G. COWAN 12.12.25

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PH4442 Advanced Ptcl. Physics Week 11

This week: (→ slide)

maybe skip → α_s from $\sigma(e^+e^- \to \text{hadrons})$ }

quark – quark scattering } QCD (finish)

Partons & PDFs

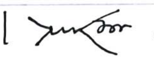
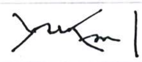
Deep Inelastic Scattering

Hadron collider physics: Drell-Yan, jets, dijets

α_s from $\sigma(e^+e^- \to \text{hadrons})$

$$\sigma(e^+e^- \to \text{hadrons}) = \sigma_{q\bar{q}} + \sigma_{q\bar{q}g} + \sigma_{q\bar{q}gg} + \cdots$$

$$\sigma_{q\bar{q}} = \frac{4\pi\alpha^2 N_c}{3E_{cm}^2} \sum_q Q_q^2$$ ← modify to include Z

from |  +  $|^2$ (real gluon emission)

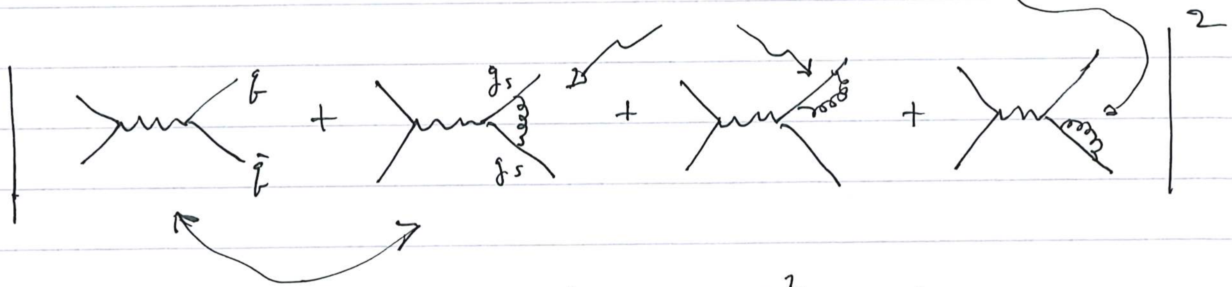
$$\sigma_{q\bar{q}g} = \int_0^1 \int_0^1 \underbrace{\frac{d^2\sigma}{dx_1\,dx_2}}\, dx_1\, dx_2 \to +\infty \;!$$

$$\hookrightarrow \sigma_{q\bar{q}} \frac{C_F \alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$ ← divergent when $x_1, x_2 \to 1$ (colinear) or $x_3 \to 0$ (infrared) $E_g \to 0$

leading-order contribution to $\sigma_{q\bar{q}g}$ at $O(\alpha_s)$

But, at same order ($O(\alpha_s)$), contributions to $\sigma_{q\bar{q}}$

from interference terms: virtual corrections



interference terms $\sim g_s^2 \sim \alpha_s$

(same as L.O. $q\bar{q}g$)

Interference terms for $\sigma_{q\bar{q}}$ also diverge, but to $\underline{-\infty}$

$$\sigma(q\bar{q}) + \sigma(q\bar{q}g) = \sigma_0\left(1 + \frac{\alpha_s}{\pi}\right)$$

Divergences (mostly) cancel, leaving finite correction

Cancellation of real/virtual divergences

works at every order of perturbation theory!

$$R = R_0\left[1 + \frac{\alpha_s}{\pi} + 1.411\left(\frac{\alpha_s}{\pi}\right)^2 - 12.8\left(\frac{\alpha_s}{\pi}\right)^3 + \dots\right]$$

$$\uparrow$$

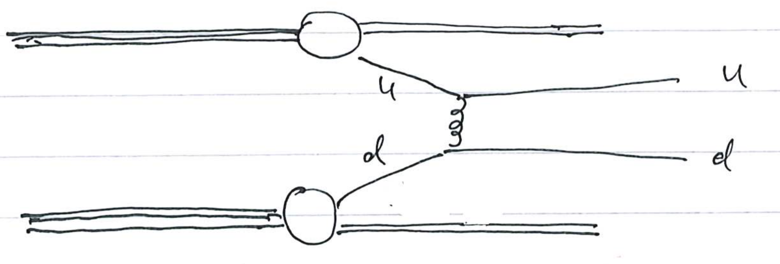
$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

From comparison of theory + measurement,

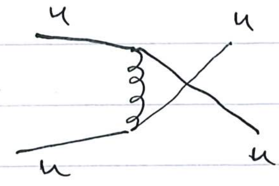
$$\alpha_s(M_Z) = 0.1200 \pm 0.0028$$

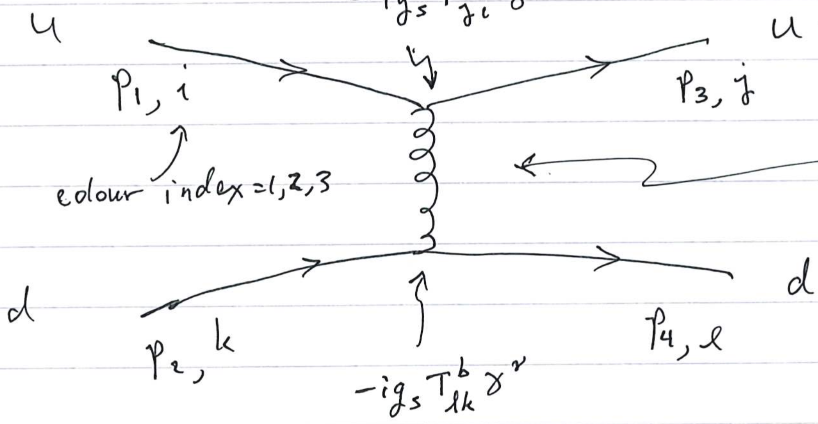
Parton - parton scattering

Consider $ud \to ud$, e.g.



different flavours avoids complication of





$$\frac{-ig_{\mu\nu} \delta^{ab}}{q^2}$$

$q = p_1 - p_3$

$$\mathcal{M} = \left[\bar{u}_j(p_3)(-ig_s T^a_{ji} \gamma^\mu) u_i(p_1) \right] \left(\frac{-i g_{\mu\nu} \delta^{ab}}{q^2} \right)$$

$q^2 = (p_1 - p_3)^2 \equiv t$

$$\times \left[\bar{u}_l(p_4)(-ig_s T^b_{lk} \gamma^\nu) u_k(p_2) \right]$$

Sum over ν and b

$$\mathcal{M} = -\frac{g_s^2}{q^2} T^a_{ji} T^a_{lk} \left[\bar{u}_j(p_3) \gamma_\mu u_i(p_1) \right] \left[\bar{u}_l(p_4) \gamma^\mu u_k(p_2) \right]$$

$\mathcal{M} \sim$ same as $e^- \mu^- \to e^- \mu^-$ w/

$$e^2 \to g_s^2$$

$$+ \text{ factors } \; T^a_{ji} T^a_{lk} = \frac{1}{4} \lambda^a_{ji} \lambda^a_{lk} \Bigg\} \begin{array}{l} \text{summed over} \\ a=1,\ldots,8 \end{array}$$

Assume all $N_c^2 = 9$ colour combinations equally likely $\Rightarrow$ mult by $1/9 = \frac{1}{N_c^2}$

$\Rightarrow \langle |\mathcal{M}|^2 \rangle$ will have factor

$$C = \frac{1}{N_c^2} \sum_{i,j,k,l} \left| \sum_{a=1}^{8} T^a_{ji} T^a_{lk} \right|^2$$

$\Bigg[$ Use $(T^b_{ji})^* = T^b_{ij}$ (T Hermitian)

$$\Rightarrow \sum_{i,j,k,l} \left| \sum_a T^a_{ji} T^a_{lk} \right|^2 = \sum_{i,j,k,l} \left(\sum_a T^a_{ji} T^a_{lk} \right) \left(\sum_b T^b_{ji} T^b_{lk} \right)^*$$

$$= \sum_{i,j,k,l} \sum_{a,b} T^a_{ji} (T^b_{ji})^* \, T^a_{lk} (T^b_{lk})^*$$

$$= \sum_{a,b} \left(\sum_{i,j} T^a_{ji} T^b_{ij} \right) \left(\sum_{k,l} T^a_{lk} T^b_{kl} \right)$$

$$= \sum_{a,b} \left[\mathrm{Tr}\left(T^a T^b \right) \right]^2$$

Now use property of SU(3) generators

$$\mathrm{Tr}(T^a T^b) = \underset{\uparrow \, 1/2}{T_F} \, \delta^{ab}$$

$$\Rightarrow \quad C = \frac{1}{N_c^2} \sum_{a,b}{}' \left[\mathrm{Tr}(T^a T^b) \right]^2$$

$$= \frac{T_F^2}{N_c^2} \sum_{a,b=1}^{8}{}' (\delta^{ab})^2 = \frac{8 T_F^2}{N_c^2} = \underline{\underline{\frac{2}{9}}}$$

Rest of $\langle |\mathcal{M}|^2 \rangle$ like $e^- p \to e^- p$ (Prob Sheet 3)

$$\Rightarrow \quad \langle |\mathcal{M}|^2 \rangle = 2 \underset{\uparrow \, 2/9}{C} \, g_s^4 \, \frac{s^2 + u^2}{t^2}$$

$$\left.\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 - p_3)^2 \\ u &= (p_1 - p_4)^2 \end{aligned}\right\}$$

Mandelstam variables (actually should write $\hat{s}, \hat{t}, \hat{u}$ since for partons, not proton

Use Eq. (7.43), $\lambda \approx s^2$

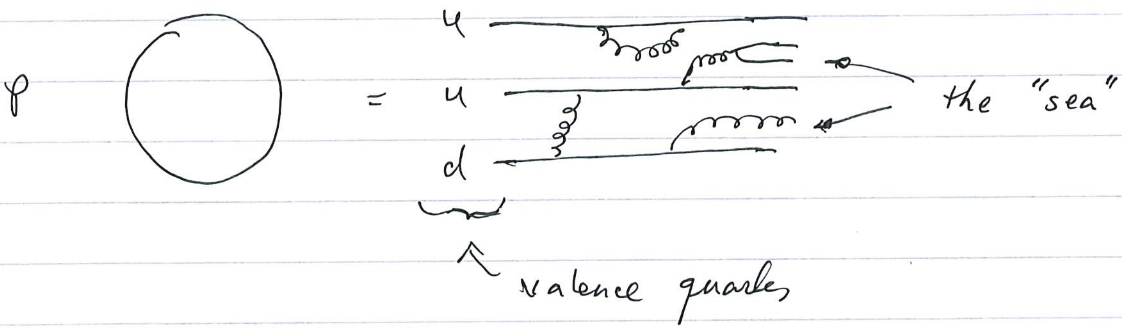
$$\Rightarrow \quad \frac{d\sigma}{dt} = \frac{1}{16\pi s^2} \langle |\mathcal{M}|^2 \rangle = \frac{4\pi}{9} \frac{\alpha_s^2}{s^2} \left(\frac{s^2 + u^2}{t^2} \right)$$

$ud \to ud$ one of many parton-parton reactions

$\longrightarrow$ slide.

From partons to hadrons

Parton model of hadron (e.g. proton)



"parton" = $q, \bar{q}, g$ labeled by index a

Hadron A has four-momentum p_A

has four momentum in infinite momentum frame

parton $\rightarrow$ $p_a = x\, p_A$ $\leftarrow$ hadron

hadron is ultra-relativistic

four-momentum fraction in

Two effects:

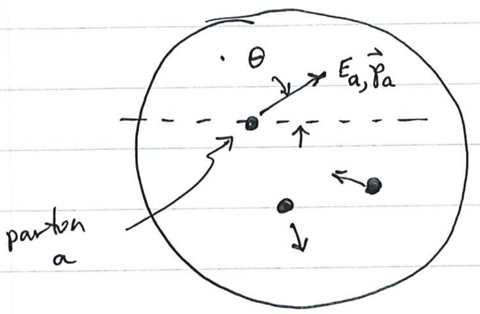
1) relativity

2) parton interactions (QCD)

why isn't $x \sim 1/3$?

First consider just relativity

proton rest frame (no prime)



proton moving along z axis (primed)

boost $\beta\gamma$ ($>>1$)

$$p_A = (M, 0, 0, 0)$$

$$p'_{A,z} = \beta\gamma M$$

$$E'_A \approx p'_{A,z}$$

$$x = \frac{p'_{a,z}}{p'_{A,z}} = \frac{\beta\gamma E_a + \gamma p_{a,z}}{\beta\gamma E_A + \gamma p_{A,z}} = \frac{\beta\gamma E_a + \gamma\sqrt{E_a^2 - m_a^2}\cos\theta}{\beta\gamma M + \gamma \cdot 0}$$

$$= \frac{E_a + \frac{1}{\beta}\sqrt{E_a^2 - m_a^2}\cos\theta}{M}$$

$$\rightarrow \frac{E_a + \sqrt{E_a^2 - m_a^2}\cos\theta}{M} \quad \text{for } \beta \to 1$$

If partons non-relativistic in proton rest frame,

$$E_a \approx m_a \quad \Rightarrow \quad x \to \frac{m_a}{M} \quad (\text{e.g. } \sim 1/3)$$

But if partons relativistic in proton rest frame,

i.e. $E_a >> m_a$ then

$$x \to \frac{E_a}{M}(1 + \cos\theta)$$

Note: $m_u = 2.2$ MeV
$m_d = 4.7$ MeV
$m_p = 938.3$ MeV

$$E_{a,max} = \frac{M}{2} \quad (\text{cons. of momentum})$$

$\Rightarrow \quad 0 < x < 1$ continuous distribution depends on E_a, θ

Parton Density Functions (PDFs)

For parton type a $(= u, d, s, \ldots g, \bar{u}, \bar{d}, \bar{s}, \ldots)$

$f_a(x)\,dx$ = (expected) # of partons of type a

with $x \in [x, x+dx]$

Effect #2 : QCD interactions

$f_a(x) \longrightarrow f_a(x, Q^2)$ ← cannot predict w/ perturbative QCD

"Q^2 = four-momentum transfer (squared)"

precise defn depends on process, e.g., $e^- p \to e^- + X$, $Q^2 = -q^2$

Deep Inelastic Sc.

q^2 of exchanged γ

But perturb. QCD predicts evolution w/ Q^2:

Altarelli-Parisi splitting func.

DGLAP equations

$$\frac{\partial f_a(x,Q^2)}{\partial \ln Q^2} = \sum_b \int_x^1 \frac{dz}{z} P_{ab}(z, \alpha_s(Q^2))\, f_b\!\left(\frac{x}{z}, Q^2\right)$$

interp: prob to resolve parton b as a @ Q^2

Below $f_a(x)$ understood to mean PDF is evolved via DGLAP to appropriate Q^2.

PDFs obey certain "sum rules", e.g. <u>baryon # sum rule</u>:

For proton:

$$\int_0^1 [f_u(x) - f_{\bar{u}}(x)]\,dx = 2$$

$$\int_0^1 [f_d(x) - f_{\bar{d}}(x)]\,dx = 1$$

$$\int_0^1 [f_Q(x) - f_{\bar{Q}}(x)]\,dx = 0, \quad Q = s, c, \ldots$$

and

$$\int_0^1 x\left(\sum_q [f_q(x) + f_{\bar{q}}(x)] + f_g(x)\right)dx = 1$$

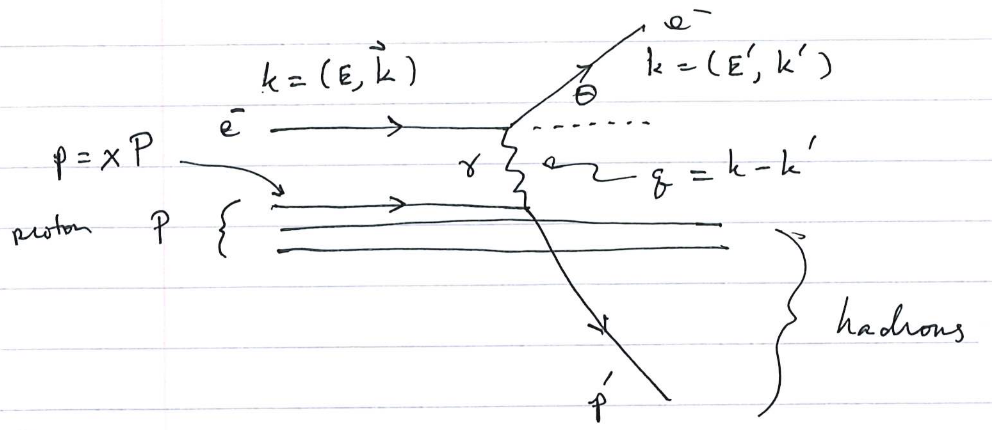
<u>Momentum sum rule</u>

(sum of momentum fractions of all types = 1).

Deep Inelastic Scattering (DIS)

$e^- p \to e^- + X$

↑ hadrons



Four-momentum transfer is

$$q^2 = (k-k')^2 = k^2 + k'^2 - 2k\cdot k'$$

$$= 2m_e^2 - 2EE' + 2\vec{k}\cdot\vec{k}'$$

neglect

$$\simeq -2EE'(1-\cos\theta) = -4EE'\sin^2\frac{\theta}{2}$$

Since $q^2 < 0$ define $Q^2 = -q^2$.

For e^--quark $\to$ e^--quark, amplitude is

$$\mathcal{M} = \left[\bar{u}(k')(ie\gamma_\mu)u(k)\right]\left(\frac{-ig^{\mu\nu}}{q^2}\right)\left[\bar{u}(p')(-ieQ_q\gamma_\nu)u(p)\right]$$

Q_q: $\frac{2}{3}$ or $-\frac{1}{3}$

skip to here →

$$= \frac{-e^2 Q_q}{q^2}\left[\bar{u}(k')\gamma_\mu u(k)\right]\left[\bar{u}(p')\gamma^\mu u(p)\right]$$

Usual technology to ave/sum over spins:

$$\langle|\mathcal{M}|^2\rangle = 32\pi^2\alpha^2\frac{(2\hat{s}^2 + 2\hat{s}q^2 + q^4)}{q^4},$$

$\hat{s} = (k+p)^2 \gg m_q^2$

↑ hat since e^--quark E_{cm}^2.

Use Eq. (7.43) for $\frac{d\sigma}{dt}$, $t = (k-k')^2 = q^2$

$$\Rightarrow \quad \frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{\hat{s}}\right)^2\right]$$

$$\hat{s} = (k + \underset{\text{quark}}{p})^2 = (k + x\underset{\text{proton}}{P})^2$$

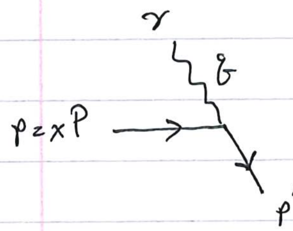
$$= \underbrace{m_e^2}_{\text{neglect}} + x^2 \underbrace{M^2}_{\text{proton mass}} + 2x\,k\cdot P \approx 2x\,k\cdot P \qquad (1)$$

E_{cm} of e^- - proton system is

$$s = (k+P)^2 = m_e^2 + M^2 + 2k\cdot P \approx 2k\cdot P \qquad (2)$$

Compare (1) + (2) $\longrightarrow$ $\underline{\hat{s} = x\,s}$

Momentum fraction x can be measured from E', θ:



$$\Rightarrow \quad xP + q = p'$$

$$\Rightarrow \quad (xP + q)^2 = p'^2$$

$$\Rightarrow \quad \cancel{\underbrace{(xP)^2}_{= p^2 = m_q^2}} + 2x P\cdot q + q^2 = \cancel{m_q^2}$$

$$\Rightarrow \quad x = -\frac{q^2}{2P\cdot q} = \frac{Q^2}{2P\cdot q} \equiv \text{"Bjorken } x\text{"}$$

measurable from E', θ

In proton rest frame, $P = (M, 0, 0, 0)$

$\Rightarrow P \cdot q = P \cdot (k - k') = M(E - E') = M\nu$,

where $\nu = E - E'$ = energy lost by e^- in lab frame

$$\Rightarrow \quad Q^2 = 4EE' \sin^2 \frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

Using $Q^2 = -q^2$, $\hat{s} = xs$, rewrite $\frac{d\sigma}{dq^2}$ as

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 Q_q^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]$$

= cross section given a target quark w/ $p = xP$.

But we can't fix the x of target quarks in proton

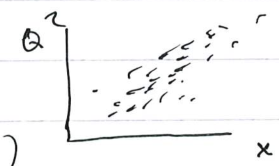
\# of quarks w/ $x \in [x, x + dx] = f_q(x)\,dx$

Antiquarks also contribute: $Q_{\bar{q}}^2 = Q_q^2$ ($\sim$same diagram)

$\Rightarrow$ double differential cross section

$$\frac{d^2\sigma}{dx\,dQ^2} = \sum_q{}' \frac{2\pi\alpha^2 Q_q^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]\left[f_q(x) + f_{\bar{q}}(x)\right]$$

$\rightarrow$ plots: H1 event,



$\Rightarrow$ measure (particular combo of)

$f_q(x) + f_{\bar{q}}(x)$

Measuring PDFs.

$e^- p \to e^- + X$ gives info on combo of $f_u + f_{\bar{u}}$, $f_d + f_{\bar{d}}$

To disentangle $q, \bar{q}$ & contributions of different processes, need info from more processes, e.g.

$e^- n$? ← use $D = np$ ("isoscalar" target)

neutrino scattering – recall

$$\sigma(\nu_\mu d \to \mu^- u) = \sigma(\nu_\mu \bar{u} \to \mu^- \bar{d}) = \frac{G_F^2 \cos^2\theta_c \, \hat{s}}{\pi}$$

$$\sigma(\bar{\nu}_\mu u \to \mu^+ d) = \sigma(\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}) = \frac{G_F^2 \cos^2\theta_c \, \hat{s}}{3\pi}$$

Here also $\hat{s} = x s$

E', θ easy to measure from outgoing μ,

but neutrino E has broad distribution

⇒ measure hadrons in final state → $E = \underbrace{E_\mu}_{E'} + E_{had}$

⇒ can measure x for each event.

$$\rightarrow \quad \frac{d\sigma}{dx}(\nu_\mu p \to \mu^- + X) = \frac{G_F^2 \cos^2\theta_c \, s x}{\pi}\left[f_d(x) + f_{\bar{u}}(x)\right]$$

$$\frac{d\sigma}{dx}(\bar{\nu}_\mu p \to \mu^+ + X) = \frac{G_F^2 \cos^2\theta_c \, s x}{3\pi}\left[f_u(x) + f_{\bar{d}}(x)\right]$$

Complete PDF fit uses many processes : DIS, neutrino, $pp \to \mu^+\mu^- + X$ (Drell-Yan), $pp \to Z/W + X$, ...

together w/ DGLAP to evolve PDFs to appropriate Q^2.

→ plots (CTEQ fits)

Hadron Collider Physics

e.g. pp @ LHC

$p\bar{p}$ @ Tevatron

x_1, x_2 = 4-momentum fractions of colliding hadrons A, B (measurable)

At lowest order in QCD,

factorization theorem:

$$\frac{d^2\sigma}{dx_1\,dx_2}(AB \to X) = \underbrace{\sum_{a,b}}_{\text{sum over parton types that contribute to } AB\to X} \sigma(ab \to X \mid \underbrace{\hat{s}}_{E_{cm}^2 \text{ of parton-parton system}} = x_1 x_2 s)\, f_{A,a}(x_1, Q^2)\, f_{B,b}(x_2, Q^2)$$

$Q^2 \approx \hat{s}$

Focus on LHC: $A = B$ = proton or drop subscript

Total cross section

$$\sigma(AB \to X) = \int_0^1\!\!\int_0^1 \frac{d^2\sigma}{dx_1\,dx_2}\,dx_1\,dx_2$$

or often convert $\frac{d^2\sigma}{dx_1\,dx_2}$ to different differential xs,

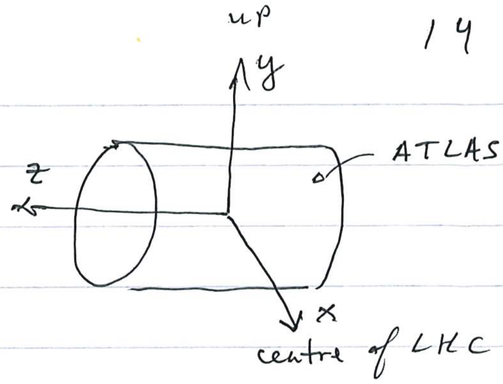
e.g. u, v are some functions of x_1, x_2

$$\frac{d^2\sigma}{du\,dv} = |J|\,\frac{d^2\sigma}{dx_1\,dx_2}$$

or

$$\frac{d^2\sigma}{du\,dv} = \iint \frac{d^2\sigma}{dx_1\,dx_2}\,\delta(u - u(x_1,x_2))\,\delta(v - v(x_1,x_2))$$

Hadron Collider basics



$$p_A = (E_p, 0, 0, E_p)$$

$$p_B = (E_p, 0, 0, -E_p)$$

$$E_p = \frac{E_{cm}}{2}, \quad E_{cm} \sim \text{up to } 13.6 \text{ TeV}$$

$$s = (p_A + p_B)^2 = \overbrace{p_A^2 + p_B^2}^{2m_p^2 \to \text{neglect}} + 2\, p_A \cdot p_B$$

$$\hat{s} = (x_1 p_A + x_2 p_B)^2 = \underbrace{x_1^2 p_A^2 + x_2^2 p_B^2}_{\text{neglect}} + 2 x_1 x_2\, p_A \cdot p_B$$

$$\Rightarrow \boxed{\hat{s} = x_1 x_2 s}$$

Total p_z, E of colliding partons is

$$p_z = x_1 p_{A,z} + x_2 p_{B,z} = E_p (x_1 - x_2)$$

$$E = E_p (x_1 + x_2)$$

$\Rightarrow$ rapidity of two-parton system is

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \frac{x_1}{x_2} = \frac{1}{2} \ln \frac{1+\beta}{1-\beta}$$

Or equivalently

$$x_1 = \sqrt{\tau}\, e^{y}$$

$$x_2 = \sqrt{\tau}\, e^{-y}$$

where $\tau = \dfrac{\hat{s}}{s} = x_1 x_2$

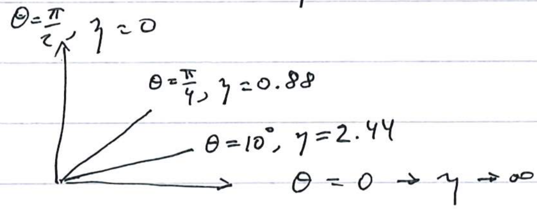
For an individual ptcl w/ outgoing ∡s θ, ϕ & energy E, momentum $\vec{p}$, define rapidity in same way $y = \frac{1}{2} \ln\left(\frac{E+p_z}{E-p_z}\right)$

In ultra-rel. limit, $y \to -\ln \tan\frac{\theta}{2} \equiv \eta$

↑ "pseudo rapidity"

or equiv, $\cos\theta = \tanh \eta$

→ plot



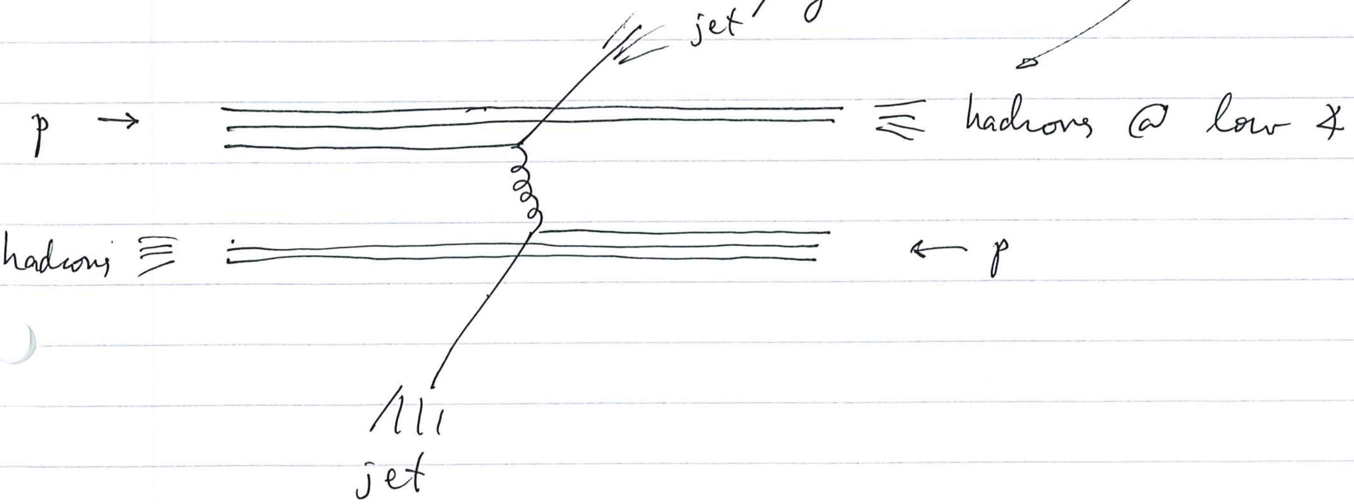
Also use transverse momentum relative to z axis

$\to \quad p_T = \sqrt{p_x^2 + p_y^2}$

$p = (E, p_x, p_y, p_z)$ equivalent to

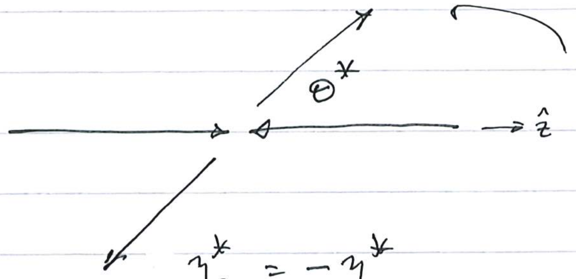
p_T, η, ϕ, M or ultra-rel means $E \approx |\vec{p}|$, $M \ll p_T$

When 2 protons collide, "remnants" continue along beam line → "underlying event".



Suppose two partons w/ x_1, x_2 collide.

In parton-parton c.m. frame

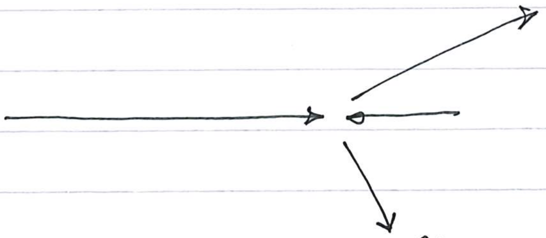


$$\eta_1^* = -\ln\tan\frac{\theta^*}{2}$$

$$\eta_2^* = -\eta_1^*$$

But x_1, x_2 in general unequal

$\Rightarrow$ in pp c.m. frame,



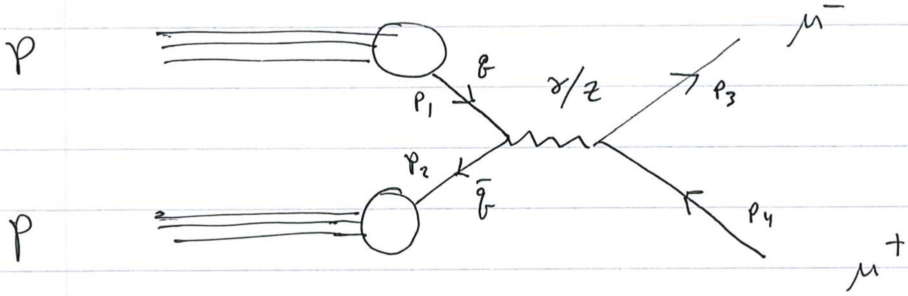
$$\eta_1 = \eta_1^* + y$$

$$y = \frac{1}{2}\ln\frac{x_1}{x_2}$$

$$\eta_2 = \eta_2^* + y \qquad = \frac{1}{2}\ln\left(\frac{1+\beta}{1-\beta}\right)$$

$\Rightarrow$ difference of (pseudo) rapidities invariant under boost along z axis.

Drell-Yan process: $pp \to l^+ l^- + X$, $l = e, \mu, \tau$



For $\sqrt{\hat{s}} \ll M_Z$, only γ exchange important

$$\sigma(q\bar{q} \to \mu^+\mu^-) = \frac{4\pi\alpha^2 Q_q^2}{3 N_c \hat{s}} \qquad (\text{cf. } e^+e^- \to q\bar{q})$$

$\hat{s} = x_1 x_2 s$

For pp collisions, need to get $\bar{q}$ from "sea"

$$\to \frac{d^2\sigma(pp \to \mu^+\mu^- + X)}{dx_1\, dx_2} = \frac{4\pi\alpha^2}{9 s x_1 x_2} \sum_q Q_q^2 \left[f_q(x_1) f_{\bar{q}}(x_2) + f_{\bar{q}}(x_1) f_q(x_2) \right]$$

(Similar for $p\bar{p} \to \mu^+\mu^- + X$ e.g. at Tevatron,

use $f_{p,q}(x) = f_{\bar{p},\bar{q}}(x)$, $f_{p,\bar{q}}(x) = f_{\bar{p},q}(x)$)

Convert to $\dfrac{d^2\sigma}{dm_{\mu\mu}\, dy}$, $m_{\mu\mu} = \sqrt{\hat{s}} = \sqrt{x_1 x_2 s}$

$$y = \frac{1}{2} \ln \frac{x_1}{x_2}$$

Jacobian $$J = \begin{vmatrix} \dfrac{\partial x_1}{\partial m_{\mu\mu}} & \dfrac{\partial x_1}{\partial y} \\[2ex] \dfrac{\partial x_2}{\partial m_{\mu\mu}} & \dfrac{\partial x_2}{\partial y} \end{vmatrix} = -\frac{2 m_{\mu\mu}}{s}$$

$$\frac{d^2\sigma(pp \to \mu\mu + X)}{dm_{\mu\mu}\, dy} = \frac{8\pi\alpha^2}{9 s m_{\mu\mu}} \sum_q Q_q^2 \left[f_q\Big(\underbrace{\tfrac{m_{\mu\mu} e^{y}}{\sqrt{s}}}_{x_1}\Big) f_{\bar{q}}\Big(\underbrace{\tfrac{m_{\mu\mu} e^{-y}}{\sqrt{s}}}_{x_2}\Big) + f_{\bar{q}}\Big(\tfrac{m_{\mu\mu} e^{y}}{\sqrt{s}}\Big) f_q\Big(\tfrac{m_{\mu\mu} e^{-y}}{\sqrt{s}}\Big) \right]$$

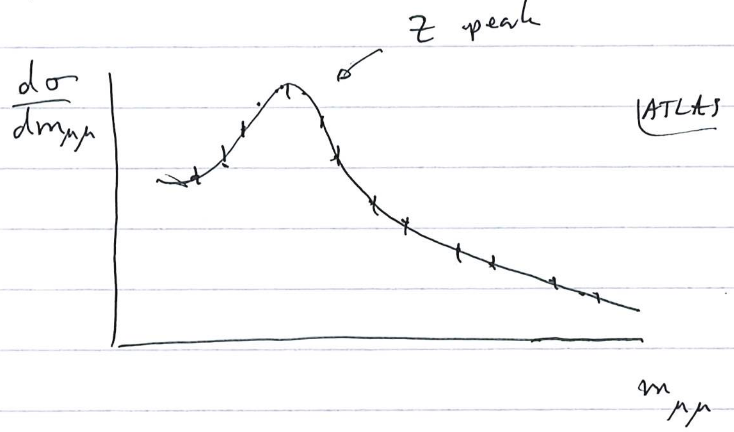
For $\frac{d\sigma}{dm_{\mu\mu}}$ integrate $\displaystyle\int_{y_{min}}^{y_{max}} \frac{d^2\sigma}{dm_{\mu\mu}\,dy}\,dy$

Need $x_1 \leq 1$, $x_2 \leq 1$ and $x_1 x_2 = \frac{m_{\mu\mu}^2}{s}$

$$\Rightarrow \quad y_{max} = \frac{1}{2}\ln\frac{x_{1,max}}{x_2} = \frac{1}{2}\ln\frac{1}{(m_{\mu\mu}^2/s)} = -\ln\left(\frac{m_{\mu\mu}}{\sqrt{s}}\right)$$

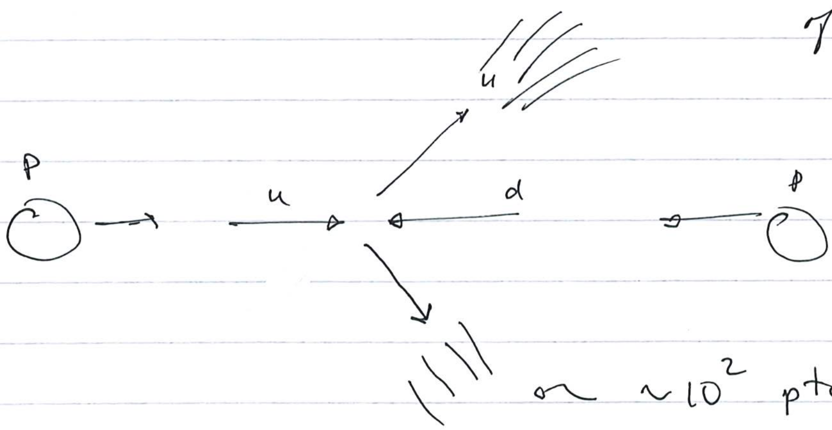
$$y_{min} = -y_{max}.$$

$\longrightarrow$ plot



Jets

Other reactions have outgoing $q, \bar{q}, g$ that we don't directly observe, rather, they produce "jets" of hadrons



or $\sim 10^2$ ptcls, $\pi^\pm$, π^0
$\hookrightarrow \gamma\gamma$

also K, p, n, some leptons
e.g. from B decay

How to define a jet

Start w/ N ptcls w/ $p_1, \ldots, p_N$ } hadrons or partons (or "reco objects")

For each pair, ∡ separation

$$\Delta R_{ij} = \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$$

$\hookrightarrow -\ln \tan \frac{\theta_i}{2}$ azimuthal ∡

Find min of:

$$d_{ij} = \min\left(p_{T_i}^{2p}, p_{T_j}^{2p}\right) \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = p_{T_i}^{2p}$$ ← distance to beamline

p = a power (1, 0, −1, see below)

R = cone size, choose value in ~ 0.4 ... 1.0

skinny jets ... fat jets

$$p = \begin{cases} 1 & k_T \text{ algorithm} \\ 0 & \text{Cambridge / Aachen} \\ -1 & \text{"anti-}k_T\text{"} \end{cases}$$

← default (→ round jets, see plot)

- If smallest d_{ij} or d_{iB} is a d_{ij}, merge $i+j$ ($p = p_i + p_j$) "E" recombination scheme

  & Replace i & j by merged pseudoptcl.
- If smallest d_{ij} or d_{iB} is a d_{iB}, remove it from list & call it a <u>jet</u>
- Iterate until no ptcls left in list.

Dijet production

From $\frac{d\sigma}{dt}$ found for $ud \to ud$, use $Q^2 = -q^2 = -t$

parton level

$$\Rightarrow \frac{d\hat{\sigma}}{dQ^2} = \frac{4\pi}{9}\frac{\alpha_s^2}{Q^4}\left[1 + \left(1 - \frac{Q^2}{\hat{s}}\right)^2\right]$$

(used $\hat{s}+\hat{t}+\hat{u} \approx 0$ for ultra-rel., hat for parton level.)

$$\hat{s} = x_1 x_2 s$$

u in either p, d in other

$$\Rightarrow \frac{d^3\sigma(pp\to ud)}{dQ^2\,dx_1\,dx_2} = \left.\frac{d\hat{\sigma}}{dQ^2}\right|_{\hat{s}=x_1x_2s} \times 2 f_u(x_1) f_d(x_2)$$

Transform from $x_1, x_2, Q^2 \to m_{jj}, y, \eta^*$

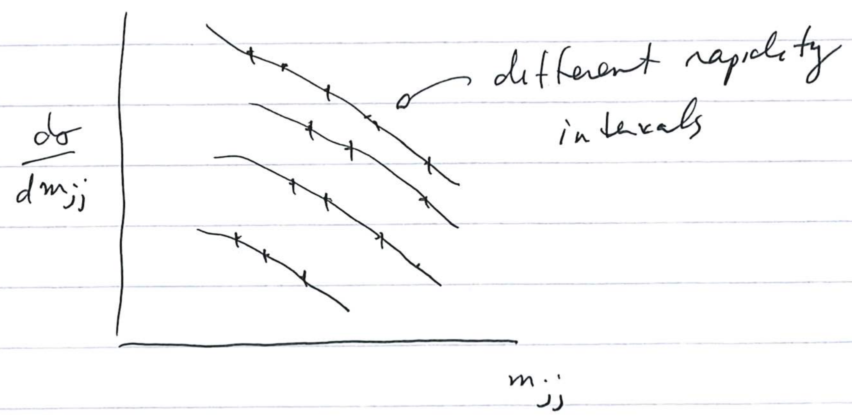
jet-jet mass; rapidity of qq in lab.; pseudorap of qq in their c.m.

$\to$ Jacobian

$\to \frac{d^3\sigma}{dm_{jj}\,d\eta^*\,dy}$ (see notes)

$ud \to ud$ only one of many (~dozen) parton-parton processes; dominant at LHC is $gg \to gg$

$\to$ plot



Data in excellent agreement w/ QCD prediction.