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PH4442 Adv. Ptcl. Physics Week 10

Recap Higgs mech: ($\to$ slides)

Need to include ptcl. masses & not break gauge sym.

$\to$ Lagrangian gets complex scalar doublet & potential

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_H = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

$$D_\mu = \partial_\mu + ig\vec{T}\cdot\vec{W}_\mu + i\frac{g'}{2} Y B_\mu$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 , \quad \lambda > 0, \ \mu^2 < 0$$

To keep photon massless, only ϕ^0 gets

nonzero vacuum expectation value (VEV):

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Expand about vacuum state:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \eta + i\phi_4 \end{pmatrix}$$

Put into $\mathcal{L}_H$, transform to unitary gauge

$$\left.\begin{aligned} \to \mathcal{L}_H = {} & \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 \\ & + \frac{1}{8} g^2 (W_{1\mu} + iW_{2\mu})(W_1^\mu - iW_2^\mu)(v+h)^2 \\ & + \frac{1}{8}(gW_{3\mu} - g'B_\mu)(gW_3^\mu - g'B^\mu)(v+h)^2 \end{aligned}\right\} \text{Rewrite in terms of } W^\pm, Z, \text{ expand } (v+h)^2 \ \& \ (v+h)^4 \text{ terms}$$

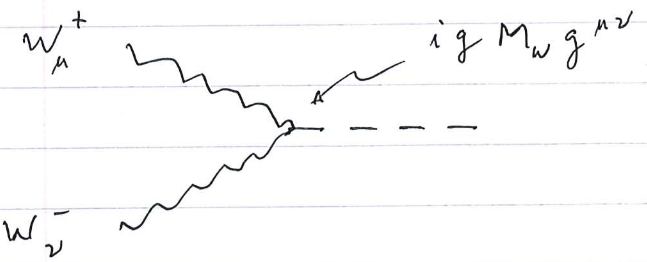
$\to \phi_1, \phi_2, \phi_4$ "eaten", $W^\pm, Z$ get mass ($M_W = M_Z \cos\theta_W$), $\eta \to h$ ~ Higgs! = real scalar boson

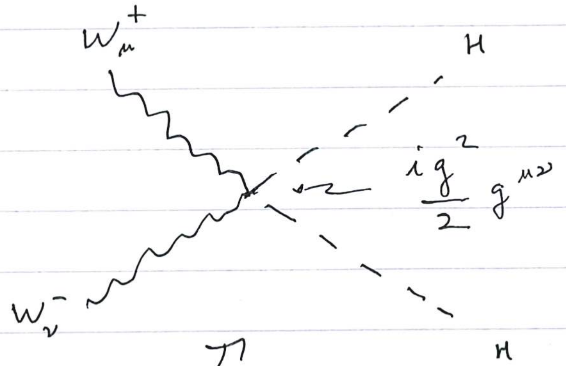
Higgs couplings to gauge bosons

Multiply out $(v+h)^2$, $(v+h)^4$ terms in $\mathcal{L}_H$,

Express W_1, W_2, W_3, B in terms of $W^\pm, Z$ (no A^μ term!)

$$\rightarrow \underbrace{\frac{1}{2} g^2 v}_{g M_W} W^{(-)}_\mu W^{(+)\mu} h + \frac{1}{4} g^2 W^{(-)}_\mu W^{(+)\mu} h^2$$

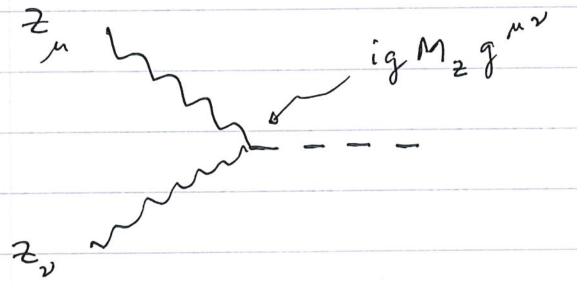


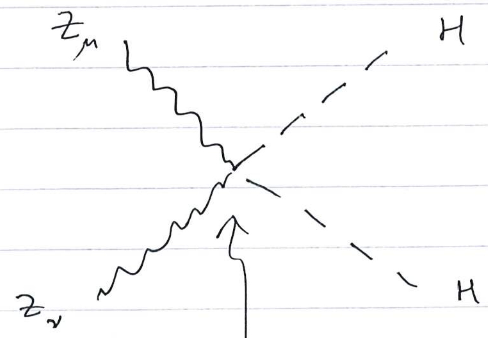


$N=2$ identical ptcls

$\Rightarrow$ sym. factor $N! = 2!$

~ same for Z but $g \rightarrow \frac{g}{\cos\theta_W}$ (or $M_W \rightarrow M_Z$)



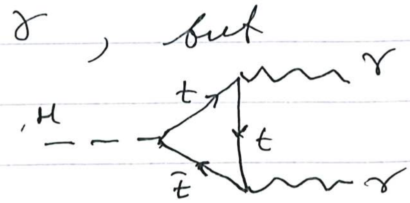


$$\frac{ig^2}{2\cos^2\theta_W} g^{\mu\nu}$$

$\Rightarrow$ both WWH & ZZH couplings $\propto M_{W/Z}$

No direct coupling to γ, but

$H \rightarrow \gamma\gamma$ through loops



Higgs coupling / mass for fermions

Recall fermion mass term in Lagrangian

$$-m\bar{\psi}\psi$$ ← OK in QED, not in SM

where $\psi = \psi_L + \psi_R$

$P_L\psi = \frac{1}{2}(1-\gamma^5)\psi$ $P_R\psi = \frac{1}{2}(1+\gamma^5)\psi$

Recall $\gamma^0\gamma^5 = -\gamma^5\gamma^0$

$\Rightarrow \quad \gamma^0 P_L = P_R\gamma^0 \, , \quad \gamma^0 P_R = P_L\gamma^0$

also $(\gamma^5)^\dagger = \gamma^5 \Rightarrow P_L = P_L^\dagger \, , \quad P_R = P_R^\dagger$

so, $$\bar{\psi}_L = (P_L\psi)^\dagger\gamma^0$$

$$= \psi^\dagger P_L\gamma^0$$

$$= \psi^\dagger\gamma^0 P_R$$

$$= \underline{\bar{\psi} P_R}$$

and similarly $\underline{\bar{\psi}_R = \bar{\psi} P_L}$

So in $-m\bar{\psi}\psi = -m\,\overline{(\psi_L + \psi_R)}\,(\psi_L + \psi_R)$,

$$\bar{\psi}_L \psi_L = \bar{\psi} P_R P_L \psi = 0$$

$$\bar{\psi}_R \psi_R = \bar{\psi} P_L P_R \psi = 0$$

$\Rightarrow$ only $\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$ survive

$$-m\bar{\psi}\psi = -m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

But $SU(2)_L$ part of gauge trans different for ψ_L, ψ_R

$\Rightarrow$ mass term not gauge invariant

But, we can construct a gauge invariant term w/ Higgs mech that will give masses to fermions! Use complex doublet:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Under $SU(2)$

$$\phi' = U_T \phi$$

$$U_T = \exp\left(ig\vec{\alpha}(x)\cdot\vec{T}\right)$$

Consider charged lepton l (ν_l will remain massless)

$$L = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \qquad \text{where} \qquad l = \psi_l$$

↑ use name of ptcl. for wave-function

Under $SU(2)$,

$$L' = U_T L$$

$$\bar{L}' = (U_T L)^\dagger \gamma^0 = L^\dagger \gamma^0 U_T^\dagger = \bar{L} U_T^\dagger$$

↑ multiplies both ν_l and l

U_T unitary $\Rightarrow$ $\bar{L}'\phi' = \bar{L} U_T^\dagger U_T \phi = \bar{L}\phi$

is invariant under $SU(2)$

Right-chiral lepton $R = P_R \psi_l = l_R$ has $T_3 = 0$ (isosinglet)

i.e. does not change under $SU(2) \Rightarrow (\bar{L}\phi)R$ invariant under $SU(2)$

Under $U(1)_Y$, $\quad R' = U_Y R$

$$U_Y = e^{iY\theta}, \qquad \theta = g'\beta(x)$$

$$Y = 2(Q - t_3) \quad \sim \text{hypercharge}$$

$\Rightarrow$ $Y_\phi = 1$, $\quad Y_L = -1$, $\quad Y_R = -2$

Consider $(\bar{L}\phi)R$ terms transform under $U(1)_Y$:

$$\bar{L} = L^\dagger \gamma^0 \to \bar{L}' = (e^{\overset{Y_L}{-} i\theta} L)^\dagger \gamma^0 = \bar{L} e^{i\theta}$$

$$\phi \to \phi' = e^{i\theta}\phi$$

$$R \to R' = e^{-2i\theta} R$$

$\Rightarrow$ $(\bar{L}\phi)R$ invariant under $U(1)_Y$

$\Rightarrow$ $(\bar{L}\phi)R$ invariant under $\underline{SU(2) \times U(1)}$

So we can add $(\bar{L}\phi)R$ to Lagrangian without violating any fundamental symmetry (gauge, Lorentz)

E.g. for charged lepton ℓ, add term

$$\mathcal{L}_\ell = -y_\ell \left[\bar{L}\phi R + (\bar{L}\phi R)^\dagger \right]$$

Yukawa coupling

Hermitian conjugate – ensures $\mathcal{L}$ real.

Repeat Higgs mech $\rightarrow$ unitary gauge

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

Upper ($t_3 = 1/2$) comp. 0 $\Rightarrow$ terms in L

involving ν_ℓ vanish, what survives is

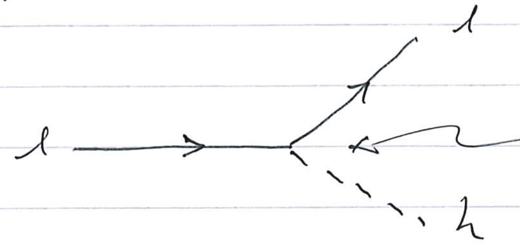
$$\mathcal{L}_\ell = \underbrace{-\frac{y_\ell}{\sqrt{2}} v \overbrace{(\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L)}^{\bar{\ell}\ell}}_{\text{mass term: } m_\ell = \frac{y_\ell v}{\sqrt{2}}} \underbrace{- \frac{y_\ell}{\sqrt{2}} h (\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L)}_{\text{interaction term}}$$

Set $\frac{y_\ell v}{\sqrt{2}}$ to observed m_ℓ $\Rightarrow$ $y_\ell = \frac{\sqrt{2}\, m_\ell}{v}$

So we get a gauge invar.-mass term but no predicted relation between masses

y_l = free param for each l, set by m_l
$\left(y_l = \frac{\sqrt{2}\, m_l}{v}\right)$

$-\frac{y_l}{\sqrt{2}} h l \bar{l}$ = interaction term between l & h



$-\frac{i y_l}{\sqrt{2}} = -\frac{i m_l}{v} \propto m_l$

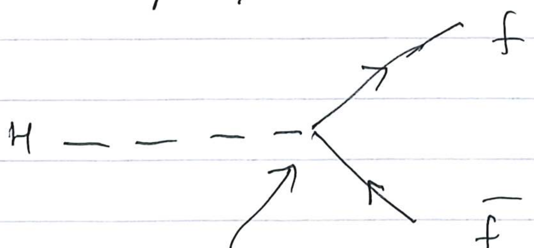
Because $\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ — $\langle 0 | \phi^+ | 0 \rangle$

cannot use same procedure for neutrino masses
& " " " " " up-type quarks

But, by trick w/ "conjugate doublet"

$$\phi_c = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}$$

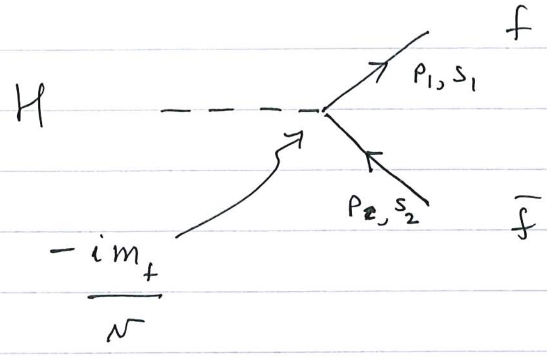
wind up w/ same result for all fermions



$y_f = \frac{\sqrt{2}\, m_f}{v}$

$-\frac{i m_f}{v}$

Rate $\Gamma(H \to f\bar{f})$



Feynman rule for external scalar: $\underline{\underline{1}}$

$$\Rightarrow \mathcal{M} = \frac{-im_f}{v} \bar{u}(p_1) v(p_2)$$

Square & sum over s_1, s_2 (usual Casimir trick)

$$\langle |\mathcal{M}|^2 \rangle = \frac{m_f^2}{v^2} \mathrm{Tr}\left[(\not{p}_1 + m_f)(\not{p}_2 - m_f)\right]$$

since v spinor

$\mathrm{Tr}(\not{p}_1 m_f) = 0$ (odd # of γs)

$\mathrm{Tr}(\not{p}_1 \not{p}_2) = 4 p_1 \cdot p_2$, $\mathrm{Tr}\, I_4 = 4$

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = \frac{m_f^2}{v^2}(4 p_1 \cdot p_2 - 4 m_f^2)$$

Use $E_{cm}^2 = m_H^2 = (p_1 + p_2)^2 = 2m_f^2 + 2 p_1 \cdot p_2$

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = \frac{2 m_f^2 m_H^2}{v^2}\left(1 - \frac{4 m_f^2}{m_H^2}\right)$$

In Higgs rest frame, $p_2 \longleftarrow \bullet \longrightarrow p_1$

$$|\vec{p}_1| = |\vec{p}_2| \equiv p^* = \frac{m_H}{2}\left(1 - \frac{4 m_f^2}{m_H^2}\right)^{1/2}$$

Use eq. (7.27) for isotropic two-body decay

$$\Gamma(H \to f\bar{f}) = \frac{p^*}{8\pi m_H^2} \langle |\mathcal{M}|^2 \rangle$$

$$= \frac{m_f^2 m_H}{8\pi v^2} \left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$$

$\Gamma \to 0$ for $H \to t\bar{t}$ since $m_H = 125$ GeV,

$2m_t = 2 \times 173$ GeV

But for all other fermions can neglect $\left(1 - \frac{4m_f^2}{m_H^2}\right)^{3/2}$ ($\lesssim 1\%$ for $b\bar{b}$)

For quarks, include factor $N_c = 3$ (colour)

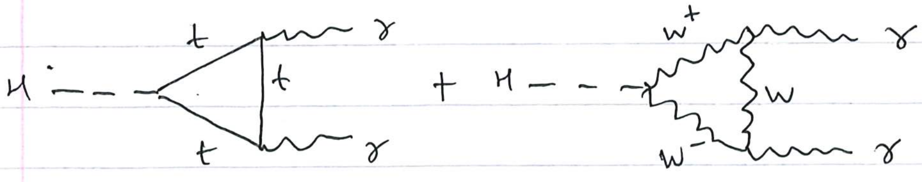
$\Rightarrow \Gamma_{bb} : \Gamma_{cc} : \Gamma_{\tau\tau} : \Gamma_{\mu\mu} = 3m_b^2 : 3m_c^2 : m_\tau^2 : m_\mu^2$

For e.g. $H \to ZZ, WW$, calc more complicated

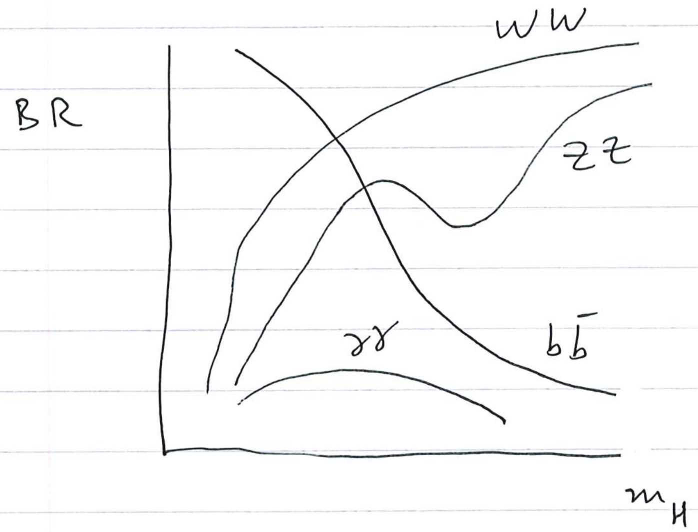
since $m_H = 125$, $M_W = 80.4$, $M_Z = 91.2$

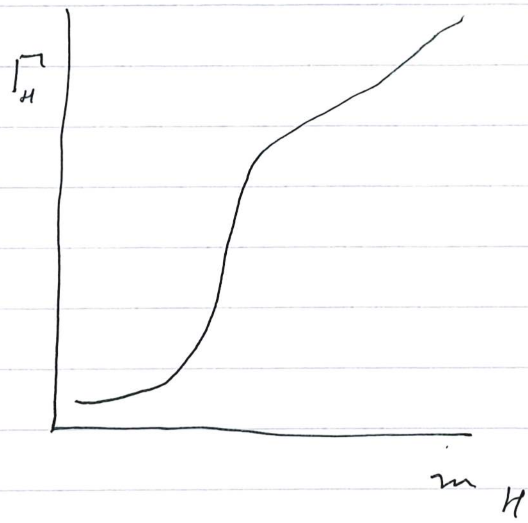
$\Rightarrow$ only $H \to WW^*$ or $H \to ZZ^*$ — virtual (off-shell)

For $H \to \gamma\gamma$, no direct coupling but



small $BR = 2.3 \times 10^{-3}$ but important channel (low background)





→ slide

Plot relevant before m_H known.

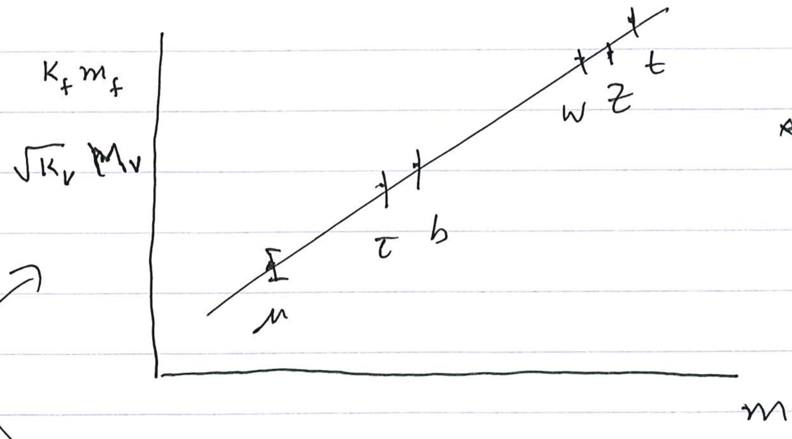
At $m_H = 125.$ GeV, $\Gamma_H = 4.1$ MeV (SM pred.)

i.e. << mass resolution

But, w/ interference effects in $H \to ZZ$
↳ ↳ leptons

get $\Gamma_H = 4.5^{+3.0}_{-2.2}$ MeV (ATLAS)

measure coupling modifiers κ



← amazing agreement

log scales

QCD

See PH3520 Notes

History:

1950s → lots of hadrons

1964 (Gell-Mann & Zweig) quarks

1964 (Greenberg) $\Delta^{++} = uuu$

$J = 3/2$ ($\measuredangle$ dist. in $\pi^+ p$ scattering)

$L = 0$ (lowest mass uuu state)

↳ spatial wave func. symmetric

⇒ must have spins aligned $\uparrow\uparrow\uparrow$

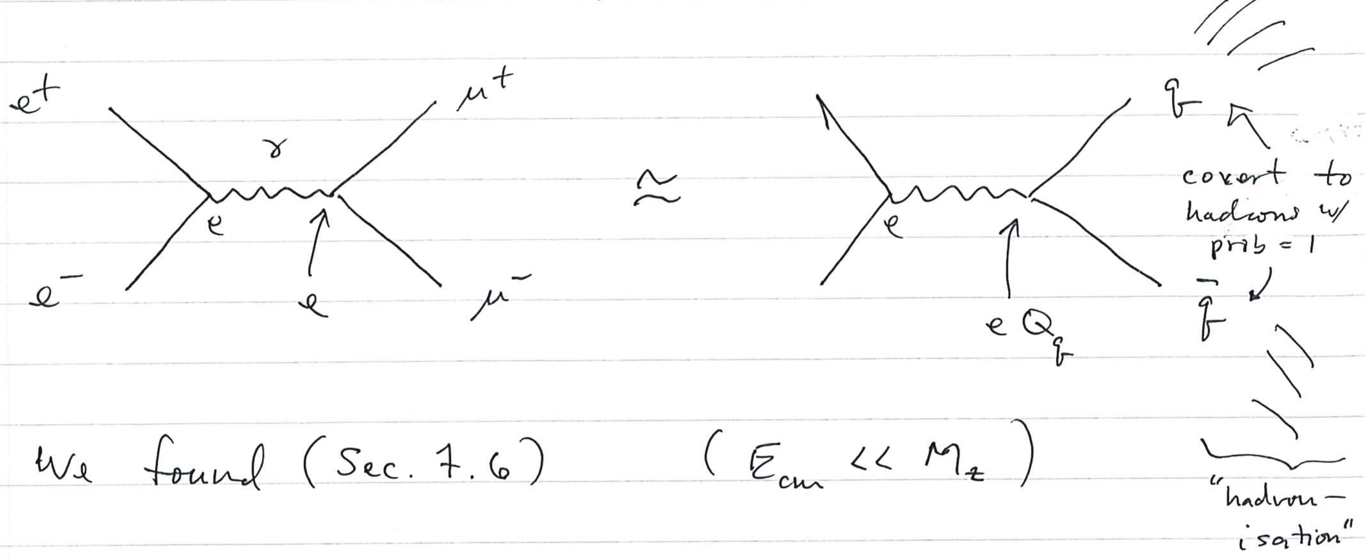
⇒ spin wave func. also symmetric

But quarks fermions – Pauli: must have antisymmetric total wave func.

⇒ extra degree of freedom "colour"

$N_c \geq 3$ → symmetry OK

$e^+e^- \to$ hadrons



We found (Sec. 7.6) $\quad (E_{cm} \ll M_Z)$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3E_{cm}^2}$$

$$\sigma(e^+e^- \to q\bar{q}) = \frac{4\pi\alpha^2 Q_q^2}{3E_{cm}^2} \qquad Q_q = \begin{cases} 2/3 & u,c,t \\ -1/3 & d,s,b \end{cases}$$

$$\Rightarrow R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

all $q\bar{q}$ except $t\bar{t}$ ← $m_t = 173\,\text{GeV} > M_Z$

$$= \sum_{\substack{u,d,s,c,b \\ (\text{no } t \text{ yet})}}' Q_q^2 = 2\times\left(\frac{2}{3}\right)^2 + 3\times\left(-\frac{1}{3}\right)^2 = \frac{11}{9}$$

But measured values of $R \simeq 4$ ($\rightarrow$ plot)

But if N_c colours,

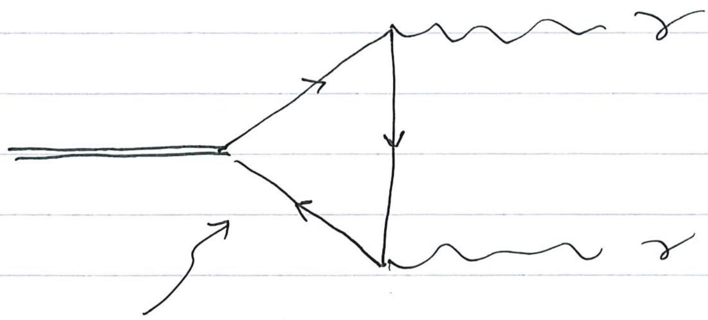
$$R = N_c \sum_q' Q_q^2 = \frac{11}{3} = 3.7 \quad \text{for } N_c = 3$$

not quite 4 but close

Later, QCD predicts R very accurately
+ excellent agreement w/ experiment

More evidence for colour:

$\pi^0 \to \gamma\gamma$



$\mathcal{M} \propto N_c$

$\Rightarrow \Gamma(\pi^0 \to \gamma\gamma) \propto N_c^2$

For $N_c = 3 \longrightarrow \Gamma(\pi^0 \to \gamma\gamma) = 7.6$ eV (pred.)

measured $= 7.7 \pm 0.6$ eV

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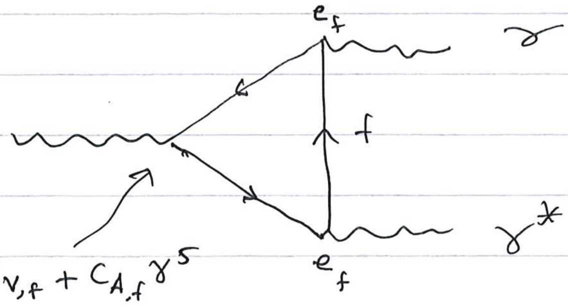
skip

Triangle anomalies, e.g.

Adler - Bell - Jackiw anomaly



$c_{V,f} + c_{A,f}\gamma^5$

or one γ off shell

In closed loop, c_{V_f} contrib. cancels

Diagram not gauge invariant unless

$$\sum_f e_f^2 c_{A_f} = \frac{1}{2}\left[-1 + N_c\left(\frac{4}{9} - \frac{1}{9}\right)\right] = 0$$

$\Rightarrow$ works for $N_c = 3$!

$(+ Q_u = \frac{2}{3},\ Q_d = -1/3)$

QCD as an SU(3) gauge theory

Take: $N_c = 3$ colours

Yang-Mills → renormalisable theory, works for electro(weak)

So... let

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

(1, 2, 3 → red, green, blue)

SU(3) local gauge trans.

$$q \to q' = U_c\, q$$

sum repeated indices

$$U_c = \exp\left[i g_s \alpha^a(x) T^a \right]$$

U_c: 3×3 unitary matrix, det = +1.

g_s: gauge coupling → $\alpha_s = g_s^2/4\pi$ (cf. $\alpha = \frac{e^2}{4\pi}$)

$\alpha^a(x)$: arb. func. of x

T^a = generators of SU(3) (Hermitian)

SU(N) has N^2-1 generators

⇒ $N_c^2 - 1 = \underline{8}$ generators for $N_c = 3$

Choose $T^a = \frac{\lambda^a}{2}$, $a = 1, \ldots, 8$

λ^a = Gell-Mann matrices (→ slide)

Yang-Mills recipe: gauge field for each generator

$$\rightarrow 8 \text{ gluons } \quad G^a_\mu \qquad a = 1, \dots, 8$$

transform under SU(3) (adjoint rep.)

$$G^a_\mu \rightarrow G'^a_\mu = G^a_\mu - \partial_\mu \alpha^a - g_s f^{abc} \alpha^b G^c_\mu$$

α^a, α^b infinitesimal

Covariant deriv is

$$D_\mu = \partial_\mu + i g_s T^a G^a_\mu$$

$\rightarrow$ QCD Lagrangian:

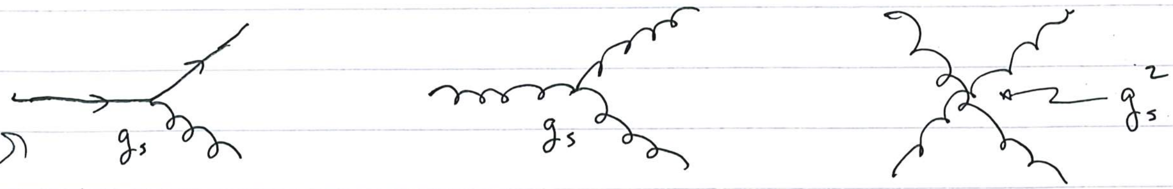
$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q{}' \bar{q} (i \not{D} - m_q) q$$

$$\doteq (\quad) + \sum_q{}' \sum_{j,k=1}^{3}{}' \bar{q}_j \left[(i \not{\partial} - m_q) \delta_{jk} - g_s \not{G}^a T^a_{jk} \right] q_k$$

u, c, t, d, s, b

where $F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s f^{abc} G^b_\mu G^c_\nu$

Gives fundamental vertices ($\rightarrow$ plots)



Same coupling for <u>all flavours</u>

Confinement

Original motivation for quarks was to explain hadron states:

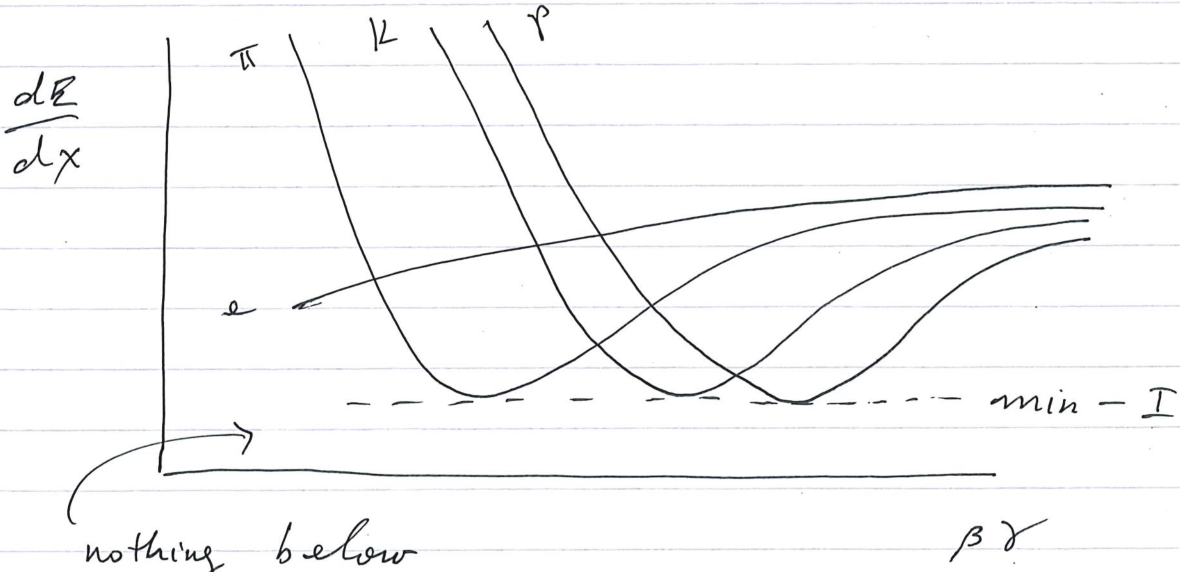
$$\text{baryon} = qqq$$
$$\text{meson} = q\bar{q}$$

(+ glueballs, tetraquarks, pentaquarks ...)

$\Rightarrow$ should be possible to see a free quark ?

$$Q_q = -1/3,\ 2/3$$

$$\Rightarrow \left(\frac{dE}{dx}\right)_{ion} \propto Q_q^2$$



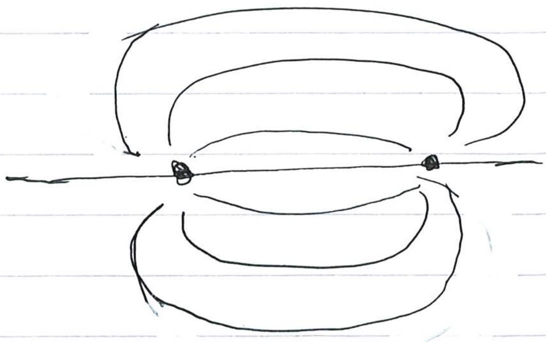
nothing below minimum ionising.

(→ slides)

Field config between charges

e^+e^- : $V_c(r) = -\frac{\alpha}{r}$

$F_c = -\frac{\alpha}{r^2}$



$q\bar{q}$: $V_{QCD}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr$

$F_{QCD} = -\frac{4}{3}\frac{\alpha_s}{r^2} - k$

approaches const.

flux tube

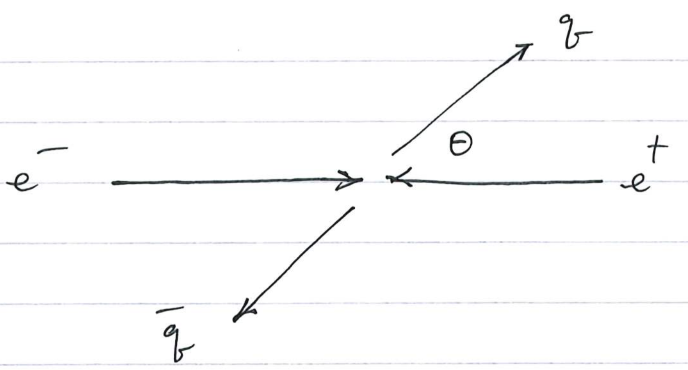
≈ "string"

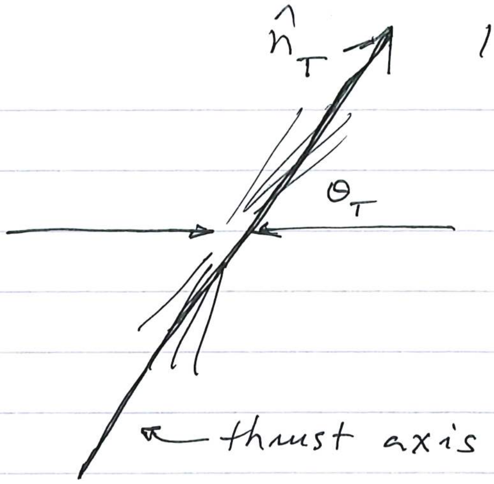
$\Rightarrow$ $q\bar{q}$ cannot escape!

$\rightarrow$ hadron production slide,

2-jet slide

But in $e^+e^- \to q\bar{q}$, the "jets" should retain direction/energy of $q, \bar{q}$





$$T = \frac{\sum_i' |\vec{p}_i \cdot \hat{n}_T|}{\sum_i' |\vec{p}_i|}$$

choose $\hat{n}_T$ to maximise T

thrust

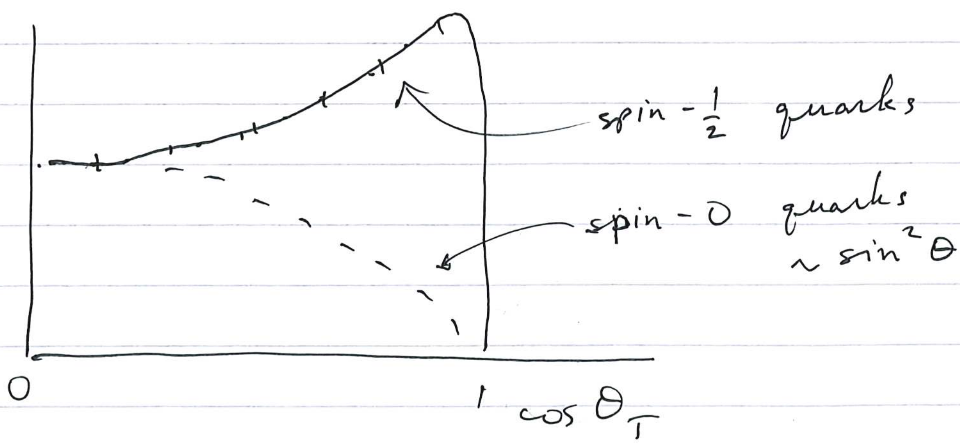
sum over all measured $\vec{p}_i$

Recall for $q\bar{q}$ (like $\mu^+\mu^-$)

$$\frac{d\sigma}{d\cos\theta} \sim 1 + \cos^2\theta \qquad (E_{cm} \ll M_Z, \text{ or } E_{cm} = M_Z)$$

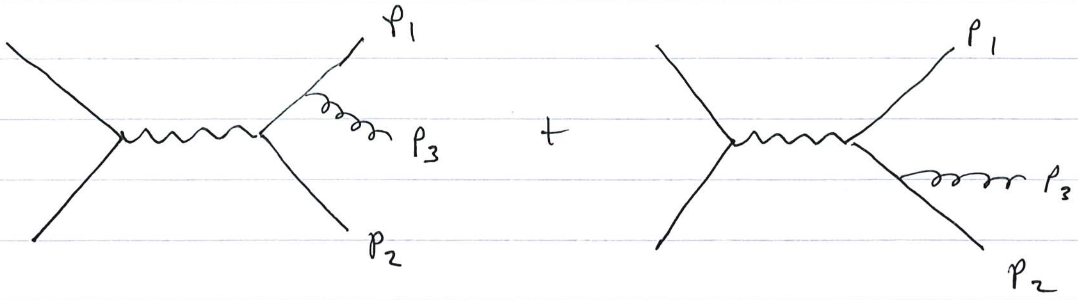
but can't distinguish $q, \bar{q}$

$$\Rightarrow \frac{d\sigma}{d\cos\theta_T} \sim 1 + \cos^2\theta_T, \qquad 0 \le \theta_T \le \frac{\pi}{2}$$



$\rightarrow$ slide

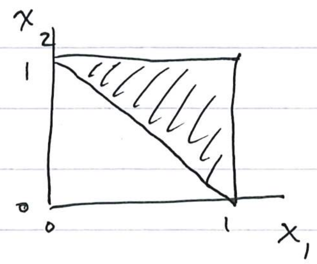
$$e^+e^- \to q\bar{q}g$$



Let $x_i = \dfrac{2E_i}{E_{cm}}$ so $0 \le x_i \le 1$

only 2 indep.

$\sum_{i=1}^{3} E_i = E_{cm} \Rightarrow \sum_i x_i = 2$



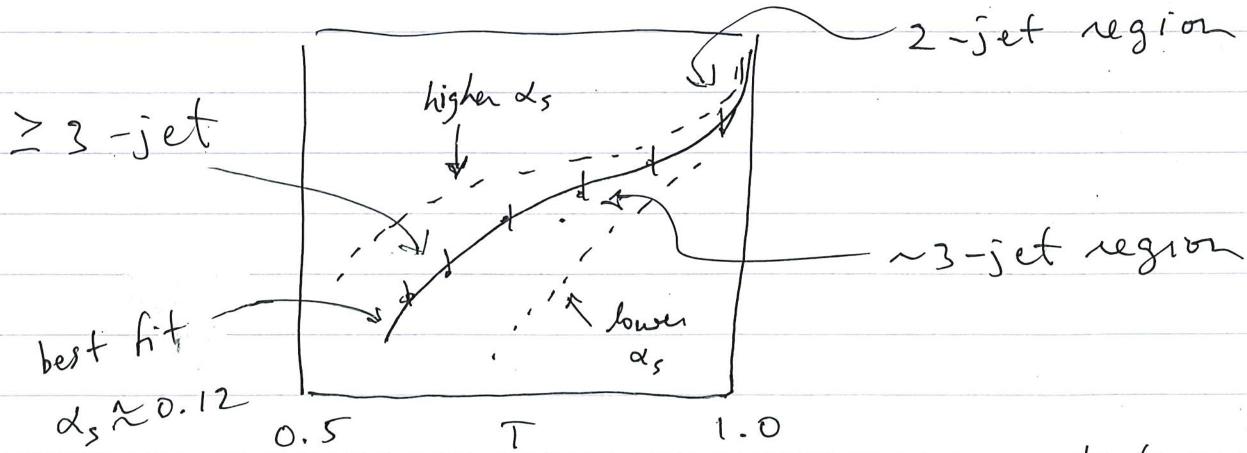
L.O. QCD calc

$$\frac{d^2\sigma}{dx_1\,dx_2} = \sigma_{q\bar{q}} \frac{\overset{4/3}{C_F}\,\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

or for thrust $T = T(x_1, x_2) = \max\{x_i\}$

$$\to \frac{d\sigma}{dT} = \int_0^1 dx_1 \int_{1-x_1}^{1} \frac{d^2\sigma}{dx_1\,dx_2}\,\delta(T - T(x_1,x_2))\,dx_2$$

$$= \sigma_{q\bar{q}} C_F \frac{\alpha_s}{2\pi}\left[\frac{2(3T^2-3T+2)}{T(1-T)} \ln\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{1-T}\right]$$



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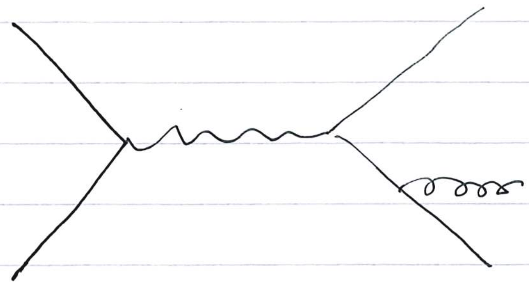

$$\mathcal{M} = \left[\bar{v}(k_2)(-ie\gamma_\mu)u(k_1)\right]\frac{-ig^{\mu\nu}}{q^2}$$

$$\times\left[\bar{u}(p_1)(-ig_s T^a \not{\varepsilon})\frac{i(\not{p}_1+\not{p}_3)}{(p_1+p_3)^2}(-ie\gamma_\nu)v(p_2)\right.$$

$$\left.+\ \bar{u}(p_1)(-ie\gamma^\mu)\frac{i(\not{p}_2-\not{p}_3)}{(p_2-p_3)^2}(-ig_s T^a\not{\varepsilon})v(p_2)\right]$$