lambda \left|H\right|^2 \left|\phi\right|^2$. A one-loop correction to the Higgs mass squared results and has the form

$$\Delta m_H^2 = \frac{\lambda}{16\pi^2}\left[\Lambda_{\mathrm{UV}}^2 - 2m_\phi^2 \,\ln\left(\frac{\Lambda_{\mathrm{UV}}}{m_\phi}\right) + \ldots\right], \tag{3.1}$$

where Λ_{UV} is an ultraviolet momentum cutoff introduced to regulate the loop integral. The interpretation of this cut-off and its effect should be treated with care. It can be interpreted as the energy scale at which new physics is present that alters the high-energy behaviour of the theory. It is possible to use dimensional regularisation instead of a cut-off to regulate the loop integral. In this case the quadratic piece in Λ_{UV} will not be present. This, however, this does not remove the piece that is proportional to m_ϕ^2. If the mass of this scalar is very large then once again large corrections to the Higgs mass will be generated.

It is this sensitivity to heavy degrees of freedom that is the core of the hierarchy problem. Any heavy particle coupled to the Higgs will contribute a correction to the Higgs mass. Moreover, it is the heaviest of these that will give the largest correction. We expect, in most reasonable scenarios, that there will be new particles at scales of order the Planck scale, $M_{\mathrm{pl}} = 2.4 \times 10^{18}$ GeV. If this is the case, then we have the problem of explaining why the Higgs mass is $m_H \sim 125$ GeV and not nearer the Planck mass.

Supersymmetry can in principle solve this hierarchy problem as the quantum corrections can be cancelled between loop corrections containing fermions and bosons. For example, if we introduce a fermion-Higgs coupling of the form $\mathcal{L} = \lambda' \overline{\psi}\psi H$, the Higgs mass receives a correction of the form

$$\Delta m_H^2 = -\frac{\lambda'^2}{16\pi^2}\left[\Lambda_{\mathrm{UV}}^2 + \ldots\right], \tag{3.2}$$

where the $\ldots$ contain terms proportional to the fermion mass squared. In we can identify $\lambda'^2 = \lambda$, then we can see that the quadratic pieces in the cut-off will cancel between the scalar and fermionic corrections. This cancellation must also happen in the terms that depend on the masses of the scalars and fermions. If Supersymmetry is unbroken, this cancellation does indeed happen and as a consequence the hierarchy problem is resolved.

However, it will become clear later that Supersymmetry has to be broken and as a result this cancelation is not exact and we reintroduce some corrections to the Higgs mass. If these corrections are kept small, of order the weak scale, then there is no issue. We will see later that it is becoming increasingly difficult to keep these corrections under control and avoid reintroducing the hierarchy problem.

Going beyond the hierarchy problem, Supersymmetry has a number of other features that motivate its use. In the Standard Model the gauge couplings do not unify at high energies within a GUT. In Supersymmetry , however, they do unify at or around $m_{\mathrm{GUT}} \sim 10^{16}$ GeV. Supersymmetry also predicts radiative electroweak symmetry breaking, where the Higgs mass is driven negative around the electro-weak scale inducing a vacuum expectation value for the Higgs at the correct scale. Supersymmetry can also provide dark matter candidates and plays a crucial role in string theory.

4 Supersymmetry Algebra and Representations

4.1 Poincaré symmetry and Spinors

The Poincaré group[1] corresponds to the basic symmetries of special relativity, the translations and Lorentz boosts. Generators for the Poincaré group are the Hermitian $M^{\mu\nu}$ (rotations and boosts) and P^{ρ} (translations). The algebra of the Poincaré group is defined by

$$[P^{\mu}, P^{\nu}] = 0, \quad [M^{\mu\nu}, P^{\rho}] = i\left(P^{\mu}\eta^{\nu\sigma} - P^{\nu}\eta^{\mu\sigma}\right) \tag{4.1}$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i\left(M^{\mu\sigma}\eta^{\nu\rho} + M^{\nu\rho}\eta^{\mu\sigma} - M^{\mu\rho}\eta^{\nu\sigma} - M^{\nu\sigma}\eta^{\mu\rho}\right) \tag{4.2}$$

In terms of the algebra the Lorentz group can be described by a complex $SU(2)\times SU(2)$. The generators of the Lorentz group can be expressed as

$$J_i = \frac{1}{2}\epsilon_{ijk}M_{jk}, \quad K_i = M_{0i}, \tag{4.3}$$

and the Lorentz algebra can be written in terms of J's and K's as

$$[K_i, K_j] = -i\epsilon_{ijk}J_k, \quad [J_i, K_j] = i\epsilon_{ijk}K_k \quad [J_i, J_j] = i\epsilon_{ijk}J_k. \tag{4.4}$$

Linear combinations of these generators can be written as

$$J_i^{\pm} = \frac{1}{2}\left(J_i \pm iK_i\right), \tag{4.5}$$

with commutation relations satisfying $SU(2)\times SU(2)$ algebra

$$\left[J_i^{\pm}, J_j^{\pm}\right] = i\epsilon_{ijk}J_k^{\pm}, \quad \left[J_i^{+}, J_j^{-}\right] = 0. \tag{4.6}$$

The fundamental representations are $(1/2, 0)$ which is a left handed 2 component Weyl spinor and $(0, 1/2)$ which is a right-handed 2 component Weyl spinor. We label the Weyl spinors using the Van der Waerden notation, undotted=$(1/2, 0)$ and dotted=$(0, 1/2)$, with

$$(1/2, 0) = \psi_{\alpha} \tag{4.7}$$

$$(0, 1/2) = \overline{\psi}^{\dot{\alpha}}, \tag{4.8}$$

[1] Here we only consider the orthochronous group transformations

where both dotted and undotted indices take values 1 to 2 and here the bar is just a way to label the field and nothing more. We have the following relations

$$\overline{\psi}_{\dot{\alpha}} \equiv (\psi_\alpha)^\dagger; \; \psi_\alpha \equiv (\overline{\psi}^{\dot{\alpha}})^*, \;\; \text{etc.} \tag{4.9}$$

Therefore, any particular fermionic degrees of freedom can be described equally well using a left-handed Weyl spinor (with an undotted index) or by a right-handed one (with a dotted index). By convention, all names of fermion fields are chosen so that left-handed Weyl spinors do not carry bars and right-handed Weyl spinors do.

The indices are raised and lowered using the symbols

$$\varepsilon_{\alpha\beta} = \varepsilon_{\dot{\alpha}\dot{\beta}} = -\varepsilon^{\alpha\beta} = -\varepsilon^{\dot{\alpha}\dot{\beta}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4.10}$$

acting on the spinors, for example

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta, \qquad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \qquad \overline{\chi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\overline{\chi}^{\dot{\beta}}, \qquad \overline{\chi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\overline{\chi}_{\dot{\beta}}$$

and where

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta_\alpha^\gamma, \qquad \epsilon_{\dot{\alpha}\dot{\beta}}\epsilon^{\dot{\beta}\dot{\gamma}} = \epsilon^{\dot{\gamma}\dot{\beta}}\epsilon_{\dot{\beta}\dot{\alpha}} = \delta_{\dot{\alpha}}^{\dot{\gamma}}.$$

We can write a Dirac spinor in terms of two Weyl spinners ξ_α and $(\overline{\chi})^{\dot{\alpha}} \equiv \overline{\chi}^{\dot{\alpha}}$ with two distinct indices $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$:

$$\psi_D = \begin{pmatrix} \xi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix}$$

and

$$\overline{\psi}_D = \psi_D^\dagger \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \chi^\alpha & \overline{\xi}_{\dot{\alpha}} \end{pmatrix}$$

Undotted indices are used for the first two components of a Dirac spinor and the dotted indices are used for the last two components of a Dirac spinor. Here we have that ξ is the left-handed Weyl spinor and $\overline{\chi}$ is right-handed.

We can check this by using:

$$P_L = \frac{1}{2}(1 - \gamma_5) \qquad \text{and} \qquad P_R = \frac{1}{2}(1 + \gamma_5) \qquad \text{with} \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

We have:

$$P_L \Psi_D = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}, \qquad P_R \Psi_D = \begin{pmatrix} 0 \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix}$$

As a convention, repeated spinor indices contracted like

$$^\alpha{}_\alpha \qquad \text{or} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \tag{4.11}$$

can be suppressed. In particular,

$$\xi\chi \equiv \xi^\alpha\chi_\alpha = \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta = -\chi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \chi^\beta\epsilon_{\beta\alpha}\xi^\alpha = \chi^\beta\xi_\beta \equiv \chi\xi. \tag{4.12}$$

See Appendix A for more useful identities using these objects.

We are now in a position to write the Dirac Lagrangian in terms of the Weyl spinors (and when we start to build a Supersymmetry theory, we will go the other way).

$$\begin{aligned}\mathcal{L} = \overline{\Psi}_D(i\gamma^\mu\partial - m)\Psi_D &= \begin{pmatrix} \chi^\alpha & \overline{\xi}_{\dot{\alpha}} \end{pmatrix} \left[\begin{pmatrix} 0 & i\sigma^\mu\partial_\mu \\ i\overline{\sigma}^\mu\partial_\mu & 0 \end{pmatrix} - \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \right] \begin{pmatrix} \xi_\alpha \\ \overline{\chi}^{\dot{\alpha}} \end{pmatrix} \\ &= i\overline{\chi}\,\overline{\sigma}^\mu\partial_\mu\chi + i\overline{\xi}\overline{\sigma}^\mu\partial_\mu\xi - m(\xi\chi + \overline{\xi}\,\overline{\chi})\end{aligned}$$

where the last step involves integration by parts and the identity $\chi\sigma^\mu\overline{\chi} = -\overline{\chi}\,\overline{\sigma}^\mu\chi$ etc.

A four component Majorana spinor can be obtained from the Dirac spinor by imposing the condition $\chi = \xi$ such that,

$$\Psi_D = \begin{pmatrix} \xi_\alpha \\ \overline{\xi}^{\dot{\alpha}} \end{pmatrix}, \qquad \overline{\Psi}_D = \begin{pmatrix} \xi^\alpha & \overline{\xi}_{\dot{\alpha}} \end{pmatrix}.$$

The Lagrangian for the Majorana spinor

$$\begin{aligned}\mathcal{L}_M &= \frac{i}{2}\overline{\Psi}_M\gamma^\mu\partial_\mu\Psi_M - \frac{1}{2}M\overline{\Psi}_M\Psi_M \\ &= -i\overline{\xi}\,\overline{\sigma}^\mu\partial_\mu\xi - \frac{1}{2}M\left(\xi\xi + \overline{\xi}\,\overline{\xi}\right).\end{aligned}$$

To efficiently move between the Weyl and Dirac notation we can use the chiral projection operators, $P_{L,R}$ e.g.

$$\overline{\Psi}_i P_L \Psi_j = \chi_i\xi_j \qquad \text{and} \qquad \overline{\Psi}_i P_R \Psi_j = \overline{\xi}_i\overline{\chi}_j$$

and

$$\overline{\Psi}_i\gamma^\mu P_L\Psi_j = \overline{\xi}_i\,\overline{\sigma}^\mu\xi_j \qquad \text{and} \qquad \overline{\psi}_i\gamma^\mu P_R\Psi_j = \chi_i\sigma^\mu\overline{\chi}_j = -\overline{\chi}_j\overline{\sigma}^\mu\chi_i$$

4.2 Supersymmetry algebra

Supersymmetry is the unique extension of the Poincaré group of symmetries, which is the semi-direct product of translations and Lorentz boosts. Coleman and Mandula provided a rigorous argument, which states that, given certain assumptions, the only possible symmetries of the S-matrix are Poincaré invariance and internal global symmetries related to conserved quantum numbers, e.g. electric charge. The symmetry generators for these internal symmetries are Lorentz scalars and form a Lie algebra consisting of commutation relations.

The Coleman Mandula theorem can in fact be evaded by weakening one of its assumptions. The theorem assumes that the symmetry algebra of the S-matrix involves only commutators. Introducing anti-commuting generators as well leads us to the possibility of Supersymmetry . The generators of Supersymmetry transform as the spinor representation of the Lorentz group and are therefore extension of the Poincaré space-time symmetries

A Supersymmetry transformation turns a bosonic field into a fermionic field and vice versa. Schematically we have

$$Q|\text{Boson}\rangle \propto |\text{Fermion}\rangle \;\text{ and }\; Q|\text{Fermion}\rangle \propto |\text{Boson}\rangle \tag{4.13}$$

where Q is a supersymmetry generator with spinor index ($\alpha = 1, 2$) and commutes with the Hamiltonian,

$$[Q_\alpha, H] = 0. \tag{4.14}$$

Adding these supersymmetry generators to Poincaré generators, we produce the super-Poincaré group. The full list is then

With this notation in mind, the Supersymmetry algebra can be written as

$$\begin{aligned}
\left[P_\mu, Q^I_\alpha\right] &= \left[P_\mu, \overline{Q}^I_{\dot\alpha}\right] = 0, && (4.15)\\
\left[M_{\mu\nu}, Q^I_\alpha\right] &= (\sigma_{\mu\nu})_\alpha^\beta Q^I_\beta, && (4.16)\\
\left[M_{\mu\nu}, \overline{Q}^{I\dot\alpha}\right] &= (\overline{\sigma}_{\mu\nu})^{\dot\alpha}_{\dot\beta} \overline{Q}^{I\dot\beta}, && (4.17)\\
\left\{Q^I_\alpha, \overline{Q}^J_{\dot\beta}\right\} &= 2\sigma^\mu_{\alpha\dot\beta} P_\mu \delta^{IJ}, && (4.18)\\
\left\{Q^I_\alpha, Q^J_\beta\right\} &= \epsilon_{\alpha\beta} Z^{IJ} && (4.19)\\
\left\{\overline{Q}^I_{\dot\alpha}, \overline{Q}^J_{\dot\beta}\right\} &= \epsilon_{\dot\alpha\dot\beta} \left(Z^{IJ}\right)^*, && (4.20)
\end{aligned}$$

where

$$\sigma^{\mu\nu\beta}_\alpha = \frac{i}{4}\left[\sigma^\mu_{\alpha\dot\gamma}\overline{\sigma}^{\nu\dot\gamma\beta} - \sigma^\nu_{\alpha\dot\gamma}\overline{\sigma}^{\mu\dot\gamma\beta}\right],$$

and $Z^{IJ} = -Z^{JI}$ are "central charges" (these objects commute with all generators and only exists for extended $\mathcal{N} > 1$ Supersymmetry algebras). The indices I and J label the Supersymmetry generators. For $\mathcal{N} = 1$ Supersymmetry we only have $I = J = 1$, but for extended Supersymmetry I, J run from $I, J = 1$ to N. We will focus in the main part on the case of $\mathcal{N} = 1$ Supersymmetry in these notes. See [1] for more on the algebra and representations for extended $\mathcal{N} > 1$ Supersymmetry .

The single-particle states of a supersymmetric theory fall into irreducible representations of the supersymmetry algebra, called supermultiplets. If we have two states, $|\Omega\rangle$ and $|\Omega'\rangle$, which are members of the same supermultiplet, we can transform $|\Omega\rangle$ into $|\Omega'\rangle$ (up to a space time translation or rotation) using a combination of the supersymmetry generators, Q and $\overline{Q}$.

From Equation 4.15, we can see that the (mass)^2 operator, $-P^2$, will commute through the Qs and thus each particle state of the supermultiplet will have the same mass. In fact, the supersymmetry generators commute with the Standard Model gauge group generators and so each member of the supermultiplet will have the same gauge quantum numbers.

Each supermultiplet has equal bosonic and fermionic degrees of freedom. We can show this straightforwardly. We introduce the operator $(-1)^{2S}$, where S is the spin operator. Its action on boson and fermionic states are clear

$$(-1)^{2S}|\text{Boson}\rangle = |\text{Boson}\rangle, \quad (-1)^{2S}|\text{Fermion}\rangle = -|\text{Fermion}\rangle. \tag{4.21}$$

It is easy then to show that the operator $(-1)^{2S}$ anti-commutes with Q, starting with

$Q|\text{Boson}\rangle = \text{Fermion}\rangle$ and applying $(-1)^{2S}$ from the left

$$
\begin{aligned}
(-1)^{2S} Q|\text{Boson}\rangle &= -|\text{Fermion}\rangle & (4.22)\\
\left(-Q(-1)^{2S} + \left\{(-1)^{2S}, Q\right\}\right)|\text{Boson}\rangle &= -|\text{Fermion}\rangle & (4.23)\\
-|\text{Fermion}\rangle + \left\{(-1)^{2S}, Q\right\}|\text{Boson}\rangle &= -|\text{Fermion}\rangle & (4.24)
\end{aligned}
$$

and hence $(-1)^{2S}$ anti-commutes with Q. Now consider the trace over the operator $(-1)^{2S}P^\mu$ (including each helicity state separately). Introduce a states labelled by $|i\rangle$. Applying Q or $\overline{Q}$ on this state returns another state $|i'\rangle$ with the same 4-momentum.

$$
\begin{aligned}
\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle &\sim \sum_i \langle i|(-1)^{2s}Q\overline{Q}|i\rangle + \sum_i \langle i|(-1)^{2s}\overline{Q}Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}Q\overline{Q}|i\rangle + \sum_i\sum_j \langle i|(-1)^{2s}\overline{Q}|j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s}Q\overline{Q}|i\rangle + \sum_j \langle j|Q(-1)^{2s}\overline{Q}|j\rangle \\
&= \sum_i \langle i|(-1)^{2s}Q\overline{Q}|i\rangle - \sum_j \langle j|(-1)^{2s}Q\overline{Q}|j\rangle \\
&= 0. \qquad (4.25)
\end{aligned}
$$

The first equality follows from the supersymmetry algebra relation in equation 4.18.

Now $\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle = p^\mu\ \text{Tr}[(-1)^{2s}]$ ($\text{Tr}[(-1)^{2s}$ is called the Witten index) is just proportional to the number of bosonic degrees of freedom n_B minus the number of fermionic degrees of freedom n_F in the trace, so that

$$
n_B = n_F \qquad (4.26)
$$

must hold for a given $p^\mu \neq 0$ in each supermultiplet.

Equation 4.18 provides us with an important connection between the energy of the vacuum and supersymmetry breaking. Let $|\text{Vac}\rangle$ be the vacuum state, then

$$
\sum_{\alpha=\overline{\alpha}=1}^{2} 2\sigma^\mu_{\alpha\dot{\alpha}}\langle\text{Vac}|P_\mu|\text{Vac}\rangle = 4\langle\text{Vac}|P_0|\text{Vac}\rangle = 4E_{\text{vac}}, \qquad (4.27)
$$

Now using Equation 4.18, we have

$$
\begin{aligned}
4E_{\text{vac}} &= \sum_{\alpha=\dot{\alpha}=1}^{2} \langle\text{Vac}|\left\{Q_\alpha, \overline{Q}_{\dot{\alpha}}\right\}|\text{Vac}\rangle \\
&= \sum_{\alpha=1}^{2}\langle\text{Vac}|\left(Q_\alpha(Q_\alpha)^\dagger + (Q_\alpha)^\dagger Q_\alpha\right)|\text{Vac}\rangle \\
&= \sum_{\alpha=1}^{2}\left(\left|(Q_\alpha^\dagger|\text{Vac}\rangle\right|^2 + |Q_\alpha|\text{Vac}\rangle|^2\right) \geq 0. \qquad (4.28)
\end{aligned}
$$

So we see that, in a global supersymmetric theory, the vacuum energy must be positive-definite (in local supersymmetry, this is not necessarily the case). Looking more closely at Equation 4.28 we can see that if supersymmetry is unbroken, i.e. acting on the vacuum with a supersymmetry generator gives zero, then $E_{vac} = 0$. This is the condition for unbroken supersymmetry.

From Equations 4.16 and 4.17 we have, with $M_{12} = J_3$, and $\sigma_{12} = -\frac{1}{2}\sigma_3$, the commutation relations

$$[J_3, Q_1] = \frac{1}{2} Q_1, \tag{4.29}$$

$$[J_3, Q_2] = -\frac{1}{2} Q_2. \tag{4.30}$$

Taking the adjoint of these and recalling that, schematically $[\ ,\ ]^\dagger = -[\ ,\ ]$, we have

$$\left[J_3, \overline{Q}_{\dot{1}}\right] = -\frac{1}{2}\overline{Q}_{\dot{1}}, \tag{4.31}$$

$$\left[J_3, \overline{Q}_{\dot{2}}\right] = \frac{1}{2}\overline{Q}_{\dot{2}}. \tag{4.32}$$

From this we can see that,

$$Q_1 \text{ and } \overline{Q}_{\dot{2}} \text{ raise helicity by } 1/2, \tag{4.33}$$

$$Q_2 \text{ and } \overline{Q}_{\dot{1}} \text{ lower helicity by } 1/2. \tag{4.34}$$

We would now like to construct the representations in terms of supermultiplets. Starting with massless representations, using Equation 4.18, and choosing the reference frame where all the momentum is in the z direction, we have,

$$\left\{Q_\alpha, \overline{Q}_{\dot{\beta}}\right\} = \begin{pmatrix} 0 & 0 \\ 0 & 4E \end{pmatrix}_{\alpha\dot{\beta}}. \tag{4.35}$$

Using Equation 4.18, we quickly find that, as operators, $Q_1 = \overline{Q}_{\dot{1}} = 0$. The remaining generators can be defined in terms of annihilation and creation operators, defined as

$$a = \frac{1}{\sqrt{4E}} Q_2, \quad a^\dagger = \frac{1}{\sqrt{4E}} \overline{Q}_{\dot{2}}, \tag{4.36}$$

with algebra,

$$\left\{a, a^\dagger\right\} = 1, \quad \{a, a\} = 0 = \left\{a^\dagger, a^\dagger\right\}. \tag{4.37}$$

Constructing the spin states of a supermultiplet is straightforward: first we choose the minimum spin state which is annihilated by a, say $|\lambda_0\rangle$, so that

$$a|\lambda_0\rangle = 0, \quad J_3|\lambda_0\rangle = \lambda_0|\lambda_0\rangle \tag{4.38}$$

and we then act on $|\lambda_0\rangle$ with the raising operator to get the rest of the spin states. As $a^\dagger$ anti-commutes with itself, we can only act with a single $a^\dagger$ once[2]. This means each

[2]For $\mathcal{N} > 1$ supersymmetry generators we can act with multiple raising operators provided no two are the same.

massless supermultiplet will contain two states, $|\lambda_0\rangle$ and $|\lambda_0 + 1/2\rangle$. However, in order to ensure CPT invariance, we must include $|-\lambda_0\rangle$ and $|-\lambda_0 - 1/2\rangle$ in the supermultiplet.

There are three choices of λ_0 which are of physical interest. First is the chiral multiplet with $\lambda_0 = -1/2$. So we have spin states -1/2, 0, 0 and 1/2 corresponding to a Weyl fermion and a complex scalar.

The second has $\lambda_0 = -1$, so that we have spin states -1, -1/2, 1/2 and 1. This corresponds to a massless vector boson and a Weyl fermion.

The third has, $\lambda_0 = -2$ giving states, -2, -3/2, 3/2 and 2. This corresponds to the graviton (spin states ± 2) and gravitino (spin states $\pm 3/2$). The reason we do not include the $\lambda_0 = -3/2$ case is that the gravitino only has a non-trivial action in the presence of gravity and so must be accompanied by the spin 2 graviton.

For massive supermultiplets, we can always boost to the rest frame of the states which means we can set $P_\mu = (m, 0, 0, 0)$. Plugging this into Equation 4.18, we have

$$\left\{Q_\alpha, \overline{Q}_{\dot{\beta}}\right\} = 2m\delta_{\alpha\dot{\beta}}, \tag{4.39}$$

and so, for a massive multiplet, all generators are non-trivial. Defining the creation and annihilation operators in this case, we have

$$a_\alpha = \frac{1}{\sqrt{2m}} Q_\alpha, \;\; a^\dagger_{\dot{\alpha}} = \frac{1}{\sqrt{2m}} \overline{Q}_{\dot{\alpha}}. \tag{4.40}$$

Again we have the creation and annihilation operator algebra, but from Equations 4.33 and 4.34, the creation operators are a_2 and $a^\dagger_1$. Thus for a massive supermultiplet with lowest spin state λ_0 the possible states are

$$\begin{aligned} |\lambda_0\rangle, \quad a^\dagger_2|\lambda_0\rangle &= |\lambda_0 + 1/2\rangle, \\ a_1\lambda_0\rangle &= |\lambda_0 + 1/2\rangle, \\ a^\dagger_2 a_1|\lambda_0\rangle &= |\lambda_0 + 1\rangle. \end{aligned} \tag{4.41}$$

In addition to these states, the CPT conjugates must be added to retain CPT invariance. So, using Equations 4.41 and plugging in $\lambda_0 = -1/2$, we will get spins states -1/2, 0, 0 and 1/2, which is already includes the CPT conjugates. This state is a massive chiral supermultiplet. Now, trying $\lambda_0 = -1$ and including the CPT conjugate states, we have -1, -1/2, -1/2, 0, 0, 1/2, 1/2 and 1, which is a massive vector supermultiplet containing a massive vector boson (1, 0, -1), a Dirac fermion (1/2, 1/2, -1/2, -1/2) and a real scalar (0).

So far, we have implicitly assumed that these supermultiplets have their components on shell and in doing so we satisfy the condition that the number of fermionic and bosonic degrees of freedom are equal in a single supermultiplet. This is because we have so far being consider the components as states rather than fields. As we will have processes with internal, off-shell fields we will need to worry about the degree of freedom counting for this case and how we satisfy $n_B = n_F$.

Considering fields that are off-shell, the counting of the number of physical degrees of freedom changes. We must still ensure that the number of bosonic degrees of freedom is equal to the number of fermionic degrees of freedom in each supermultiplet.

The counting for an on-shell chiral supermultiplet proceeds as: 2 physical degrees of freedom for both the Weyl fermion and complex scalar. Off-shell, however, the Weyl fermion has 2 complex components and so has 4 degrees of freedom with the complex scalar still having 2 degrees of freedom. We thus have 2 more fermionic degrees of freedom than we do scalar and so we need to balance this with two extra bosonic degrees of freedom. This is achieved by adding another complex scalar which vanishes on shell, or can be eliminated by an algebraic constraint so that it has no propagating degrees of freedom. This type of state is called an auxiliary field. In summary, the off-shell chiral supermultiplet consists of a complex scalar, A, a Weyl fermion, ψ_α, and an auxiliary field, F.

For a vector multiplet, both the vector boson and Weyl fermion have 2 on-shell degrees of freedom. However, off-shell the vector boson has 3 degrees of freedom and the Weyl fermion has 4, so we need to introduce an auxiliary field with one degree of freedom. Consequently we add a real scalar field to the vector supermultiplet. For a vector multiplet, therefore, we have a Weyl fermion, ψ_α, a vector boson, A_μ and an auxiliary real scalar field, D.

5 Superspace and Superfields

We would now like to write down the interacting supersymmetric theory. There are several ways in which this can be done. The first is to work with the components of the supermultiplets separately and investigate how they transform under these supersymmetric transformations and go on to form a set of renormalisable interactions invariant under supersymmetry. This method is outlined in [2], but here we give an introduction to the notation of superfields and superspace and use these to generate the most general renormalisable interacting supersymmetric theory.

The first step is to generalise space-time to superspace ($\mathcal{N}$=1). In terms of coordinates we write

$$x^\mu \to x^\mu, \theta_\alpha, \overline{\theta}_{\dot\alpha}, \tag{5.1}$$

where θ_α and $\overline{\theta}_{\dot\alpha}$ are the extra "super" space coordinates. These coordinates are anti-commuting spinors which do not depend on x^μ and as usual α and $\dot\alpha$ run from 1 to 2. Derivatives with respect to these superspace coordinates are given by

$$\frac{\partial\theta^\beta}{\partial\theta^\alpha} = \delta^\beta_\alpha, \;\; \frac{\partial\overline{\theta}^{\dot\beta}}{\partial\overline{\theta}^{\dot\alpha}} = \delta^{\dot\beta}_{\dot\alpha}, \tag{5.2}$$

where the derivatives themselves are anti-commuting objects,

$$\{\partial_\alpha, \partial_\beta\} = 0, \quad \{\overline{\partial}_{\dot\alpha}, \overline{\partial}_{\dot\beta}\} = 0. \tag{5.3}$$

Additional identities (listed in Appendix A but repeated here for convenience) involving the superspace coordinates are

$$\partial_\alpha \theta^\beta = \delta^\alpha_\beta, \qquad \partial^\alpha \theta_\beta = \delta^\beta_\alpha, \tag{5.4}$$

$$\overline{\partial}_{\dot\alpha} \overline{\theta}^{\dot\beta} = \delta^{\dot\alpha}_{\dot\beta}, \qquad \overline{\partial}^{\dot\alpha} \overline{\theta}_{\dot\beta} = -\delta^{\dot\beta}_{\dot\alpha} \tag{5.5}$$

$$\epsilon^{\alpha\beta} \partial_\beta = -\partial^\alpha, \qquad \epsilon^{\dot\alpha\dot\beta} \partial_{\dot\beta} = -\overline{\partial}^{\dot\alpha} \tag{5.6}$$

$$\partial_\alpha(\theta^2) = 2\theta_\alpha, \qquad \overline{\partial}_{\dot\alpha}(\overline{\theta}^2) = -2\overline{\theta}_{\dot\alpha} \tag{5.7}$$

$$\theta_\alpha \theta_\beta = \frac{1}{2}\epsilon_{\alpha\beta}\theta^2, \quad \overline{\theta}_{\dot\alpha}\overline{\theta}_{\dot\beta} = -\frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\overline{\theta}^2. \tag{5.8}$$

It is convenient to define an integral over superspace coordinates. For a single θ the integral has the form

$$\int \theta d\theta = 1, \ \int d\theta.1 = 0. \tag{5.9}$$

From this it is easy to show that in fact superspace integration is equivalent to differentiation

$$\frac{\partial h(\theta)}{\partial \theta} = \int d\theta h(\theta). \tag{5.10}$$

For $\mathcal{N}$=1 supersymmetric coordinates, θ_α and $\theta_{\dot\alpha}$, we have the definitions,

$$d^2\theta \equiv -\frac{1}{4}\varepsilon_{\alpha\beta} d\theta^\alpha d\theta^\beta, \ \ d^2\overline{\theta} \equiv -\frac{1}{4}\varepsilon^{\dot\alpha\dot\beta} d\overline{\theta}_{\dot\alpha} d\overline{\theta}_{\dot\beta} \ \text{ and } \ d^4\theta \equiv d^2\theta d^2\overline{\theta}, \tag{5.11}$$

such that

$$\int d^2\theta(\theta^2) = 1, \ \int d^2\overline{\theta}(\overline{\theta}^2) = 1 \ \text{ and } \ \int d^4\theta(\overline{\theta}^2\theta^2) = 1. \tag{5.12}$$

These integral definitions will prove useful later.

Having introduced the Grassmann variables, we can now rewrite the $\mathcal{N} = 1$ Supersymmetry algebra as a Lie algebra using only commutators,

$$\left[\eta Q, \overline{\eta}\overline{Q}\right] = 2\eta\sigma^\mu\overline{\eta}P_\mu, \ \ [\eta Q, \eta Q] = \left[\overline{\eta}\overline{Q}, \overline{\eta}\overline{Q}\right] = 0. \tag{5.13}$$

A crucial question we need to answer is how does a superfield transform under supersymmetry transformations? We wish to express the Supersymmetry generators as differential operators (as we do for the translations, orations and boosts of the Poincaré group). The usually instructive way to proceed here is to use calligraphic letters for the abstract operator and latin ones for the representation of the same operator as a differential operator in field space.

First we recall the procedure for a translation in ordinary space-time generated by the operator $\mathcal{P}_\mu$, with infinitesimal parameter a^μ on a field $\phi(x)$. This is defined as (treating ϕ as an operator)

$$\phi(x+a) = e^{-ia\mathcal{P}}\phi(x)e^{ia\mathcal{P}} = \phi(x) - ia^\mu\left[\mathcal{P}_\mu, \ \phi(x)\right] + \dots \ . \tag{5.14}$$

Alternatively we may Taylor expand the left hand side as

$$\phi(x+a) = \phi(x) + a^\mu\partial_\mu\phi(x) + \dots \ . \tag{5.15}$$

Comparing these two equations we find that

$$[\phi(x),\ \mathcal{P}_\mu] = -i\partial_\mu \phi(x) \equiv P_\mu \phi(x) \tag{5.16}$$

where we define P_μ as the differential operator representation of the generator $\mathcal{P}_\mu$. As a result of this we know that a translation of a field, $\phi(x)$, by parameter a^μ induces a change in the field as

$$\delta_a \phi(x) = \phi(x+a) - \phi(x) = ia^\mu P_\mu \phi(x), \tag{5.17}$$

to leading order in a.

Given this we can repeat the same procedure for a Supersymmetry transformation on a superfield. A translation in superspace on a superfield $\Phi(x, \theta, \overline{\theta})$ by parameters $(\eta, \overline{\eta})$, where $(\eta, \overline{\eta})$ are constant Grassmann variables is defined as

$$\Phi(x+\delta x, \theta+\delta\theta, \overline{\theta}+\delta\overline{\theta}) = e^{-i(\eta\mathcal{Q}+\overline{\eta}\overline{\mathcal{Q}})}\Phi(x, \theta, \overline{\theta})e^{i(\eta\mathcal{Q}+\overline{\eta}\overline{\mathcal{Q}})}, \tag{5.18}$$

with

$$\delta_{\eta,\overline{\eta}}\Phi(x, \theta, \overline{\theta}) \equiv \Phi(x+\delta x, \theta+\delta\theta, \overline{\theta}+\delta\overline{\theta}) - \Phi(x, \theta, \overline{\theta}) \tag{5.19}$$

We would like to find the explicit expression for the changes in x, θ and $\overline{\theta}$. In order to do this we need to employ the Baker-Campbell-Hausdorff formula for non-commuting objects, which reads

$$e^A e^B = e^{A+B+\frac{1}{2}[A,\, B]+\ldots} \tag{5.20}$$

where the ellipses represent higher commutators, which for us will vanish. With this in mind we can rewrite equation 5.18 as

$$\Phi(x+\delta x, \theta+\delta\theta, \overline{\theta}+\delta\overline{\theta}) = e^{-i(\eta\mathcal{Q}+\overline{\eta}\overline{\mathcal{Q}})}e^{-i(x\mathcal{P}+\theta\mathcal{Q}+\overline{\theta}\ \overline{\mathcal{Q}})}\Phi(0,\ 0,\ 0)e^{i(x\mathcal{P}+\theta\mathcal{Q}+\overline{\theta}\ \overline{\mathcal{Q}})}e^{i(\eta\mathcal{Q}+\overline{\eta}\overline{\mathcal{Q}})} \tag{5.21}$$

and then expand the exponentials using the Baker-Campbell-Hausdorff formula

$$\begin{aligned} &\exp\left[i(x\mathcal{P}+\theta\mathcal{Q}+\overline{\theta}\ \overline{\mathcal{Q}})\right]\exp\left[i(\eta\mathcal{Q}+\overline{\eta}\overline{\mathcal{Q}})\right] \\ &= \exp\left[i(x^\mu + i\theta\sigma^\mu\overline{\eta} - i\eta\sigma^\mu\overline{\theta})\mathcal{P}_\mu + i(\eta+\theta)\mathcal{Q} + i(\overline{\eta}+\overline{\theta})\overline{\mathcal{Q}}\right]. \end{aligned} \tag{5.22}$$

From this we can see that

$$\begin{aligned} \delta x^\mu &= i\theta\sigma^\mu\overline{\eta} - i\eta\sigma^\mu\overline{\theta} \\ \delta\theta^\alpha &= \eta^\alpha \\ \delta\overline{\theta}^{\dot{\alpha}} &= \overline{\eta}^{\dot{\alpha}}. \end{aligned} \tag{5.23}$$

We should not be surprised that the shift in the x coordinate involves superspace coordinates as we know the commutation relations of the superpoincaré algebra contains $\{Q, \overline{Q}\} \sim P_\mu$.

Now we want to look for a differential representation for the Supersymmetry generators, $\mathcal{Q}$ and $\overline{\mathcal{Q}}$. First consider equation 5.19. We can Taylor expand the first term on the right hand side and using the form of the shifts in x, θ and $\overline{\theta}$ displayed in equation 5.23 we find

$$\delta_{\eta,\overline{\eta}}\Phi(x,\theta,\overline{\theta}) = \left[\eta^{\alpha}\partial_{\alpha} + \overline{\eta}^{\dot{\alpha}}\overline{\partial}_{\dot{\alpha}} + i(\theta\sigma^{\mu}\overline{\eta} - \eta\sigma^{\mu}\overline{\theta})\partial_{\mu} + \ldots\right]\Phi(x,\theta,\overline{\theta}). \tag{5.24}$$

We can also expand $\Phi(x + \delta x, \theta + \delta\theta, \overline{\theta} + \delta\overline{\theta})$ using equation 5.18 so that equation 5.19 becomes

$$\begin{aligned}\delta_{\eta,\overline{\eta}}\Phi(x,\theta,\overline{\theta}) &= (1 - i\eta\mathcal{Q} - i\overline{\eta}\overline{\mathcal{Q}} + \ldots)\Phi(x,\theta,\overline{\theta})(1 + i\eta\mathcal{Q} + i\overline{\eta}\overline{\mathcal{Q}} + \ldots) - \Phi(x,\theta,\overline{\theta}) \\ &= -i\eta^{\alpha}\left[\mathcal{Q}_{\alpha}, \Phi(x,\theta,\overline{\theta})\right] - i\overline{\eta}_{\dot{\alpha}}\left[\overline{\mathcal{Q}}^{\dot{\alpha}}, \Phi(x,\theta,\overline{\theta})\right] + \ldots,\end{aligned} \tag{5.25}$$

Defining

$$[\Phi, \mathcal{Q}_{\alpha}] \equiv Q_{\alpha}\Phi, \qquad [\Phi, \overline{\mathcal{Q}}_{\dot{\alpha}}] \equiv \overline{Q}_{\dot{\alpha}}\Phi, \tag{5.26}$$

we find

$$\delta_{\eta,\overline{\eta}}\Phi(x,\theta,\overline{\theta}) = i(\eta Q + \overline{\eta}\overline{Q})\Phi(x,\theta,\overline{\theta}). \tag{5.27}$$

This is the form of a supersymmetric variation of a superfield. Finally comparing this with equation 5.24, we can identify the differential representations for Q and $\overline{Q}$ as

$$Q_{\alpha} = -i\partial_{\alpha} - \sigma^{\mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_{\mu} \tag{5.28}$$

$$\overline{Q}_{\dot{\alpha}} = i\overline{\partial}_{\dot{\alpha}} + \theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu}. \tag{5.29}$$

This differential representation does indeed satisfy the Supersymmetry commutation relations, $\left\{Q_{\alpha}, \overline{Q}_{\dot{\beta}}\right\} = 2\sigma^{\mu}_{\alpha\dot{\beta}}P_{\mu}$ etc. Further more, we can define a superfield as a field in superspace that transforms according to equation 5.27.

5.1 Chiral Superfields

We can define the form of a general superfield in superspace coordinates by the terminating Taylor expansion

$$\begin{aligned}\Phi(x,\theta,\overline{\theta}) = \varepsilon(x) + \theta\psi(x) + \overline{\theta}\overline{\chi}(x) + \theta^2 m(x) + \overline{\theta}^2 n(x) + \\ \theta\sigma^{\mu}\overline{\theta}v_{\mu}(x) + \theta^2\overline{\theta}\overline{\lambda}(x) + \overline{\theta}^2\theta\zeta(x) + \theta^2\overline{\theta}^2 d(x),\end{aligned} \tag{5.30}$$

where the expansion terminates due to the fact that the θs are anti-commuting.

This form of a superfield has too many components to form an irreducible representation, like a chiral or vector supermultiplet and so we must apply constraints on $\Phi(x,\theta,\overline{\theta})$. This is done by the application of the supersymmetric covariant derivative. Specifically the condition used

$$\overline{D}_{\dot{\alpha}}\Phi = 0. \tag{5.31}$$

There is also a corresponding condition for an anti-chiral superfield

$$D_{\alpha}\Phi^{\dagger} = 0, \tag{5.32}$$

where we can define

$$\overline{D}_{\dot{\alpha}} = -\overline{\partial}_{\dot{\alpha}} - i\theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu}, \tag{5.33}$$

$$D_{\alpha} = \partial_{\alpha} + i\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_{\mu}, \tag{5.34}$$

with

$$\left\{D_{\alpha}, \overline{D}_{\dot{\beta}}\right\} = -2i\sigma^{\mu}_{\alpha\dot{\beta}}\partial_{\mu}, \ \ \{D_{\alpha}, D_{\beta}\} = 0, \ \ \left\{\overline{D}_{\dot{\alpha}}, \overline{D}_{\dot{\beta}}\right\} = 0. \tag{5.35}$$

Noticing that $\overline{D}_{\dot{\alpha}}(x^{\mu} + i\theta\sigma^{\mu}\overline{\theta}) = 0$ and $\overline{D}_{\dot{\alpha}}\theta^{\alpha} = 0$, any function $\Phi(y^{\mu}, \theta)$, where $y^{\mu} = x^{\mu} + i\theta\sigma^{\mu}\overline{\theta}$, will satisfy Equation 5.31. We can expand this function using a Taylor series which again terminates thanks to the anti-commuting θs, the resulting form reads

$$\Phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{5.36}$$

$$= A(x) + i\theta\sigma^{\mu}\overline{\theta}\partial_{\mu}A(x) - \frac{1}{4}\theta^2\overline{\theta}^2\Box A(x)$$

$$+ \sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\overline{\theta} + \theta\theta F(x). \tag{5.37}$$

where the factor of $\sqrt{2}$ in equation 5.36 is there by convention. The anti-chiral superfield, using $y^{\dagger\mu} = x^{\mu} - i\theta\sigma^{\mu}\overline{\theta}$ has the form

$$\Phi^{\dagger}(y^{\dagger},\theta) = A^*(y^{\dagger}) + \sqrt{2\overline{\theta}\overline{\psi}}(y^{\dagger}) + \overline{\theta\theta}F^*(y^{\dagger}) \tag{5.38}$$

$$= A^*(x) - i\theta\sigma^{\mu}\overline{\theta}\partial_{\mu}A^*(x) - \frac{1}{4}\theta^2\overline{\theta}^2\Box A^*(x)$$

$$+ \sqrt{2\overline{\theta}\overline{\psi}}(x) + \frac{i}{\sqrt{2}}\overline{\theta}^2\theta\sigma^{\mu}\partial_{\mu}\psi(x) + \overline{\theta\theta}F^*(x). \tag{5.39}$$

The fields A and F are complex scalars and ψ is a Weyl fermion. This has the correct composition to be a chiral superfield. We can check that the condition in Equation 5.31 remains invariant under a supersymmetric transformation. It is easy to see this by using the commutation relations

$$[D_{\alpha}, \xi Q] = \left[D_{\alpha}, \overline{\xi Q}\right] = 0, \tag{5.40}$$

$$\left[\overline{D}_{\dot{\alpha}}, \xi Q\right] = \left[\overline{D}_{\dot{\alpha}}, \overline{\xi Q}\right] = 0. \tag{5.41}$$

Applying an infinitesimal supersymmetry transformation to Equation 5.31 and applying the commutation relations we have

$$\overline{D}_{\dot{\alpha}}\left[(1+\delta_{\eta})\,\Phi\right] = i\overline{D}_{\dot{\alpha}}\left(\eta Q\Phi + \overline{\eta}\overline{Q}\Phi\right) = i\eta Q\overline{D}_{\dot{\alpha}}\Phi + i\overline{\eta}\overline{Q}\overline{D}_{\dot{\alpha}}\Phi = 0, \tag{5.42}$$

and so this condition is consistent. That is, $\overline{D}_{\dot{\alpha}}\Phi$ is a supersymmetric invariant constraint we can impose on a superfield Φ to reduce the number of its components, while still having the field carrying a representation of the supersymmetry algebra

We would now like to examine how the individual components (A, ψ, F) of the superfield transform under Supersymmetry transformations. There are a number of ways to calculate the way in which the individual components travel. The easiest way is to start by expressing the Supersymmetry generators, $Q, \overline{Q}$ in terms of the variables $y, \theta, \overline{\theta}$

$$Q_\alpha = -i\partial_\alpha, \quad \overline{Q}_{\dot\alpha} = i\overline{\partial}_{\dot\alpha} + 2\theta^\alpha \sigma^\mu_{\alpha\dot\alpha}\frac{\partial}{\partial y^\mu}. \tag{5.43}$$

Now we can directly apply a Supersymmetry transformation to the superfield $\Phi(y, \theta)$ and expand,

$$\begin{aligned}
\delta_\eta \Phi(y,\theta) &= i(\eta Q + \overline{\eta}\overline{Q})\Phi(y,\theta) && (5.44)\\
&= i\left(-i\eta^\alpha\partial_\alpha + 2\theta^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{\eta}^{\dot\alpha}\frac{\partial}{\partial y^\mu}\right)(A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)) && (5.45)\\
&= \sqrt{2}\eta\psi + \theta^\alpha\left(2\eta_\alpha\, F + 2i\sigma^\mu_{\alpha\dot\alpha}\overline{\eta}^{\dot\alpha}\partial_\mu A\right) - \theta^2\,\sqrt{2}i\partial_\mu\left(\psi\sigma^\mu\overline{\eta}\right). && (5.46)
\end{aligned}$$

Comparing this with the form for the chiral superfield in equation 5.36, we can identify the shifts in the components as

$$\begin{aligned}
\delta_\eta A &= \sqrt{2}\eta\psi, && (5.47)\\
\delta_\eta \psi_\alpha &= \sqrt{2}\eta_\alpha F + \sqrt{2}i\sigma^\mu_{\alpha\dot\alpha}\overline{\eta}^{\dot\alpha}\partial_\mu A, && (5.48)\\
\delta_\eta F &= -\sqrt{2}i\partial_\mu\left(\psi\sigma^\mu\overline{\eta}\right). && (5.49)
\end{aligned}$$

The most important result of these transformations is that the F- component transforms into a total space-time derivative. This enables us to start writing down supersymetrically-invariant actions. The first thing that we could write down is simply the F-component of the chiral superfield. The F-component is the term in the chiral superfield which has a coefficient θ^2. This means that if we differentiate a chiral superfield twice with respect to θ, then we will be left with the F-component. Alternatively, in terms of an integral over θ we can express the F-term more neatly as $\int d\theta^2\Phi$. Thus, an invariant action is

$$S = \int d^4x \int d^2\theta\Phi. \tag{5.50}$$

However, this particular action is not very interesting as it is only linear and due to gauge invariance the superfield involved cannot be charged under a gauge symmetry. In order to write down an interacting theory we use the fact that a product of chiral superfields is still a chiral superfield and so it follows that

$$S = \frac{1}{4}\int d^4x \int d^2\theta\left(a\Phi + b\Phi^2 + c\Phi^3 + ...\right), \tag{5.51}$$

is a supersymmetric invariant.

The object under the theta integral is called the superpotential and is usually denoted $W(\Phi)$. As we will see, it gives rise to masses and interaction terms for both scalars and fermions. However, it does not generate any kinetic terms and so we need to look for another invariant.

We go back to the general form for a superfield in equation 5.30, repeated here for convenience

$$\begin{aligned}
Y(x,\theta,\overline{\theta}) = \varepsilon(x) + \theta\psi(x) + \overline{\theta}\overline{\chi}(x) + \theta^2 m(x) + \overline{\theta}^2 n(x) +\\
\theta\sigma^\mu\overline{\theta}v_\mu(x) + \theta^2\overline{\theta}\overline{\lambda}(x) + \overline{\theta}^2\theta\zeta(x) + \theta^2\overline{\theta}^2 d(x).
\end{aligned}$$

Examining how these components change under a Supersymmetry transformation we find that the $d(x)$ component transforms,

$$
\begin{aligned}
\delta_\eta Y(x,\theta,\overline{\theta}) &= i(\eta Q + \overline{\eta}\overline{Q})\Phi(x,\theta,\overline{\theta}) \\
&= -i\left(\eta^\alpha(i\partial_\alpha + \sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu) + \overline{\eta}^{\dot{\alpha}}(i\overline{\partial}_{\dot{\alpha}} + \theta^\beta\sigma^\mu_{\beta\dot{\alpha}}\partial_\mu)\right)\Phi(x,\theta,\overline{\theta}) \qquad (5.52) \\
&= \frac{i}{2}\theta^2\overline{\theta}^2\partial_\mu\left[\eta\sigma^\mu\overline{\lambda} - \zeta\sigma^\mu\overline{\eta}\right] + \ldots, \qquad (5.53)
\end{aligned}
$$

where we have only calculated the component with coefficient $\theta^2\overline{\theta}^2$. This shows that the d component transforms as to a total derivative

$$
\delta_\eta d = \frac{i}{2}\partial_\mu\left[\eta\sigma^\mu\overline{\lambda} - \zeta\sigma^\mu\overline{\eta}\right], \qquad (5.54)
$$

under a Supersymmetry transformation.

We started this by asking how the kinetic terms for the fields in the chiral superfields are generated. We now have a further invariant that we can construct but we need to find how this object is related to the chiral superfields. If we multiply a superfield by its anti-chiral superfield, $\Phi^\dagger\Phi$ and expand this out in terms of θs and $\overline{\theta}$s we get the full Taylor expansion shown in Equation 5.30 including a term with the coefficient $\theta^2\overline{\theta}^2$.

The result of all of this is that we can write down another invariant

$$
S_{KE} = \int d^4x \int d^4\theta \left(\Phi^\dagger\Phi\right), \qquad (5.55)
$$

where we have made use of the integral form of the derivatives to pull out the term with coefficient $\theta^2\overline{\theta}^2$. Taking a closer look at the expansion of $\Phi^\dagger\Phi$ as a function of x^μ we find that the kinetic terms for the scalar and fermionic components are generated by Equation 5.55,

$$
\begin{aligned}
S_{KE} &= \int d^4x \int d^4\theta \left(\Phi^\dagger\Phi\right) \\
&= \int d^4x \left(\frac{1}{4}A^*\Box A + \frac{1}{4}\Box A^* A - \frac{1}{2}\partial_\mu A^*\partial^\mu A + \frac{i}{2}\left(\partial_\mu\psi\sigma^\mu\overline{\psi} - \psi\sigma^\mu\partial_\mu\overline{\psi}\right) + \ldots\right) \\
&= \int d^4x \left(\partial_\mu A^*\partial^\mu A - i\psi\sigma^\mu\partial_\mu\overline{\psi} + |F|^2\right). \qquad (5.56)
\end{aligned}
$$

As we should expect, there is no kinetic term for the auxiliary field F.

We can now start to pull things together. We can identify the mass terms and interaction terms in the superpotential. As the action must be dimension 0, this means that $\int d^4\theta\left(\Phi^\dagger\Phi\right)$ must also have dimension 0.

The mass dimension of θ is -1/2, which means that a chiral superfield is dimension 1. This of course makes sense as the lowest component of a chiral superfield is the scalar component which is also dimension 1. The Weyl fermion has dimension 3/2 and the auxiliary field, F, has mass dimension 2.

Examining the form of the superpotential in Equation 5.51, we find that a has the dimensions of mass squared, b has the dimensions of mass and c is dimensionless. Higher-order terms will be non-renormalisable and in fact the linear term can be removed by shifting the superfield from $\Phi \to \Phi + C$, where C is constant of mass dimension 1. We are now in a position to write down the most general renormalisable action (for a single chiral superfield). The final form reads

$$S = \int d^4x \left(\int d^4\theta \left(\Phi^\dagger \Phi \right) + \int d^2\theta \left(\frac{1}{2} m \Phi^2 + \frac{1}{6} y \Phi^3 \right) + c.c \right). \tag{5.57}$$

This particular action has no gauge fields yet and is called the Wess-Zumino model [24]. Analysing this model further, we can write down, with the use of the expansions given Appendix B, the bosonic part of the action

$$S_{bos} = \int d^4x \left(\partial^\mu A^* \partial_\mu A + F^* F + \left(mAF + yA^2 F + c.c \right) \right). \tag{5.58}$$

We can eliminate the auxiliary field, F, using the equations of motion we have

$$F^* = -\left(mA + yA^2 \right). \tag{5.59}$$

Substituting this back into the action we have

$$S_{bos} = \int d^4x \left(\partial^\mu A^* \partial_\mu A - \left| mA + yA^2 \right|^2 \right). \tag{5.60}$$

We generate a mass for the scalar field as well as interaction terms, all determined by m and y. We identify $V(A, A^*) = \left| mA + yA^2 \right|^2 = |F|^2$ as the scalar potential. This can be generalised to include more fields

$$V_F(A_i, A_i^*) = \sum_i |F_i|^2, \tag{5.61}$$

where

$$F_i^* = \left. \frac{\partial W(\Phi_i, ..\Phi_n)}{\partial \Phi_i} \right|_{\Phi_i = A_i} \quad \text{and} \quad F_i = \left. \frac{\partial W^*(\Phi_i^\dagger, ..\Phi_n^\dagger)}{\partial \Phi_i^\dagger} \right|_{\Phi_i^\dagger = A_i^*}. \tag{5.62}$$

The masses and Yukawa couplings of the fermions can be found in a similar way by expanding out the products of the superfields and collecting the fermionic parts. Appendix B lists these expansions and it is easier to read off the fermionic interaction and mass terms.

Instead of the doing this however, and for a general superpotential, we can find the terms involving 2 fermionic fields by,

$$\int d^2\theta W(\Phi_i) = -\left. \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \right|_{\Phi_i = A_i} \psi_i \psi_j, \tag{5.63}$$

where the expression on the right will include the mass terms for the fermions as well as the Yukawa interaction terms. For the Wess-Zumino case we have,

$$\left. \frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j} \right|_{\Phi_i = A_i} = m + 2yA, \tag{5.64}$$

$$\begin{aligned} \text{which gives mass terms} \quad &\propto -\left(m\psi\psi + m^* \overline{\psi\psi} \right) \\ \text{and Yukawa terms} \quad &\propto -yA\psi\psi - y^* A^* \overline{\psi\psi}. \end{aligned}$$

In general we can therefore write

$$S_{\text{fermionic}} = \int d^4x \left[-i\psi_i\sigma^\mu\partial_\mu\overline{\psi}_i - \frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\Big|_{\Phi=A}\psi_i\psi_j - \left(\frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}\Big|_{\Phi=A}\right)^\dagger \overline{\psi}_i\overline{\psi}_j \right]. \tag{5.65}$$

5.2 Vector Superfields

If we want a phenomenologically viable theory, gauge fields must be included. We can do this by introducing the vector (or gauge) superfield. The vector superfield is a real superfield (that is $V = V^\dagger$), which we can Taylor expand as

$$\begin{aligned} V(x,\theta,\overline{\theta}) &= C + i\theta\chi - i\overline{\theta}\overline{\chi} + \theta\sigma^\mu\overline{\theta}A_\mu + \frac{i}{2}\theta^2(M+iN) - \frac{i}{2}\overline{\theta}^2(M-iN) \\ &+ \theta^2\overline{\theta}\left(\overline{\lambda} - \frac{1}{2}\overline{\sigma}^\mu\partial_\mu\chi\right) + \overline{\theta}^2\theta\left(\lambda + \frac{1}{2}\sigma^\mu\partial_\mu\overline{\chi}\right) \\ &+ \frac{1}{2}\theta^2\overline{\theta}^2\left(D - \frac{1}{2}\partial^\mu\partial_\mu C\right). \end{aligned} \tag{5.66}$$

The components C, D, M, N and A_μ must be real to satisfy $V = V^\dagger$. With 8 bosonic and 8 fermionic degrees of freedom, we can see that we have too many fields in the full expansion to form an irreducible representation (After gauge fixing, this will reduce the number of off-shell degrees of freedom to 4B+4F, which become 2B+2F on-shell (for a massless representation),as it should be the case for a massless vector multiplet of states).

First notice that $i(\Phi - \Phi^\dagger)$ is a vector superfield if Φ is a chiral superfield. We can define a supersymmetric version of an Abelian gauge transformation as,

$$V \to V + i\left(\Lambda - \Lambda^\dagger\right), \tag{5.67}$$

where Λ and $\Lambda^\dagger$ are chiral and anti-chiral superfields respectively. From Appendix B, Equation B.8, we have

$$\begin{aligned} \Lambda - \Lambda^\dagger &= A - A^* + \sqrt{2}\left(\theta\psi + \overline{\theta\psi}\right) + \theta\theta F - \overline{\theta\theta}F^*(x) \\ &+ i\theta\sigma^\mu\overline{\theta}\partial_\mu(A + A^*) + \frac{i}{\sqrt{2}}\theta\theta\overline{\theta}\overline{\sigma}^\mu\partial_\mu\psi \\ &- \frac{i}{\sqrt{2}}\overline{\theta\theta}\theta\sigma^\mu\partial_\mu\overline{\psi} - \frac{1}{4}\theta^2\overline{\theta}^2\Box(A - A^*). \end{aligned} \tag{5.68}$$

From this we can see how all the components have transformed

$$C \to C + i(A - A^*) \tag{5.69}$$
$$\chi \to \chi + \sqrt{2}\psi \tag{5.70}$$
$$A_\mu \to A_\mu - \partial_\mu(A + A^*) \tag{5.71}$$
$$M + iN \to M + iN + 2F \tag{5.72}$$
$$\lambda \to \lambda \tag{5.73}$$
$$D \to D. \tag{5.74}$$

For the vector field, A_μ, the effect of the supersymmetric gauge transformation is to shift $A_\mu \to A_\mu - \partial_\mu(A+A^*)$, which is the correct form for a (Abelian) gauge transformation of a vector field[3].

We can chose components of Λ (A, ψ and F) in such a way as to set C, M, N, χ to zero as well as one degree of freedom from A_μ. This particular gauge choice is called the Wess-Zumino gauge and we are left with the vector superfield

$$V_{WZ} = \theta\sigma^\mu\overline{\theta}A_\mu(x) + \theta^2\overline{\theta}\overline{\lambda}(x) + \overline{\theta}^2\theta\lambda(x) + \frac{1}{2}\theta^2\overline{\theta}^2 D(x), \tag{5.75}$$

where A_μ the vector boson has mass dimension 1, λ the Weyl fermion (or gaugino) has mass dimension 3/2 and the auxiliary field, D, has mass dimension 2. Moreover, the is superfield has 4 off-shell bosonic degrees of freedom (3 and 1 for A_μ and D respectively) and 4 off-shell fermionic degrees of freedom (λ). On shell, we have 2 bosonic from the on-shell vector boson, A_μ (0 from the Auxiliary field) and 2 on-shell fermionic degrees of freedom from the Weyl spinor.

Powers of V_{WZ} are given by

$$V_{\mathrm{WZ}}^2 = \frac{1}{2}\theta^2\overline{\theta}^2 A^\mu A_\mu, \;\; V_{\mathrm{WZ}}^3 = 0, \tag{5.76}$$

with all higher powers also 0.

Note that the Wess Zumino gauge is not supersymmetric. However, under a combination of a supersymmetry and a generalised gauge transformation we can end up with a vector field in Wess Zumino gauge.

Under an infinitesimal supersymmetry transformation the components of the vector superfield transform (in Wess-Zumino gauge) according to

$$\delta_\eta A_\mu = \eta\sigma^\mu\overline{\lambda} + \lambda\overline{\sigma}^\mu\overline{\eta}, \tag{5.77}$$

$$\delta_\eta \lambda_\alpha = \frac{i}{2}\left(\sigma^\mu\overline{\sigma}^\nu\eta\right)_\alpha F_{\mu\nu} + \eta_\alpha D, \tag{5.78}$$

$$\delta_\eta D = i\partial_\mu\left[\eta\sigma^\mu\overline{\lambda} - \lambda\sigma^\mu\overline{\eta}\right]. \tag{5.79}$$

Now we understand the general mechanism of introducing a Supersymmetry gauge transformation we can now start to introduce the terms one would usually expect to find. The first of which is the interaction of the matter fields with the components of the vector superfield (that is the gauge boson and its associated fermion state, the gaugino).

Considering to begin with an abelian gauge symmetry. In the non-supersymmetric case for a complex scalar field $\varphi(x)$ coupled to an abelian gauge field, A_μ via a covariant derivative $D_\mu = \partial_\mu - iqA_\mu$ we have the following under a gauge transformation

$$\varphi \to e^{iq\alpha(x)}\varphi, \tag{5.80}$$

$$A_\mu \to A_\mu + \partial_\mu\alpha(x), \tag{5.81}$$

where we are assuming a local $U(1)$ with charge q and local parameter $\alpha(x)$.

[3] Assuming A_μ is charged under the gauge symmetry.

Under supersymmetry we need to generalise these fields are promoted to a chiral superfield Φ and a vector superfield V. To construct a gauge invariant quantity out of these objects we impose the following transformation properties

$$\Phi \to e^{2iq\Lambda}\Phi, \tag{5.82}$$

$$V \to V - i\left(\Lambda - \Lambda^\dagger\right). \tag{5.83}$$

With these transformations it is easy to see that the object

$$\Phi^\dagger e^{2qV}\Phi = \text{ gauge invariant.} \tag{5.84}$$

Here, Λ is the chiral superfield defining the generalised gauge transformations. As stated before, the product of any number of chiral superfields is still a chiral superfield and as a consequence the object $\exp(2iq\Lambda)\Phi$ is also a chiral superfield.

A further term we need is the kinetic terms or field strength for the gauge fields. In a non-supersymmetric theory we have

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\nu, \tag{5.85}$$

as the abelian field strength. The supersymmetric generalisation of this is

$$W_\alpha = -\frac{1}{4}\overline{D}\overline{D}D_\alpha V, \tag{5.86}$$

where the D and $\overline{D}$s are the super covariant derivatives and as a reminder have forms

$$\overline{D}_{\dot{\alpha}} = -\overline{\partial}_{\dot{\alpha}} - i\theta^\beta \sigma^\mu_{\beta\dot{\alpha}}\partial_\mu, \tag{5.87}$$

$$D_\alpha = \partial_\alpha + i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu. \tag{5.88}$$

The supersymmetric field strengths are chiral (and anti-chiral respectively) and supergauge invariant. It is easiest to show the chirality of these objects by expressing them in their component form in the Wess-Zumino gauge with the change of variable $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$. The covariant derivatives in this case are then

$$\begin{aligned} D_\alpha &= \partial_\alpha + 2i\sigma^\mu_{\alpha\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\frac{\partial}{\partial y^\mu}, \\ \overline{D}_{\dot{\alpha}} &= -\overline{\partial}_{\dot{\alpha}}. \end{aligned} \tag{5.89}$$

The expression for the vector superfield in these coordinates and in the WZ-gauge is

$$V_{\text{WZ}}(y,\theta,\overline{\theta}) = \theta\sigma^\mu\overline{\theta}A_\mu(y) + \theta^2\overline{\theta\lambda}(y) + \overline{\theta}^2\theta\lambda(y) + \frac{1}{2}\theta^2\overline{\theta}^2\left[D(y) - i\partial^\mu A_\mu(y)\right]. \tag{5.90}$$

Now we can calculate the supersymmetric field strengths and the result is

$$W_\alpha(y,\theta) = \lambda_\alpha(y) + \theta_\alpha D(y) + (\sigma^{\mu\nu})^\beta_\alpha\theta_\beta F_{\mu\nu} - i\theta^2\sigma^\mu_{\alpha\dot{\beta}}\partial_\mu\overline{\lambda}^{\dot{\beta}}(y). \tag{5.91}$$

It is then clear first of all that $W_\alpha(y,\theta)$ is generalised gauge invariant as each of the components (λ, D and $F_{\mu\nu}$) are gauge invariant. It is also an easy task to now prove that $W_\alpha(y,\theta)$ is chiral as it is only a function of y and θ and with $\overline{D}_{\dot{\alpha}} = -\overline{\partial}_{\dot{\alpha}}$ we have

$$\overline{D}_{\dot{\alpha}}W_\alpha(y,\theta) = 0. \tag{5.92}$$

5.2.1 Non-abelian gauge symmetry

We now extend our discussion to non-abelian gauge groups. Introducing T^a as the hermitian generators of the non-abelian gauge symmetry we now that

$$\Lambda = \Lambda_a T^a, \qquad V = V_a T^a, \qquad \left[T^a, T^b\right] = i f^{abc} T_c. \tag{5.93}$$

Just like in the Abelian case, we would like to keep the quantity $\Phi^\dagger e^{2qV}\Phi$ invariant under the transformation $\Phi \to e^{iq\Lambda}\Phi$. For the non-abelian case, the corresponding transformation for the vector superfield is more complicated. It is clear that we need

$$e^{2V} \to e^{i\Lambda^\dagger} e^{2V} e^{-i\Lambda}. \tag{5.94}$$

to establish how V transforms we need to employ the Baker-Campbell-Hausdorff formula for non-commuting objects

$$e^X e^Y = e^{X+Y+\frac{1}{2}[X,Y]+\cdots}. \tag{5.95}$$

Given this we find from equation 5.94 that V needs to transform as

$$V \to V - \frac{i}{2}(\Lambda - \Lambda^\dagger) - \frac{i}{2}[V, \Lambda + \Lambda^\dagger]. \tag{5.96}$$

The field strength superfield for a non-abelian gauge symmetry is also modified and has the form

$$W_\alpha = -\frac{1}{8}\overline{DD}\left(e^{-2V} D_\alpha e^{2V}\right) \tag{5.97}$$

The field strength superfield transforms under the generalised gauge transformation in analogy with the non-susy field strength tensor (that is $F_{\mu\nu} \to U F_{\mu\nu} U^{-1}$), i.e. covariantly

$$W_\alpha \to e^{i\Lambda} W_\alpha e^{-i\Lambda}. \tag{5.98}$$

In the Wess-Zumino gauge, the supersymmetric field strengths has the form

$$W_\alpha(y,\theta) = \lambda_\alpha(y) + \theta_\alpha D(y) (\sigma^{\mu\nu})_\alpha^{\beta} \theta_\beta F_{\mu\nu} - i\theta^2 \sigma^\mu_{\alpha\dot{\beta}} D_\mu \overline{\lambda}^{\dot{\beta}}(y), \tag{5.99}$$

$$\tag{5.100}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i\left[A_\mu, A_\nu\right] \tag{5.101}$$

$$D_\mu \overline{\lambda} = \partial_\mu \overline{\lambda} + i\left[A_\mu, \overline{\lambda}\right] \tag{5.102}$$

and where $F_{\mu\nu} = F^a_{\mu\nu} T^a$, etc.

5.2.2 Abelian vector superfield Lagrangian.

We would like to construct the Lagrangian for the vector superfield and its components interactions with the matter fields. There are two terms that are added to the Lagrangian. In a non-supersymmetric theory we ensure gauge invariance for a scalar field charged under a local $U(1)$ by introducing the covariant derivative that acts on the scalar state as

$$D_\mu \varphi = \partial_\mu - iqA_\mu \tag{5.103}$$

with Lagrangian density

$$\mathcal{L} = D^\mu \varphi (D_\mu \varphi)^* + \dots . \tag{5.104}$$

Under a gauge transformation

$$\varphi \to e^{iq\alpha(x)}, \qquad A_\mu \to A_\mu + \partial_\mu \alpha(x). \tag{5.105}$$

In the non-supersymmetric case we also have the kinetic term for A_μ as

$$\mathcal{L} = \dots \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{5.106}$$

In Supersymmetry , we have already seen the term that generates the kinetic term for the matter chiral superfields. This terms is called the Käbler potential, $K = \Phi^\dagger \Phi$ and it is not invariant under the supersymmetric version of a gauge transformation,

$$\Phi \to e^{2iq\Lambda}\Phi, \quad \Phi^\dagger \Phi \to \Phi^\dagger e^{2iq(\Lambda^\dagger - \Lambda)} \Phi. \tag{5.107}$$

Analogously to the non-supersymmetric case we must modify this term to ensure gauge invariants. We of course have already seen the object we need involving the vector superfield. The Kähler potential is modified to read

$$K = \Phi^\dagger e^{2qV} \Phi, \tag{5.108}$$

with the vector superfield, V, transforming under the generalised gauge transformation as

$$V \to V - i(\Lambda - \Lambda^\dagger). \tag{5.109}$$

The next thing we need is the kinetic terms for the components of the vector superfield, namely the vector boson and Weyl fermion. We again follow the non-supersymmetric case and construct the kinetic terms from the field strengths. Recall, that the product of two chiral superfields is itself a chiral superfield and that we must still have terms that are invariant under supersymmetric transformations tells us that the kinetic terms for the vector superfield components must come from a superpotential term. The term in the Lagrangian is then

$$\mathcal{L}_{\text{kinetic}} = \int d^2\theta \; f(\phi) W^\alpha W_\alpha, \tag{5.110}$$

where $f(\phi)$ is called the gauge kinetic function and is a function of some chiral superfield or superfields. The scalar component of this (non-dynamic) superfield will gain an expectation value through some dynamics and as a result the gauge kinetic function will become a possibly complex constant. We will come back to this in the non-abelian case.

A further and important addition in the supersymmetric case compared to the non-supersymmetric case a term know as the Fayet-Iliopoulos term. This terms is invariant under supersymmetric and abelian gauge transformations and has the form

$$\mathcal{L}_{\mathrm{FI}} = \int d^4\theta \xi V = \frac{1}{2}\xi D(x). \tag{5.111}$$

This term is only allowed in abelian gauge theories as otherwise the D component would transform under the gauge transformation where as for a $U(1)$ it does not.

The resulting Lagrangian for Super-QED is then

$$\mathcal{L} = \int d^4\theta \left[\sum_i \Phi_i^\dagger e^{2q_i V}\Phi_i + \xi V\right] + \left(\int d^2\theta \; [W(\Phi_i) + fW^\alpha W_\alpha] + \text{h.c.}\right), \tag{5.112}$$

where $W(\Phi_i)$ is the superpotential, which is a function of at least two superfields, otherwise it is zero. We have already examined the component expansion of the superpotential. Let us expand the other terms we have constructed involving the vector superfield. Firstly, the Kähler potential

$$\begin{aligned}\int d^4\theta \Phi^\dagger e^{2qV}\Phi &= \int d^4\theta \; \Phi^\dagger(1+2qV+2q^2V^2)\Phi = F^*F + \partial_\mu A\partial^\mu A^* - i\overline{\psi}\overline{\sigma}^\mu D_\mu\psi \\ &+ qA^\mu(iA^*\partial_\mu A - iA\partial_\mu A^*) + \sqrt{2}q(A\overline{\lambda\psi} + A^*\lambda\psi) + q(D + qA_\mu A^\mu)\left|A\right|^2 \\ &= F^*F + \left|D_\mu A\right|^2 - i\overline{\psi}\overline{\sigma}^\mu D_\mu\psi + \sqrt{2}q(A\overline{\lambda\psi} + A^*\lambda\psi) + qD\left|A\right|^2,\end{aligned} \tag{5.113}$$

where $D_\mu = \partial_\mu - iqA_\mu$. We notice that there are no kinetic terms for the field D, as we expected given its role as an auxiliary field in the vector superfield.

Now let us examine the $W^\alpha W_\alpha$ term, fixing $f = \frac{1}{4g^2}$:

$$\int d^2\theta f \; W^\alpha W_\alpha + \text{h.c} = \frac{1}{2g^2}D^2 - \frac{1}{4g^2}F^{\mu\nu}F_{\mu\nu} - \frac{i}{g^2}\lambda\sigma^\mu\partial_\mu\overline{\lambda}. \tag{5.114}$$

If we also include the FI contribution, the terms involving the D field are

$$\mathcal{L}_D = qD\left|A\right|^2 + \frac{1}{2g^2}D^2 + \frac{1}{2}\xi D. \tag{5.115}$$

We know that D is an auxiliary field and so we can eliminate it using the equations of motion,

$$\frac{\delta\mathcal{S}_D}{\delta D} = 0, \rightarrow \; D = -g^2\left(\frac{\xi}{2} + q\left|A\right|^2\right). \tag{5.116}$$

Substituting this back into $\mathcal{L}_D$, we get

$$\mathcal{L}_D = -\frac{g^2}{2}\left(\frac{\xi}{2} + q\left|A\right|^2\right)^2 \equiv -V_D(A, A^*), \tag{5.117}$$

where $V_D(A, A^*)$ is a scalar potential (careful of the clash in notation). Generalising to multiple fields and combining with the scalar potential from eliminating the auxiliary field F, $V_F(A_i, A_i^*)$, we can write down the total scalar potential for super QED

$$V(A) = V_F(A_i, A_i^*) + V_D(A_i, A_i^*) = \sum_i \left| \frac{\partial W}{\partial \Phi_i}\Big|_{\Phi_i = A_i} \right|^2 + \sum_i \frac{g^2}{2} \left(\frac{\xi}{2} + q_i \left| A_i \right|^2 \right)^2 \quad (5.118)$$

5.2.3 Non-abelian vector superfield Lagrangian.

For the non-Abelian case we can follow closely the procedure for the Abelian case. The kinetic terms for the matter fields contained within the chiral superfields and their interactions with the vector superfield are once again generated by the Käbler potential with form

$$\mathcal{L} = \int d^4\theta \ \mathrm{Tr}(\Phi^\dagger e^{2V} \Phi) \quad (5.119)$$

is the gauge invariant kinetic term for chiral superfields in any representation of the gauge groups with

$$\Lambda = \Lambda^a T_R^a, \ \ V = V^a T_R^a, \quad (5.120)$$

where R donates the way in which the chiral superfields transform under the gauge symmetry and T_R^a are the hermitian generators for that representation. The trace is over the gauge indices.

The non-abelian version of the kinetic terms for the vector superfield is straightforward, we write

$$\mathcal{L}_{\text{kinetic}} = \int d^2\theta f(\Phi) \ \mathrm{Tr}(W^\alpha W_\alpha) + \text{h.c}, \quad (5.121)$$

where once again the $f(\Phi)$ is the gauge kinetic function, the superfield Φ is sometimes called the gauge coupling superfield. If we assume that, with some convenient choice of parameters that

$$f = \frac{1}{8\pi i}\tau, \quad \tau \equiv \frac{\Theta}{2\pi} + \frac{4\pi i}{g_a^2}. \quad (5.122)$$

where the Θ is a CP-violating parameter, whose effect is to include a total derivative term in the Lagrangian density. In the non-Abelian case, this can have physical effects due to topologically non-trivial field configurations (instantons). This is something you should have already seen and is intimately connected to the strong CP problem and its potential solution in terms of axions.

Putting it all together the full action for a non-abelian gauge symmetry is

$$S = \int d^4x \left[\int d^4\theta \ \mathrm{Tr}(\Phi^\dagger e^{2V} \Phi) + \int d^2\theta \left(W(\Phi) + \frac{1}{8\pi i}\tau \ \mathrm{Tr}(W^\alpha W_\alpha) + \text{h.c} \right) \right] \quad (5.123)$$

If there is more than one gauge symmetry then we sum over the different groups.

Looking at the component expansion of the $W^\alpha W_\alpha$ term we have

$$\int d^2\theta \frac{1}{8\pi i}\tau \; \mathrm{Tr}(W^\alpha W_\alpha) + \mathrm{h.c} = \mathrm{Tr}\left(\frac{1}{g^2}D^2 - \frac{1}{2g^2}F^{\mu\nu}F_{\mu\nu} - \frac{2i}{g^2}\lambda\sigma^\mu D_\mu\overline{\lambda} + \frac{\Theta}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right), \tag{5.124}$$

where once again the trace is over the gauge indices and $D_\mu\overline{\lambda} = \partial_\mu\overline{\lambda} + i\left[A_\mu, \overline{\lambda}\right]$. Once again we find the kinetic terms for the gauge boson, A_μ and its supersymmetric partner weyl fermion, or gaugino λ. In addition we identify the usual CP-violating topological term associated with non-abelian gauge symmetries. We note that for a non-abelian symmetry there is no FI term due to the fact that the D field transforms under the non-abelian gauge symmetry where as for an abelian symmetry it does not. We also note that there is no kinetic term for the D field as expected.

The component expansion for the term generating the interactions between the component of the superfields (charged under the non-abelian gauge symmetry) and the vector superfield as well as the kinetic terms for ϕ reads

$$\begin{aligned}\int d^4\theta \mathrm{Tr}\left(\Phi^\dagger e^{2V}\Phi\right) &= F^*F + |D_\mu A|^2 - i\overline{\psi}\overline{\sigma}^\mu D_\mu\psi \\ &- \sqrt{2}(\overline{\lambda}^a(\overline{\psi}T^a A) + (A^*T^a\psi)\lambda) + D^a(A^*T^a A),\end{aligned} \tag{5.125}$$

where $D_\mu = \partial_\mu - iA^a_\mu T^a$. If we now collect up the D-field dependent terms we have

$$\mathcal{L}_D = \frac{1}{2g^2}D^a D^a + D^a(A^*T^a A). \tag{5.126}$$

The equations of motion for D are now

$$D^a = -g^2(A^*T^a A), \tag{5.127}$$

and so our D-term potential in this case is

$$V_D = \frac{g^2}{2}\left(A^*T^a A\right)^2. \tag{5.128}$$

Of course if there are multiple gauge symmetries we need to sum over them all to get the total D term potential.

5.3 R-Symmetry

It is worth noting at this point, that a supersymmetric theory can have a special global U(1) symmetry called an R-symmetry. Under an R-symmetry the θ variables transform as

$$\theta \to e^{i\beta}\theta, \;\; \overline{\theta} \to e^{-i\beta}\overline{\theta}, \tag{5.129}$$

where β is a constant parametrising the phase transformation. Recalling that an integral over θ is effectively the same operation as a differentiation with respect to θ we see that the θ measure transforms according to

$$d\theta \to e^{-i\beta}d\theta, \;\; d\overline{\theta} \to e^{i\beta}d\overline{\theta}. \tag{5.130}$$

It follows that the Supersymmetry generators transform as

$$Q \rightarrow e^{-i\beta} Q \qquad \overline{Q} \rightarrow e^{i\beta} \overline{Q}, \tag{5.131}$$

which in turn implies that the Supersymmetry generators have R-charges -1 and +1 respectively and so do not commute with the R-symmetry

$$[R, Q] = -Q \quad [R, \overline{Q}] = \overline{Q}. \tag{5.132}$$

A chiral superfield transforms under the R-symmetry as

$$\Phi(x^\mu, \theta) \rightarrow e^{in\beta} \Phi_i(x^\mu, e^{i\beta}\theta), \tag{5.133}$$

where n is the R-charge of the chiral superfield. In terms of components, this means we have the transformation laws

$$\phi \rightarrow e^{in\beta}\phi, \;\; \psi \rightarrow e^{i(n-1)\beta}\psi, \;\; F \rightarrow e^{i(n-2)\beta} F, \tag{5.134}$$

for the scalar, fermionic and F components respectively. The Kinetic terms for the chiral superfields ($\Phi^\dagger\Phi$) are automatically invariant under this R-symmetry, but the superpotential is only invariant if it transforms as

$$W \rightarrow e^{2i\beta} W. \tag{5.135}$$

Gauge vector superfields will always have an R-charge of zero since they are real. However, the components will transform. In the WZ gauge we have

$$A^\mu \rightarrow A^\mu, \qquad \lambda \rightarrow e^{i\beta}\lambda, \qquad D \rightarrow D \tag{5.136}$$

under the R-symmetry transformation.

If this minimal supersymmetric model is to describe the real world, additional terms must be added which break supersymmetry, generating mass differences between the known fermions, scalar and vector bosons and their superpartners (this will be discussed more below). For example, the gaugino mass term $m_\lambda \lambda\lambda$ will break the continuous R-symmetry. The result of this breaking may leave the Lagrangian invariant under a discrete remnant of the R-symmetry called R-parity. It can be defined as

$$R = (-1)^{3(B-L)+2s}. \tag{5.137}$$

All quarks, leptons and the Higgs bosons are even under R-parity where as all superpartners are odd. An extremely beneficial consequence of R-parity is that if it is conserved and as a consequence the lightest supersymmetric particle will be stable. This means that supersymmetry can provide a dark matter candidate.

6 The MSSM

Now we know how to write down a supersymmetric action, we would like to specify the particle content of our theory in terms of supermultiplets. We are of course guided by the particle content of the Standard Model and, as the name suggests, the Minimal Supersymmetric Standard Model (MSSM) is the minimal supersymmetric extension of the Standard Model. Each Standard Model particle appears as a component of either a chiral or vector supermultiplet and is accompanied by a superpartner with spin differing by half integer. Only chiral multiplets can contain fermions with chirality and as all the Standard Model fermions have this property, they are all contained in chiral supermultiplets.

As the left-handed and right-handed components of the Standard Model fermions transform differently under the gauge symmetries they must appear in different supermultiplets. As stated before, the Supersymmetry generators commute with the standard model gauge groups. Thus, each left-handed and right-handed fermion will have its own spin-0 partner.

In Table 1 is a list of all the chiral superfields we have in the MSSM. Also listed is how the chiral superfields transform under $SU(3)_C \times SU(2)_L$ along with their $U(1)_Y$ charges.

Chiral Superfield		Spin 0	Spin 1/2	$(SU(3)_C, SU(2)_L, U(1)_Y)$
Quarks/Squarks with 3 families	Q	$(\tilde{u}_L\ \tilde{d}_L) \equiv \tilde{Q}_L$	$(u_L\ d_L)$	$(\mathbf{3},\mathbf{2},\frac{1}{6})$
	U	$\tilde{u}_R^*$	$\overline{u_R}$	$(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3})$
	D	$\tilde{d}_R^*$	$\overline{d_R}$	$(\overline{\mathbf{3}},\mathbf{1},\frac{1}{3})$
Leptons/Sleptons with 3 families	L	$(\tilde{\nu}_L\ \tilde{e}_L) \equiv \tilde{L}_L$	$(\nu_L\ e_L)$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$
	E	$\tilde{e}_R^*$	$\overline{e_R}$	$(\mathbf{1},\mathbf{1},1)$
Higgs/Higgsinos	H_u	$(H_u^+\ H_u^0)$	$(\tilde{H}_u^+\ \tilde{H}_u^0) \equiv \tilde{H}_u$	$(\mathbf{1},\mathbf{2},\frac{1}{2})$
	H_d	$(H_d^0\ H_d^-)$	$(\tilde{H}_d^0\ \tilde{H}_d^-) \equiv \tilde{H}_d$	$(\mathbf{1},\mathbf{2},-\frac{1}{2})$

Table 1. Assignments of chiral superfields.

One thing we notice straight away is that there are now two Higgs doublets. There are two reasons for this. The first is anomaly cancellation. There would be a triangular gauge anomaly if we did not have two Higgs fields. The triangular anomaly in question is the $SU(2)_L^2 U(1)_Y$ anomaly. The anomaly coefficient is proportional to $\mathrm{Tr}[T_3^2 Y]$, where T_3 is the third component of weak isospin and the trace is over all Weyl fermions transforming non-trivially under $SU(2)_L$, which includes the Higgsinos. Using the hypercharge assignments in Table 1[4], we can see that this trace is zero and there is no anomaly. If we only had one Higgsino, then this would not be the case. The second reason is that we need the second Higgs field in order to write down a Yukawa interaction in the superpotential for the up-like quarks. The superpotential can only contain superfields and not anti-superfields (that is it is holomorphic) and so we are unable to use the conjugate of the single Higgs field to write down the up-like Yukawas as in the Standard Model .

[4]As all the gauginos have $Y = 0$, they do not contribute to the chiral anomaly, even though they are charged under $SU(2)_L$

The gauge bosons of the Standard Model must be contained in vector supermultiplets, with their fermionic partners, the gauginos. The superpartners of the gluons are the gluinos, the superpartners of the W's and B are the winos and bino respectively. After electroweak symmetry breaking the W^0 and B^0 mix to form the mass eigenstates Z^0 and γ. The corresponding superpartners are the zino ($\tilde{Z}^0$) and the photino ($\tilde{\gamma}$). In Table 2 we list the assignments for the vector (gauge) superfields in the MSSM.

Gauge Superfield	Spin 1/2	Spin 1	$(SU(3)_C, SU(2)_L, U(1)_Y)$
Gluino/Gluon	$\tilde{g}$	g	**(8,1,0)**
Wino/W bosons	$\tilde{W}^{\pm}$ $\tilde{W}^0$	$W^{\pm}$ W^0	**(1,3,0)**
Bino/B Boson	$\tilde{B}^0$	B^0	**(1,1,0)**

Table 2. Assignments of gauge superfields

We are now in a position to write down the superpotential of the MSSM. We know how to construct a superpotential and using the gauge assignments in Table 1, we can write down renormalisable gauge-invariant operators. The MSSM superpotential, suppressing generation indices, is given as

$$W_{MSSM} = Uy_uQH_u + Dy_dQH_d + Ey_eLH_d + \mu H_uH_d, \tag{6.1}$$

where we see that there is only one gauge invariant mass term and three Yukawa interaction terms. These Yukawa terms will generate supersymmetric masses for quarks, charged leptons, squarks and charged sleptons, after electroweak symmetry breaking.

Using only the chiral superfields of the MSSM, we can add to the superpotential other renormalisable terms that are invariant under the Standard Model gauge group. These terms are

$$\Delta W = \varepsilon_i L_i H_u + \lambda_{ijk} L_i L_j E_k + \lambda'_{ijk} L_i Q_j D_k + \lambda''_{ijk} U_i D_j D_k. \tag{6.2}$$

Each term in the above violates a global symmetry. The first three terms violate lepton number while the last violates baryon number. Experimental bounds on lepton and baryon number conservation restrict the sizes of these terms to be tiny if not zero. For example, limits on proton decay places very stringent bounds on the sizes of λ'_{ijk} and λ''_{ijk}. Each of these terms violate the global R-symmetry but unlike terms that are needed for supersymmetry breaking (see later), they do not leave the Lagrangian invariant under R-parity. Thus, one way to avoid these terms is to insist that the Lagrangian is invariant under R-parity.

We have already seen that each supermultiplet contains a scalar and a Weyl fermion and that, in the limit of unbroken supersymmetry, their masses are forced to be identical. This is clearly in complete contrast to what experiments have, or, in this case, have not, seen. If supersymmetry were exact, we would have already seen most of the superpartners by now. This leads us to conclude that supersymmetry must be broken.

Fortunately, there are clues to the nature of supersymmetry breaking. One of these can be obtained by re-examining the hierarchy problem. In order to avoid the quadratic divergences in the Higgs mass, we needed to cancel the quadratic divergence coming from a fermion loop with one coming from a scalar loop. A complete cancellation occurs if we have unbroken supersymmetry. When supersymmetry is broken, we still need the quadratic divergences to cancel and we can still do this provided the dimensionless couplings do not break Supersymmetry . This means that supersymmetry should only be broken by parameters with positive mass dimension, otherwise we would not cancel the quadratic divergence. This kind of symmetry breaking is referred to as soft breaking. We can parameterise the soft breaking by splitting the Lagrangian into two contributions

$$\mathcal{L} = \mathcal{L}_{SUSY} + \mathcal{L}_{soft}, \tag{6.3}$$

where $\mathcal{L}_{SUSY}$ preserves supersymmetry and $\mathcal{L}_{soft}$ breaks supersymmetry softly. If the largest soft scalar mass in the softly broken sector is, m_{soft}, then we will get corrections to the Higgs mass of the form

$$\Delta m_H^2 = m_{soft}^2 \left(\frac{\lambda}{16\pi^2} \ln[\Lambda/m_{soft}^2] \right), \tag{6.4}$$

where λ is a dimensionless coupling. The splitting between the Standard Model particles and their superpartners will be m_{soft} and so the superpartners cannot be too heavy, otherwise we will re-introduce a hierarchy, this time between the scale of the superpartners and the Higgs mass removing one of the main motivations of supersymmetry. Given the status of Supersymmetry searches this scenario seems to be emerging. Questions about how one quantitatively measures the degree of fine tuning in a model and indeed how much fine tuning one can bare are becoming more and more important.

In addition to solving the hierarchy problem and providing potential dark matter candidates, there is another aspect of supersymmetry that is highly attractive. The three gauge couplings have been measured at the Z-pole but when we increase the energy at which they are measured their values will change, i.e they are energy-dependent quantities. The way in which they change with energy is described by the renormalisation group equations (RGE). The relevant RGE for the gauge couplings reads

$$\frac{dg_a}{dt} = \frac{1}{16\pi^2} b_a g_a^3, \tag{6.5}$$

where $t = \ln(Q/Q_0)$, with Q the RG scale and Q_0 in this case will be the Z-pole. In the Standard Model, $b_a^{SM} = (41/10, -19/6, -7)$ and solving Equation 6.5, we find that, at no scale, Q, do the three couplings have the same value. In the MSSM, however, $b_a^{MSSM} = (33/5, 1, -3)$, the difference coming from the extra MSSM particles appearing in loops. If we then assume that the superpartners have an average mass of around m_{soft}, we can run the standard model gauge couplings from their experimentally measured values up to m_{soft} using the b_a^{SM} parameters. Then run the gauge couplings from m_{soft} up to high energy scales using the b_a^{MSSM} parameters. The striking result is that all three standard model gauge couplings converge to close to the same value in the ultra violet. In addition, the energy at which they converge is well below the Planck scale. This is a remarkable result. Since we have fixed the scale of m_{soft}, using naturalness arguments, to be near the electroweak scale, gauge coupling unification is a natural prediction of supersymmetry.

7 Breaking Supersymmetry

It is clear that a realistic model must contain supersymmetry breaking in order to explain experimental data. We ignore the possibility of so called hard breaking or explicit breaking. This form of supersymmetry breaking is ugly and has various problems when we consider the UV region of the theory. This suggests that supersymmetry is broken spontaneously. This means that although we have a Lagrangian that is exactly supersymmetric, the vacuum state is not. There are various ways of spontaneously breaking supersymmetry but here we employ a simple way in which we can parametrise the effects of supersymmetry breaking in the MSSM. In the context of effective field theories, one can do this by adding soft terms to the Lagrangian which break supersymmetry but do not break the electroweak symmetry (or any other Standard Model gauge group). Considering only renormalisable soft terms, we can add the following soft mass terms for the squarks and sleptons,

$$\begin{aligned}\mathcal{L}_{soft} \supset &- m_{\tilde{q}}^2|\tilde{Q}_L|^2 - m_{\tilde{L}}^2|\tilde{L}_L|^2 \\ &- m_{\tilde{u}}^2|\tilde{u}_R|^2 - m_{\tilde{d}}^2|\tilde{d}_R|^2 - m_{\tilde{e}}^2|\tilde{e}_R|^2,\end{aligned} \tag{7.1}$$

where, $m_{\tilde{Q}}$ and $m_{\tilde{L}}$ are the left-handed squark and left-handed slepton doublet soft masses respectively, $m_{\tilde{u}}^2$, $m_{\tilde{d}}^2$ and $m_{\tilde{e}}^2$ are the right-handed up-like squark, right-handed down-like squark and right-handed charged lepton soft mass squareds respectively, which are in general 3×3 Hermitian matrices in flavour space. We do not want to break the Standard Model gauge group so all states in an $SU(2)_L$ doublet or $SU(3)_c$ triplet will have the same Supersymmetry breaking soft mass. In the case of the left-handed doublets, both the up-like and down-like squarks have equal soft masses. The same is true for the charged slepton and sneutrino doublet. In addition to the squark and slepton soft masses the gauginos have soft mass terms

$$\mathcal{L}_{soft} \supset - \frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g}\right) + c.c, \tag{7.2}$$

where M_1, M_2 and M_3 are the masses of the Bino, Wino and gluinos respectively. We also have Higgs mass terms,

$$\mathcal{L}_{soft} \supset -m_{H_u}^2 H_u^\dagger H_u - m_{H_d}^2 H_d^\dagger H_d - BH_dH_u, \tag{7.3}$$

where $m_{H_u}^2$ and $m_{H_d}^2$ are the squared soft masses for the two Higgs doublets and B is a mass dimension 2 parameter, giving a mixing between the two Higgs doublets. In addition to the soft masses, we can write down tri-linear scalar interaction terms, or A-terms: they are

$$\mathcal{L}_{soft} \supset -(\tilde{u}_R\mathbf{A_u}\tilde{Q}_LH_u + \tilde{d}_R\mathbf{A_d}\tilde{Q}_LH_d + \tilde{e}_R\mathbf{A_e}\tilde{L}_LH_d) + c.c, \tag{7.4}$$

where the couplings $\mathbf{A_i}$ have mass dimension 1 and are, in general, complex 3×3 matrices in flavour space. In order to be "free" from fine tuning in the Higgs mass, we require that all mass dimension 2 objects have magnitudes $\sim m_{susy}^2$ and objects with mass dimension 1 have magnitudes $\sim m_{susy}$, with m_{susy} not much bigger than 10^3 GeV. As mentioned above, we are now facing the possibly that some soft terms maybe bigger than this scale.

It is clear that when we include these soft terms, we introduce a huge number of extra unknown parameters. In general these new parameters involve a mixing of flavours as well as CP violation. Experimentally, we can constrain the amount of flavour and CP violation allowed in the sfermion mass matrices by looking for various types of decays. In particular, looking at limits on branching ratios for flavour changing neutral current (FCNC) and CP violating processes. The experimental information coming from FCNCs and CP violating phenomena can be translated into upper bounds on the ratios $\delta_{ij} \equiv \Delta_{ij}/\tilde{m}$, where Δ_{ij} denotes off-diagonal entries in the sfermion mass matrices (i.e mass terms relating sfermions with different flavour, but the same electric charge) and $\tilde{m} = \sqrt{m_i m_j}$, is the average sfermion mass.

In order to avoid these experimental constraints we can either assume that the soft mass scale is a lot heavier than 1 TeV, this way diagrams that give rise to FCNCs are suppressed, however it is very difficult to avoid reintroducing fine tuning in the Higgs sector.

A further option is the squark and slepton soft mass matrices are assumed to have small off-diagonal entries. In fact, it is usually assumed that these matrices are diagonal. The tri-linear couplings, A_u, A_d, A_e, are also assumed to have specific forms. Again, these can be diagonal or an alternative is to assume they have the same flavour structure as the corresponding Yukawa couplings, thereby allowing only the third generation of squarks and sleptons to have large A couplings.

To avoid possible CP violating phases, we must assume that the soft parameters do not introduce additional phases. For the squark and slepton matrices we have assumed they are diagonal and since they have to be hermitian, the diagonal components have to be real. As for the tri-linear scalar couplings and the gaugino masses, if we want to restrict the CP-violating parameters solely to the CKM mixing matrix we can assume they have a phase of 0 or π At this stage there is no compelling theoretical reason to assume these forms but it is hoped that the mechanism which breaks supersymmetry will either enforce this somehow or will determine another way to avoid the experimental constraints. It is thus important to explore some aspects of supersymmetry breaking and ways in which it can be communicated to the MSSM.

8 F- and D-term Breaking.

So far we have introduced soft terms which have broken supersymmetry explicitly. We now need to explain how these terms are produced as a result of spontaneous supersymmetry breaking. The soft breaking we have described so far is an effective field theory description which encodes the effects of spontaneous supersymmetry breaking at low energy. We generically expect that the soft terms will be generated indirectly or radiatively. It is necessary that the fields responsible for breaking supersymmetry live in a “hidden sector”, which have zero or very small direct couplings to the “visible” MSSM sector. These two sectors however, must share some interactions and it is via these interactions that supersymmetry breaking is communicated. There are various ways in which this general set up can be realised but unfortunately there is no time to go into detail here.

If global supersymmetry is broken spontaneously, the vacuum state is not invariant under supersymmetric transformations. Moreover, the vacuum state must have non-zero energy, which, if we ignore any space-time effects or fermion condensates, implies that the scalar potential is non zero at its minimum. The full scalar potential has the form

$$V = V_D + V_F = \sum_a \frac{1}{2g^2} |D^a|^2 + \sum_i |F_i|^2 , \tag{8.1}$$

which implies that if any F_is and/or D^as gain non-zero VEVs then supersymmetry will be spontaneously broken. More concisely, the condition is that if, for any value of the fields, the equations, $F_i = 0$ and $D^a = 0$ cannot be satisfied simultaneously then supersymmetry is broken.

We know that the generators responsible for supersymmetric transformations are fermionic. Therefore, when we break supersymmetry we expect to see a massless Nambu-Goldstone fermion (goldstino) for every broken fermionic generator. This is in analogy to breaking bosonic generators and producing massless Nambu-Goldstone bosons.

As an example, consider the case where supersymmetry is broken by a non zero F component expectation value. Using the supersymmetric transformations of the components of a chiral multiplet, Equations 5.47, 5.48 and 5.49, in the limit where the fields are stationary we have the variations

$$\delta A_i \propto \xi\psi_i, \quad \delta\psi_{i\alpha} \propto \xi_\alpha F_i, \quad \delta F_i = 0. \tag{8.2}$$

We do not want to break Lorentz symmetry and so we are not permitted to allow ψ_i to gain a non zero VEV. Setting $\psi_i = 0$ we are left with, $\delta\psi_{i\alpha} \propto \xi_\alpha F_i$, as the only possible non-zero change. This confirms that if we wish to break supersymmetry this way, then a non zero value for F is the way to do it [5]. So if $F_i \neq 0$, then from Equation 8.2 we see that it is ψ_i which changes under the supersymmetry transformation and is consequently the goldstino.

A further example is to consider the case where the D^a components gain non zero VEVs. In this case, it is pertinent to look at how the gauge multiplet transforms under supersymmetric transformations. With stationary fields and ensuring Lorentz symmetry is not broken by setting λ^a and A^a_μ to zero, and using Equations 5.77, 5.78, and 5.79, we have,

$$\delta A^a_\mu = 0, \quad \delta\lambda^a_\alpha \propto \eta_\alpha D^a, \quad \delta D^a = 0. \tag{8.3}$$

We see that if we allow D to gain a VEV then supersymmetry is broken. In this case the gaugino, λ^a, is the goldstino as it receives a shift in value from the supersymmetry transformation. In the next section we examine how these D and F component VEVs can be generated.

[5] As F is a function of A_is, we might expect that some of these A_is may get VEVs as well. However, giving a scalar component a VEV does not automatically break supersymmetry, which is clear from the supersymmetric transformation laws in Equation 8.2. If the scalar field, A_i, carries any global or gauge charges then the corresponding symmetry will be spontaneously broken when A_i gets a VEV.

8.1 F-Breaking

F-term breaking models are called O'Raifeartaigh models [8]. The idea is to write down a superpotential involving chiral superfields that generates a non zero expectation value for at least one F_i. This type of supersymmetry breaking model is subject to a sum rule or supertrace given by

$$Str\mathcal{M}^2 \equiv \sum_{J=0,1/2} (-1)^{2J}(2J+1)m_J^2 = 0 \tag{8.4}$$

where J is the spin of the particle or sparticle. This constraint on the masses means that in these types of model some of the scalars are lighter than their fermionic partners. This is clearly in conflict with what is observed in experiments.

As an example the minimal O'Raifeartaigh model requires three chiral superfields, where at least one must be a gauge singlet. The superpotential has the form

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2, \tag{8.5}$$

where k is a mass dimension 2 parameter, m is a mass and y is a dimensionless coupling. Here we assume that Φ_1 is a gauge singlet. However, we do not assume anything about the gauge structure of this theory, other than that the terms in Equation 8.5 are gauge invariant. This superpotential is invariant under one global phase symmetry, which in this case is an R-Symmetry. Under this R-symmetry, the three chiral superfields transform according to

$$\begin{aligned}
\Phi_1(x,\theta) &\longrightarrow e^{2i\alpha}\Phi_1(x, e^{\alpha}\theta) \\
\Phi_2(x,\theta) &\longrightarrow e^{2i\alpha}\Phi_2(x, e^{\alpha}\theta) \\
\Phi_3(x,\theta) &\longrightarrow \Phi_3(x, e^{\alpha}\theta),
\end{aligned} \tag{8.6}$$

where α parameterises the phase transformation [6]. Choosing k, y and m to be real and solving the F equations of motion we arrive at the following relations

$$F_1 = k - \frac{y}{2}A_3^*, \;\; F_2 = -mA_3^*, \;\; F_3 = -mA_2^* - yA_1^*A_3^*. \tag{8.7}$$

It is clear that no configuration of scalar VEVs will simultaneously set all three Fs to zero, consequently supersymmetry is broken. Using Equations 8.1 and 8.7, we find the scalar potential has the form

$$V = \left|k - \frac{y}{2}A_3\right|^2 + |mA_3|^2 + |mA_2 + yA_1A_3|^2 . \tag{8.8}$$

Minimising this potential with respect to the three fields, we have

$$\frac{\partial V}{\partial A_3} = -\frac{yA_3}{2}(k - \frac{y}{2}A_3^{*2}) + m^2A_3^* + yA_1(mA_2^* + yA_1^*A_3^*) = 0, \tag{8.9}$$

$$\frac{\partial V}{\partial A_2} = m(mA_2^* + yA_1^*A_3^*) = 0, \tag{8.10}$$

$$\frac{\partial V}{\partial A_1} = yA_3(mA_2^* + yA_1^*A_3^*) = 0. \tag{8.11}$$

[6] Note: The R-charges given are the R-charges for the scalar components of the chiral superfields, correspondingly the R-charges for the fermionic components are R[ψ_i] =R[Φ_i] $-$ 1

The last two conditions above are the same, which implies that one of the scalar fields will be undetermined as we have only two independent equations. This is an example of a flat direction in the scalar potential. A flat direction is a surface in the space of the scalar fields along which the scalar potential does not change. In this example, provided we satisfy the two conditions above by solving for two of the 3 fields, the undetermined field can have any value and the scalar potential will still be at a minimum. The three conditions above, reduce to the two equations

$$-\frac{y}{2}A_3(k-\frac{y}{2}A_3^*)+m^2A_3^*=0, \tag{8.12}$$

$$mA_2^*+yA_1^*A_3^*=0. \tag{8.13}$$

Equation 8.12 has two different solutions depending on the relative sizes of the parameters. The first solution is given by

$$\begin{aligned}\text{Solution 1. For } m^2 > ky \\ \Rightarrow\ & A_3=0,\ \ A_2=0,\ \ A_1=\text{undetermined},\\ \Rightarrow\ & F_2=F_3=0,\ \ F_1=k,\\ \Rightarrow\ & V=k^2.\end{aligned} \tag{8.14}$$

As expected, the potential is non-zero and supersymmetry is broken due to one of the F-terms gaining a non-zero expectation value. We see that the R-symmetry is also spontaneously broken, (except at $A_1=0$) and consequently we would expect to see not only the goldstino arising from supersymmetry breaking but also a Goldstone boson as we have a broken bosonic generator. For solution 1, the goldstino is ψ_1. This is easy to see as it is only F_1 that receives a non-zero VEV and so it is only ψ_1 which is shifted under the supersymmetry transformations. This is further backed up by the superpotential, which only generates a mass term, $m\psi_2\psi_3$, with ψ_1 remaining massless.

Expanding the scalar fields in terms of their real and imaginary components, i.e.

$$A_1=\frac{1}{\sqrt{2}}(v_1+a_1+ib_1),\ \ A_2=\frac{1}{\sqrt{2}}(a_2+ib_2),\ \ A_2=\frac{1}{\sqrt{2}}(a_3+ib_3), \tag{8.15}$$

where v_1 is the undetermined VEV of A_1, we can write down the terms in the potential quadratic in the excitations

$$\begin{pmatrix} a_2 & a_3 \end{pmatrix}\begin{pmatrix} \frac{m^2}{2} & \frac{myv_1}{2} \\ \frac{myv_1}{2} & \frac{m^2}{2}-\frac{yk}{2}+\frac{y^2v_1^2}{4}\end{pmatrix}\begin{pmatrix} a_2 \\ a_3\end{pmatrix} \tag{8.16}$$

$$\begin{pmatrix} b_2 & b_3 \end{pmatrix}\begin{pmatrix} \frac{m^2}{2} & \frac{myv_1}{2} \\ \frac{myv_1}{2} & \frac{m^2}{2}+\frac{yk}{2}+\frac{y^2v_1^2}{4}\end{pmatrix}\begin{pmatrix} b_2 \\ b_3\end{pmatrix}, \tag{8.17}$$

where we notice that there are no quadratic terms for a_1, b_1. Diagonalising the above matrices and taking the second derivatives of the scalar potential with respect to the mass eigenvectors, we have four massive scalars. The 6 scalar masses are as follows

$$\begin{aligned}
&\frac{1}{16}\{4m^2+y\big[v_1^2y-2k^2-\big(8m^2v_1^2+(v_1^2y-2k)^2\big)^{1/2}\big]\},\\
&\frac{1}{16}\{4m^2+y\big[v_1^2y-2k^2+\big(8m^2v_1^2+(v_1^2y-2k)^2\big)^{1/2}\big]\},\\
&\frac{1}{16}\{4m^2+y\big[v_1^2y+2k^2-\big(8m^2v_1^2+(v_1^2y-2k)^2\big)^{1/2}\big]\},\\
&\frac{1}{16}\{4m^2+y\big[v_1^2y+2k^2+\big(8m^2v_1^2+(v_1^2y-2k)^2\big)^{1/2}\big]\},0,0.
\end{aligned} \tag{8.18}$$

As discussed above, we expect to see a massless Goldstone boson. It is clear that this corresponds to one of the degrees of freedom associated with A_1 as this contains the two massless scalar degrees of freedom. If we express A_1 as $\varsigma e^{i\vartheta}$ and examine how this transforms under the broken $U(1)_R$ generator, we see that after giving A_1 its VEV ϑ picks up an extra contribution [7], hence ϑ is the Goldstone boson in this case. The other massless degree of freedom in the above is generated as a result of the flat direction associated with A_1: any value of A_1 gives the same potential energy at tree-level. This flat direction is an accidental feature of the tree-level scalar potential and is removed with quantum corrections, that is we generate a mass for the radial component of A_1 via the one-loop effective potential.

Looking back at the superpotential, we see that we have two terms that contribute to fermion masses. They are $m\Phi_2\Phi_3$ and $\frac{y}{2}\Phi_1\Phi_3^2$. For the first term it is easy to see where the fermion mass terms comes from, the second generates a mass term when we replace the scalar component of Φ_1 with its VEV giving the mass terms,

$$m\psi_2\psi_3+\frac{y}{2\sqrt{2}}v_1\psi_3\psi_3, \tag{8.19}$$

giving two Dirac masses after diagonalisation, namely

$$\frac{1}{4\sqrt{2}}(v_1y+[8m^3+v_1^2y^2]^{1/2}) \text{ and } \frac{1}{4\sqrt{2}}(v_1y-[8m^3+v_1^2y^2]^{1/2}). \tag{8.20}$$

As discussed above, ψ_1 is massless and in this case the goldstino. Evaluating the supertrace for the above scalar and fermion masses we see that it vanishes, as expected.

It is worth noting, for completeness, that when A_1 has a zero VEV, the R-symmetry is not broken. This means that we do not have a Goldstone boson. In this case, we have scalar masses,

$$0,0,m^2,m^2,m^2-yk,m^2+yk, \tag{8.21}$$

and fermion masses,

$$0,m,m. \tag{8.22}$$

In this configuration, the two massless scalar degrees of freedom both arise due to the flat direction. It is easy to see that the supertrace condition is still satisfied.

[7] It is trivial to see that no other scalar has the same property, as they have zero VEVs.

There is another, more complicated solution, to the constraint in Equation 8.12 which corresponds to the opposite condition stated in Equation 8.14. That is,

Solution 2. For $m^2 < ky$

$$\Rightarrow\ A_3 = \frac{\sqrt{2}}{y}(ky - m^2)^{1/2},\ \ A_2 = -\sqrt{2}A_1\left(\frac{ky}{m^2}\right),\ \ A_1 = \text{undetermined},$$
$$\Rightarrow\ F_3 = 0,\ \ F_2 = -m\left[\frac{\sqrt{2}}{y}(ky - m^2)^{1/2}\right],\ \ F_1 = \frac{m^2}{y}$$
$$\Rightarrow\ V = \frac{m^2}{y}\left(2k - \frac{m^2}{y}\right). \tag{8.23}$$

Again, we proceed in working out the scalar and fermion masses and again we expect to see a Goldstino and a Goldstone boson. Due to the fact that there are two F components, F_1 and F_2, that gain VEVs, the goldstino in this case is a linear combination of ψ_1 and ψ_2. In the scalar sector we again have 2 massless degrees of freedom, one of which, is the Goldstone boson. Furthermore, the supertrace is zero for this configuration also. It is clear that the supertrace is always zero for F-term breaking and so restricts the phenomenology of this mechanism.

8.2 D-term or Fayet-Illiopoulos Breaking

D-term breaking can proceed via the mechanism introduced by Fayet and Illiopoulos [9, 10]. It involves the introduction of a non-anomolous Abelian gauge symmetry and centres on the dynamics of the associated gauge multiplet, V. Due to the Abelian nature of the gauge symmetry, we are permitted to write down a term linear in the D field [8] contained within the Abelian gauge multiplet and it is this term that can lead to supersymmetry breaking. This mechanism is governed by the same supertrace as Equation 8.4, with the modification

$$\text{Str}\mathcal{M}^2 \equiv gD\sum_i q_i, \tag{8.24}$$

where g is the coupling constant of the Abelian gauge symmetry and q_i is the charge of the ith Weyl fermion under the Abelian gauge symmetry. This sum is zero if the U(1) anomaly (i.e the $U(1)^3$ triangle anomaly) is cancelled in a vector-like way, e.g, if we have a number of pairs of fermion fields with equal and opposite charges under the U(1). The simplest example is a U(1) gauge theory interacting with two chiral superfields Φ_1 and Φ_2

[8]Under a general infinitesimal gauge transformation, D^a transforms as $\delta_{gauge}D^a = gf^{abc}D^a\Lambda^c$, where f^{abc} is a structure constant specific to the gauge group and Λ^c is an infinitesimal parameter of the gauge transformation.). For an Abelian U(1), the structure constants f^{abc} are zero, hence a linear term in D^a is gauge invariant.

which have opposite charges under the U(1) [9, 10]. The Lagrangian in this case is,

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{i}{g^2}\overline{\lambda}\sigma^{\mu}\partial_{\mu}\lambda + \frac{1}{2g^2}D^2 \\ &+ \sum_{i=1}^{2}\{(\mathcal{D}_{\mu}A_i)^*(\mathcal{D}_{\mu}A_i) + i\overline{\psi}_i\sigma^{\mu}\mathcal{D}_{\mu}\psi_i + F_i^*F_i\} \\ &+ D(A_1^*A_1 - A_2^*A_2) - \{\sqrt{2}(A_1^*\lambda\psi_1 - A_2^*\lambda\psi_2)\} + c.c \\ &+ m(A_1F_2 + A_2F_1 - \psi_1\psi_2) + c.c - \varepsilon\mu^2 D, \end{aligned} \tag{8.25}$$

where as usual A_i and ψ_i are the scalar and fermionic components of the chiral superfield, Φ_i. The first line of Equation 8.25 is simply the Lagrangian for the gauge supermultiplet. The second and third lines consists of the usual terms which result from the Kähler potential term, $\Phi_i^{\dagger}e^{2qV}\Phi_i$ for a $U(1)$ gauge theory for a chiral superfield with charge q under the $U(1)$. In the fourth line, the first group of terms come from the mass term in the superpotential of the form $m\Phi_1\Phi_2$. The final term in Equation 8.25 is the most important for this discussion and is the advertised gauge invariant linear term in D, where $\varepsilon = \pm 1$ and μ^2 is a positive mass squared. The U(1) version of the covariant derivatives acting on the scalar and fermion fields are given by $\mathcal{D}_{\mu}A_i = (\partial_{\mu} - q_iA_{\mu})A_i$ and $\mathcal{D}_{\mu}\psi_i = (\partial_{\mu} - q_iA_{\mu})\psi_i$ respectively. The first step of the analysis is to evaluate the equations of motion for the auxiliary fields, D and F. Solving the equations of motion for D and F we find the conditions,

$$\begin{aligned} D &= g^2\left(\epsilon\mu^2 - (A_1^*A_1 - A_2^*A_2)\right) \\ F_1 &= -mA_1^*, \quad F_2 = -mA_1^*. \end{aligned} \tag{8.26}$$

Clearly, these three equations of motion conditions cannot simultaneously be set to zero. Consequently, supersymmetry is broken. The next stage of the analysis is to investigate the mass spectra of the fermions and scalars in the theory. We can easily write down the form of the scalar potential. Using Equations 8.1 and 8.26, we have,

$$V_{scalar} = \frac{g^2}{2}\{(A_1^*A_1 - A_2^*A_2) - \varepsilon\mu^2\}^2 + |mA_1|^2 + |mA_2|^2. \tag{8.27}$$

Minimising this scalar potential leads to the conditions,

$$\begin{aligned} A_1^*\{g^2(A_1^*A_1 - A_2^*A_2 - \varepsilon\mu^2) + |m|^2\} &= 0 \\ A_2^*\{g^2(A_1^*A_1 - A_2^*A_2 - \varepsilon\mu^2) - |m|^2\} &= 0. \end{aligned} \tag{8.28}$$

There are two types of possible solution, the first is when $A_1 = A_2 = 0$ and the second is when only one of these is zero. In the first case, when we set $A_1 = A_2 = 0$, and evaluate F, D and the scalar potential, we have,

$$D = g^2\varepsilon\mu^2, \;\; F_1 = F_2 = 0, \;\; V_0 = \frac{g^2\mu^4}{2}. \tag{8.29}$$

Due to the fact that the two scalar components do not gain expectation values, the gauge theory is unbroken. It is only supersymmetry that is broken in this case. From the discussion in section 8, we can easily see that the gaugino, λ, becomes the goldstino.

The mass spectra are straight forward to work out. The only fermion mass term is the original Dirac mass term, $m\psi_1\psi_2$, and reading of the scalar masses from the scalar potential we have two complex scalars with masses $m^2+\varepsilon\mu^2 g^2$ and $m^2-\varepsilon\mu^2 g^2$. Evaluating the sum rule of Equation 8.24, we find that it indeed vanishes. This is easy to see, as we have,

$$Str\mathcal{M} = gD(q_1+q_2). \tag{8.30}$$

In this example $q_1 = -q_2$ and so the sum rule is not changed from zero. Although this is only a toy model, we can see from this that the sum rule will pose phenomenological problems, as it does for F-breaking, for the spectrum of particle and sparticle masses as it is so constrictive.

There is a second configuration of scalar VEVs, occurring when we assign a non-zero VEV to A_1. If this is the case, we break both gauge symmetry and R-symmetry, but keep a linear combination invariant. This means that we are, effectively, only breaking one generator. Due to the fact that we are breaking a gauge symmetry, the Nambu-Goldstone boson will be eaten by the gauge boson in the unitary gauge, i.e. the gauge boson will now gain a mass.

Setting $A_2 = 0$ and $A_1 = v$ and minimising the potential, we have the condition,

$$v^2 = \frac{1}{g^2}(g^2\varepsilon\mu^2 - m^2). \tag{8.31}$$

If we assume that v is real and taking g as positive, we have that $\varepsilon = 1$ and $g^2\mu^2 > m^2$. Evaluating the D and F terms at the minimum of the scalar potential, we have,

$$D = m^2, \;\; F_1 = 0, \;\; F_2 = -\frac{m}{g}(g^2\mu^2 - m^2)^{1/2}. \tag{8.32}$$

Again we break supersymmetry, but this time the resultant Goldstino is a linear superposition of ψ_2 and the gaugino, λ. Furthermore, if we calculate the scalar and fermion masses and plug them into the sum rule, we find that it is again satisfied.

It is now clear that spontaneous supersymmetry breaking (dynamical or not) requires us to extend the MSSM. The ultimate supersymmetry-breaking order parameter cannot belong to any of the MSSM supermultiplets; a D-term VEV for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no candidate gauge singlet whose F component could develop a vev. This suggest that supersymmetry is broken in some hidden sector and this breaking is communicated to the MSSM/visible sector via some messengers. The form of this communication can be gravity mediated, gauge mediated or anomaly mediated. Unfortunately there is not time to go into these in this set of lectures. See [2] for more details on the ways in which Supersymmetry breaking can be communicated to the visible sector.

A Spinor Manipulation

These identities can prove useful when manipulating spinors. They are mostly taken from Lykken, [1] but some are from Martin's Supersymmetry primer [2]. Most appear throughout the main text of the lecture notes but we collect them together here for ease of reference.

Spinor indices are raised ad lowered according to

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta, \quad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta, \quad \overline{\chi}_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\overline{\chi}^{\dot\beta}, \quad \overline{\chi}^{\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\overline{\chi}_{\dot\beta} \tag{A.1}$$

and where

$$\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta^\gamma_\alpha, \quad \epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \epsilon^{\dot\gamma\dot\beta}\epsilon_{\dot\beta\dot\alpha} = \delta^{\dot\gamma}_{\dot\alpha}. \tag{A.2}$$

$$\epsilon_{\alpha\beta}\epsilon^{\delta\gamma} = \delta^\gamma_\alpha\delta^\delta_\beta - \delta^\delta_\alpha\delta^\gamma_\beta, \quad \epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\delta\dot\gamma} = \delta^{\dot\gamma}_{\dot\alpha}\delta^{\dot\delta}_{\dot\beta} - \delta^{\dot\delta}_{\dot\alpha}\delta^{\dot\gamma}_{\dot\beta}. \tag{A.3}$$

We can manipulate the form of the Lorentz scalar combination of to spinors

$$\begin{aligned} \psi\chi = \psi^\alpha\chi_\alpha &= -\psi_\alpha\chi^\alpha = \chi^\alpha\psi_\alpha = \chi\psi \\ \overline{\psi}\overline{\chi} = \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha} &= -\overline{\psi}^{\dot\alpha}\overline{\chi}_{\dot\alpha} = \overline{\chi}_{\dot\alpha}\overline{\psi}^{\dot\alpha} = \overline{\chi}\overline{\psi} \end{aligned} \tag{A.4}$$

We also have:

$$\begin{aligned} (\chi\psi)^\dagger &= (\chi^\alpha\psi_\alpha)^\dagger = \overline{\psi}_{\dot\alpha}\overline{\chi}^{\dot\alpha} = \overline{\chi}\overline{\psi} \\ (\chi\sigma^m\overline{\psi})^\dagger &= \psi\sigma^m\overline{\chi} = \text{Lorentz vector} \end{aligned} \tag{A.5}$$

Other useful relations are:

$$\begin{aligned} \psi^\alpha\psi^\beta &= -\frac{1}{2}\epsilon^{\alpha\beta}\psi\psi \quad , \\ \psi_\alpha\psi_\beta &= \frac{1}{2}\epsilon_{\alpha\beta}\psi\psi \quad , \\ \overline{\psi}^{\dot\alpha}\overline{\psi}^{\dot\beta} &= \frac{1}{2}\epsilon^{\dot\alpha\dot\beta}\overline{\psi\psi} \quad , \\ \overline{\psi}_{\dot\alpha}\overline{\psi}_{\dot\beta} &= -\frac{1}{2}\epsilon_{\dot\alpha\dot\beta}\overline{\psi\psi} \quad . \end{aligned} \tag{A.6}$$

Pauli Matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.7}$$

From which we define:

$$\begin{aligned} \sigma^\mu &= (I, \vec\sigma) = \overline{\sigma}_\mu, \\ \overline{\sigma}^\mu &= (I, -\vec\sigma) = \sigma_\mu, \end{aligned} \tag{A.8}$$

where I denotes the 2x2 identity matrix. Note that in these definitions “bar” **does not** indicate complex conjugation.

$$\sigma^\mu \text{ has undotted-dotted indices: } \sigma^\mu_{\alpha\dot\beta}$$

$$\overline{\sigma}^\mu \text{ has dotted-undotted indices: } \overline{\sigma}^{\mu\dot\alpha\beta}$$

We also have the completeness relations:

$$\begin{aligned} \operatorname{tr} \sigma^\mu \overline{\sigma}^\nu &= 2\eta^{\mu\nu}, \\ \sigma^\mu_{\alpha\dot\beta} \overline{\sigma}^{\dot\gamma\delta}_\mu &= 2\delta^\delta_\alpha \delta^{\dot\gamma}_{\dot\beta}. \end{aligned} \tag{A.9}$$

We can move between σ^μ and $\overline{\sigma}^\mu$ following symbols:

$$\overline{\sigma}^{\mu\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\gamma}\epsilon^{\beta\delta}\sigma^\mu_{\delta\dot\gamma}; \quad \sigma^\mu_{\alpha\dot\beta} = \epsilon_{\dot\delta\dot\beta}\epsilon_{\gamma\alpha}\overline{\sigma}^{\mu\dot\delta\gamma}. \tag{A.10}$$

A useful trick that is to perform a "fake" conversion of an undotted to a dotted index or vice versa using the fact that σ^0 and $\overline{\sigma}^0$ are just the identity matrix:

$$\psi^\alpha = (\overline{\psi}_{\dot\beta})^* \overline{\sigma}^{0\dot\beta\alpha}; \qquad \overline{\psi}^{\dot\alpha} = (\psi_\beta)^* \sigma^{0\beta\dot\alpha}. \tag{A.11}$$

Because the Pauli matrices anti-commute, i.e.

$$\{\sigma^i, \sigma^j\} = 2\delta^{ij} I \qquad i, j = 1, 2, 3 \tag{A.12}$$

we have the relations:

$$\begin{aligned} (\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu)_\alpha^{\ \beta} &= 2\eta^{\mu\nu}\delta_\alpha^\beta, \\ (\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu)^{\dot\alpha}_{\ \dot\beta} &= 2\eta^{\mu\nu}\delta^{\dot\alpha}_{\dot\beta}. \end{aligned} \tag{A.13}$$

The SL(2,C) generators are defined as

$$\begin{aligned} \sigma^{\mu\nu\ \beta}_{\alpha} &= \frac{i}{4}\left[\sigma^\mu_{\alpha\dot\gamma}\overline{\sigma}^{\nu\dot\gamma\beta} - \sigma^\nu_{\alpha\dot\gamma}\overline{\sigma}^{\mu\dot\gamma\beta}\right] \\ \overline{\sigma}^{\mu\nu\dot\alpha}_{\quad\ \dot\beta} &= \frac{i}{4}\left[\overline{\sigma}^{\mu\dot\alpha\gamma}\sigma^\nu_{\gamma\dot\beta} - \overline{\sigma}^{\nu\dot\alpha\gamma}\sigma^\mu_{\gamma\dot\beta}\right] \end{aligned} \tag{A.14}$$

We have:

$$\begin{aligned} \epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma} &= 2i\sigma^{\mu\nu}; \qquad \text{self dual } (1,0) \\ \epsilon^{\mu\nu\rho\sigma}\overline{\sigma}_{\rho\sigma} &= -2i\overline{\sigma}^{\mu\nu}; \qquad \text{anti self dual } (0,1) \end{aligned} \tag{A.15}$$

And thus the trace relation:

$$\operatorname{tr}\left[\sigma^{\mu\nu}\sigma^{\rho\sigma}\right] = \frac{1}{2}\left(\eta^{\mu\rho}\eta^{\nu\sigma} - \eta^{\mu\sigma}\eta^{\nu\rho}\right) + \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}. \tag{A.16}$$

Superspace Identities

$$\partial_\alpha \theta^\beta = \delta^\alpha_\beta \tag{A.17}$$
$$\partial^\alpha \theta_\beta = \delta^\beta_\alpha \tag{A.18}$$
$$\overline{\partial}_{\dot\alpha} \overline{\theta}^{\dot\beta} = \delta^{\dot\alpha}_{\dot\beta} \tag{A.19}$$
$$\overline{\partial}^{\dot\alpha} \overline{\theta}_{\dot\beta} = \delta^{\dot\beta}_{\dot\alpha} \tag{A.20}$$
$$\epsilon^{\alpha\beta}\partial_\beta = -\partial^\alpha \tag{A.21}$$
$$\epsilon^{\dot\alpha\dot\beta}\partial_{\dot\beta} = -\overline{\partial}^{\dot\alpha} \tag{A.22}$$
$$\partial_\alpha(\theta^2) = 2\theta_\alpha \tag{A.23}$$
$$\overline{\partial}_{\dot\alpha}(\overline{\theta}^2) = -2\overline{\theta}_{\dot\alpha} \tag{A.24}$$

Fierz identities

$$(\theta\phi)(\theta\psi) = -\frac{1}{2}(\theta\theta)(\phi\psi) \tag{A.25}$$

$$(\overline{\theta\phi})(\overline{\theta\psi}) = -\frac{1}{2}(\overline{\phi\psi})(\overline{\theta\theta}) \tag{A.26}$$

$$\phi\sigma^\mu\overline{\chi} = -\overline{\chi\sigma}^\mu\phi \tag{A.27}$$

$$\phi\sigma_\mu\overline{\chi} = -\overline{\chi\sigma}_\mu\phi \tag{A.28}$$

$$(\theta\sigma^\mu\overline{\theta})(\theta\sigma^\nu\overline{\theta}) = \frac{1}{2}\eta^{\mu\nu}(\theta\theta)(\overline{\theta\theta}) \tag{A.29}$$

$$(\sigma^\mu\overline{\theta})_\alpha(\theta\sigma^\nu\overline{\theta}) = \frac{1}{2}\eta^{\mu\nu}\theta_\alpha(\overline{\theta\theta}) - i(\sigma^{\mu\nu}\theta)_\alpha(\overline{\theta\theta}) \tag{A.30}$$

$$(\theta\phi)(\overline{\theta\psi}) = \frac{1}{2}(\theta\sigma^\mu\overline{\theta})(\phi\sigma_\mu\overline{\psi}) \tag{A.31}$$

$$(\overline{\theta\psi})(\theta\phi) = \frac{1}{2}(\theta\sigma^\mu\overline{\theta})(\phi\sigma_\mu\overline{\psi}) = (\theta\phi)(\overline{\theta\psi}) \tag{A.32}$$

Other identities that may be useful:

$$\xi\sigma^\mu\overline{\sigma}^\nu\chi = \chi\sigma^\nu\overline{\sigma}^\mu\xi = (\chi^\dagger\overline{\sigma}^\nu\sigma^\mu\xi^\dagger)^* = (\xi^\dagger\overline{\sigma}^\mu\sigma^\nu\chi^\dagger)^*, \tag{A.33}$$

and the Fierz rearrangement identity:

$$\chi_\alpha(\xi\eta) = -\xi_\alpha(\eta\chi) - \eta_\alpha(\chi\xi), \tag{A.34}$$

and the reduction identities

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\,\overline{\sigma}^{\dot{\beta}\beta}_{\mu} \;=\; 2\delta^{\beta}_{\alpha}\delta^{\dot{\beta}}_{\dot{\alpha}}, \tag{A.35}$$

$$\sigma^{\mu}_{\alpha\dot{\alpha}}\,\sigma_{\mu\beta\dot{\beta}} \;=\; 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}, \tag{A.36}$$

$$\overline{\sigma}^{\mu\dot{\alpha}\alpha}\,\overline{\sigma}^{\dot{\beta}\beta}_{\mu} \;=\; 2\epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}, \tag{A.37}$$

$$\left[\sigma^{\mu}\overline{\sigma}^{\nu}+\sigma^{\nu}\overline{\sigma}^{\mu}\right]_{\alpha}^{\;\;\beta} \;=\; 2\eta^{\mu\nu}\delta^{\beta}_{\alpha}, \tag{A.38}$$

$$\left[\overline{\sigma}^{\mu}\sigma^{\nu}+\overline{\sigma}^{\nu}\sigma^{\mu}\right]^{\dot{\beta}}_{\;\;\dot{\alpha}} \;=\; 2\eta^{\mu\nu}\delta^{\dot{\beta}}_{\dot{\alpha}}, \tag{A.39}$$

$$\overline{\sigma}^{\mu}\sigma^{\nu}\overline{\sigma}^{\rho} = \eta^{\mu\nu}\overline{\sigma}^{\rho} + \eta^{\nu\rho}\overline{\sigma}^{\mu} - \eta^{\mu\rho}\overline{\sigma}^{\nu} - i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_{\kappa}, \tag{A.40}$$

$$\sigma^{\mu}\overline{\sigma}^{\nu}\sigma^{\rho} = \eta^{\mu\nu}\sigma^{\rho} + \eta^{\nu\rho}\sigma^{\mu} - \eta^{\mu\rho}\sigma^{\nu} + i\epsilon^{\mu\nu\rho\kappa}\sigma_{\kappa}, \tag{A.41}$$

where $\epsilon^{\mu\nu\rho\kappa}$ is the totally antisymmetric tensor with $\epsilon^{0123}=+1$.

B Superfields

In this Appendix is a list of useful superfield expansions in their full component glory, using $y^{\mu}=x^{\mu}+i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}$, we have for chiral superfields Φ_i,

$$\Phi \;=\; A(y)+\sqrt{2}\theta^{\alpha}\psi_{\alpha}(y)+\theta^{\alpha}\theta_{\alpha}F(y) \tag{B.1}$$

$$\begin{aligned} &= A(x)+i\theta^{\alpha}\sigma^{mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_{\mu}A(x)-\frac{1}{4}\theta^{2}\overline{\theta}^{2}\Box A(x)\\ &+\sqrt{2}\theta^{\alpha}\psi_{\alpha}(x)-\frac{i}{\sqrt{2}}\theta^{\alpha}\theta_{\alpha}\partial_{\mu}\psi^{\beta}(x)\sigma^{\mu}_{\beta\dot{\gamma}}\overline{\theta}^{\dot{\gamma}}+\theta^{\alpha}\theta_{\alpha}F(x). \end{aligned} \tag{B.2}$$

$$\Phi^{\dagger} \;=\; A^{*}(y)+\sqrt{2}\overline{\theta}_{\dot{\alpha}}\overline{\psi}^{\dot{\alpha}}(y)+\overline{\theta}_{\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}F^{*}(y) \tag{B.3}$$

$$\begin{aligned} &= A^{*}(x)-i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_{\mu}A^{*}(x)-\frac{1}{4}\theta^{2}\overline{\theta}^{2}\Box A^{*}(x)\\ &+\sqrt{2}\overline{\theta}_{\dot{\alpha}}\overline{\psi}^{\dot{\alpha}}(x)+\frac{i}{\sqrt{2}}\overline{\theta}_{\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}\theta^{\beta}\sigma^{\mu}_{\beta\dot{\gamma}}\partial_{\mu}\overline{\psi}^{\dot{\gamma}}(x)+\overline{\theta}_{\dot{\alpha}}\overline{\theta}^{\dot{\alpha}}F^{*}(x). \end{aligned} \tag{B.4}$$

$$\begin{aligned} \Phi_i\Phi_j \;=\;& A_i(y)A_j(y)\\ &+\sqrt{2}\theta^{\alpha}\left[\psi_{\alpha i}(y)A_j(y)+A_i(y)\psi_{\alpha j}(y)\right]\\ &+\theta^{\alpha}\theta_{\alpha}\left[A_i(y)F_j(y)+F_i(y)A_j(y)-\psi^{\alpha}_{i}(y)\psi_{\alpha j}(y)\right]. \end{aligned} \tag{B.5}$$

$$\begin{aligned} \Phi_i\Phi_j\Phi_k \;=\;& A_i(y)A_j(y)A_k(y)\\ &+\sqrt{2}\theta^{\alpha}\left[\psi_{\alpha i}(y)A_j(y)A_k(y)+\psi_{\alpha i}(y)A_k(y)A_j(y)+\psi_{\alpha k}(y)A_j(y)A_i(y)\right]\\ &+\theta^{\alpha}\theta_{\alpha}\left[F_i(y)A_j(y)A_k(y)+F_j(y)A_k(y)A_i(y)+F_k(y)A_i(y)A_j(y)\right]\\ &-\theta^{\alpha}\theta_{\alpha}\left[\psi^{\alpha}_{i}\psi_{j\alpha}A_k(y)+\psi^{\alpha}_{j}\psi_{k\alpha}A_i(y)+\psi^{\alpha}_{k}\psi_{i\alpha}A_j(y)\right]. \end{aligned} \tag{B.6}$$

$$\begin{aligned}
\Phi_i^\dagger\Phi_j =\ & A_i^*(x)A_j(x) + \sqrt{2}\theta^\alpha\psi_{j\alpha}(x)A_i^*(x) + \sqrt{2}\overline{\theta}_{\dot\alpha}\overline{\psi}_i^{\dot\alpha}(x)A_i(x) \\
& + \theta^\alpha\theta_\alpha A_i^*(x)F_j(x) + \overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}F_i^*(x)A_j(x) \\
& + \theta^\alpha\overline{\theta}^{\dot\alpha}\left[i\sigma^\mu_{\alpha\dot\alpha}\left(A_i^*(x)\partial_\mu A_j(x) - \partial_\mu A_i^*(x)A_j(x)\right) - 2\overline{\psi}_{i\dot\alpha}(x)\psi_{i\alpha}(x)\right] \\
& + \theta^\alpha\theta_\alpha\overline{\theta}^{\dot\alpha}\left[\frac{i}{\sqrt{2}}\sigma^\mu_{\alpha\dot\alpha}\left(A_i^*(x)\partial_\mu\psi_j^\alpha - \partial_\mu A_i^*(x)\psi_j^\alpha(x)\right) - \sqrt{2}F_j(x)\overline{\psi}_{i\dot\alpha}\right] \\
& + \overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}\theta^\alpha\left[-\frac{i}{\sqrt{2}}\sigma^\mu_{\alpha\dot\alpha}\left(\overline{\psi}_i^{\dot\alpha}(x)\partial_\mu A_j(x) - \partial_\mu\overline{\psi}_i^{\dot\alpha}(x)A_j(x)\right) + \sqrt{2}F_i^*(x)\psi_{j\alpha}(x)\right] \\
& + \theta^\alpha\theta_\alpha\overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}\left[F_i^*(x)F_j(x) - \frac{1}{4}A_i^*(x)\Box A_j(x) - \frac{1}{4}\Box A_i^*(x)A_j(x) + \frac{1}{2}\partial_\mu A_i^*(x)\partial^\mu A_j(x)\right] \\
& + \theta^\alpha\theta_\alpha\overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}\left[\frac{i}{2}\partial_\mu\overline{\psi}_{i\dot\alpha}(x)\overline{\sigma}^{\mu\dot\alpha\beta}\psi_{j\beta}(x) - \frac{i}{2}\overline{\psi}_{i\dot\alpha}(x)\overline{\sigma}^{\mu\dot\alpha\beta}\partial_\mu\psi_{j\beta}(x)\right]. \qquad (B.7)
\end{aligned}$$

In section 5 we wanted to eliminate some of the gauge degrees of freedom from a vector superfield, we defined a supersymmetric version of an Abelian gauge transformation, as $V \to V + \frac{i}{2}\left(\Phi - \Phi^\dagger\right)$, where $\Phi - \Phi^\dagger$ is given by,

$$\begin{aligned}
\Phi - \Phi^\dagger =\ & A(x) - A^*(x) + \sqrt{2}\left(\theta^\alpha\psi_\alpha(x) - \overline{\theta}_{\dot\alpha}\overline{\psi}^{\dot\alpha}(x)\right) + \theta^\alpha\theta_\alpha F - \overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}F^*(x) \\
& + i\theta^\alpha\sigma^\mu_{\alpha\dot\beta}\overline{\theta}^{\dot\beta}\partial_\mu\left(A(x) + A^*(x)\right) + \frac{i}{\sqrt{2}}\theta^\alpha\theta_\alpha\overline{\theta}_{\dot\alpha}\overline{\sigma}^{\mu\dot\alpha\beta}\partial_\mu\psi_\alpha(x) \\
& - \frac{i}{\sqrt{2}}\overline{\theta}_{\dot\alpha}\overline{\theta}^{\dot\alpha}\theta^\beta\sigma^\mu_{\beta\dot\gamma}\partial_\mu\overline{\psi}^{\dot\gamma}(x) - \frac{1}{4}\theta^2\overline{\theta}^2\Box\left(A(x) - A^*(x)\right). \qquad (B.8)
\end{aligned}$$

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