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1

PH4442 Adv. Ptcl. Physics Week 7

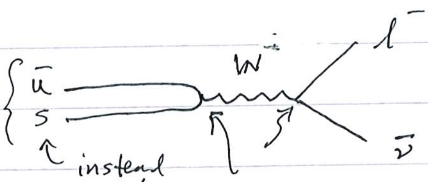
CKM Matrix

Last week found:

$$\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell) = \frac{G_F^2}{8\pi m_\pi^3} f_\pi^2 m_\ell^2 (m_\pi^2 - m_\ell^2)^2$$

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^+ \to \mu^- \bar{\nu}_\mu)} = 1.2 \times 10^{-4} \Rightarrow \text{major success for V-A theory}$$

Kaon undergoes similar decays: K^-



instead of d

same couplings ?

Expect: $$R_\ell = \frac{\Gamma(K^- \to \ell^- \bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^- \bar{\nu}_\ell)} = \frac{f_K^2}{f_\pi^2} \frac{m_\pi^3}{m_K^3} \frac{(m_K^2 - m_\ell^2)^2}{(m_\pi^2 - m_\ell^2)^2}$$

In 1960s, not clear what to assume for f_K / f_π

For now suppose $f_K = f_\pi$

$m_{\pi^\pm} = 139.6$ MeV
$m_{K^\pm} = 493.7$ MeV
$m_\mu = 105.7$ MeV
$m_e = 0.511$ MeV

$\Rightarrow$ Predict: $R_e = 3.54$, $R_\mu = 17.7$

$$R_\ell = \frac{\Gamma(K \to \ell\nu)}{\Gamma(\pi \to \ell\nu)} = \frac{B(K \to \ell\nu)}{B(\pi \to \ell\nu)} \frac{\Gamma_K}{\Gamma_\pi} = \frac{B(K \to \ell\nu)}{B(\pi \to \ell\nu)} \frac{\tau_\pi}{\tau_K}$$

branching ratios; total decay rates; mean lifetimes

τ_π ← 2.60×10^{-8} s

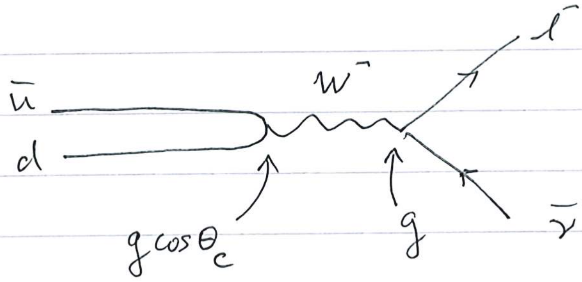
τ_K ← 1.24×10^{-8} s

$\Rightarrow$ measure: $R_e = 0.270$, $R_\mu = 1.34$

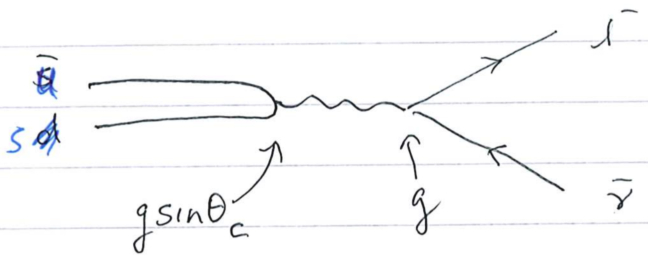
$\Rightarrow$ Predicted / measured ≈ 13 (both e and μ)

Cabibbo 1963 (translated into present language of u, d, s quarks)

$\pi^- \to \ell^- \bar{\nu}$



$K^- \to \ell \bar{\nu}$



I.e. coupling g applies to vertex $\bar{u}\, W d'$

i.e. $|d'\rangle$

"weak eigenstates"

$$\begin{cases} d' = d\cos\theta_c + s\sin\theta_c \\ s' = -d\sin\theta_c + s\cos\theta_c \end{cases}$$

← orthogonal to d'

But π & K contain d, s (mass eigenstates)

$$d = d'\cos\theta_c - s'\sin\theta_c$$

$$s = d'\sin\theta_c + s'\cos\theta_c$$

$\Rightarrow$ in $\pi \to \ell\bar{\nu}$, $g \to g\cos\theta_c$

in $K \to \ell\bar{\nu}$, $g \to g\sin\theta_c$

$$\Rightarrow \frac{\Gamma(K\to \ell\nu)}{\Gamma(\pi\to\ell\nu)} = \tan^2\theta_c \, \frac{f_K^2}{f_\pi^2}\, \frac{m_\pi^3}{m_K^3}\, \frac{(m_K^2 - m_\ell^2)^2}{(m_\pi^2 - m_\ell^2)^2}$$

Compare to measured $\Gamma(K \to l\nu)/\Gamma(\pi \to l\nu)$

$$\Rightarrow \theta_c = 15.4^\circ \qquad (\sin\theta_c = 0.27)$$

More refined analysis including

$$\frac{f_K}{f_\pi} = 1.19 \qquad \text{(from lattice QCD)}$$

$$\Rightarrow \theta_c = 12.8^\circ \qquad (\sin\theta_c = 0.221,\ \cos\theta_c = 0.975)$$

slides ?

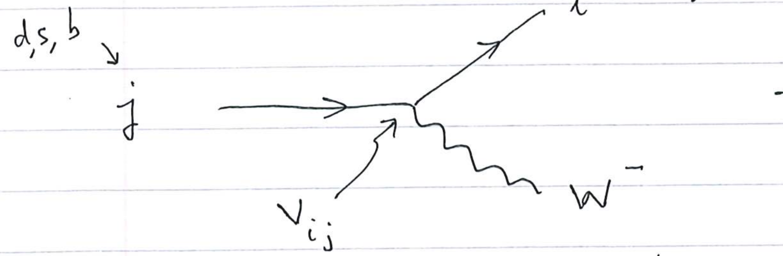
Generalise to 3 families (after discovery of b, before t!)
Kobayashi + Maskawa 1973

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

"flavour eigenstates"

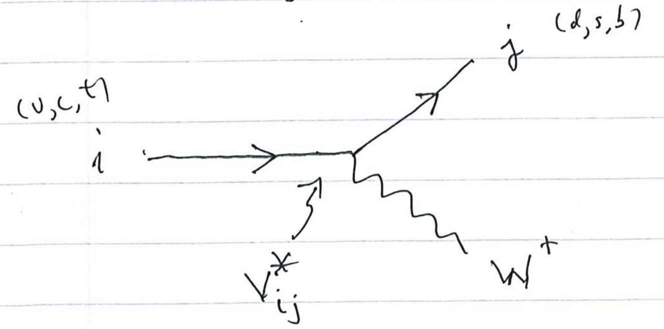
CKM matrix (<u>unitary</u>)

"mass eigenstates"



$$-\frac{ig}{\sqrt{2}} V_{ij} \gamma^\mu \frac{1}{2}(1-\gamma^5)$$

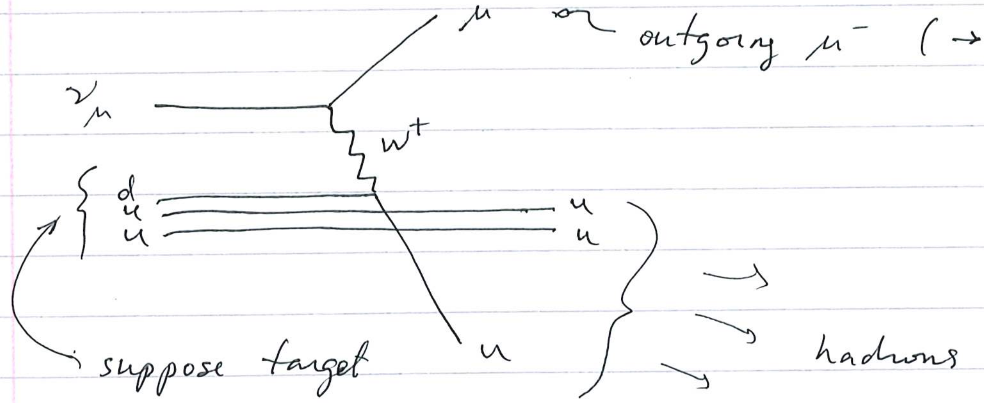
V_{ij} when i (= u,c,t) at head of arrow



$$-\frac{ig}{\sqrt{2}} V_{ij}^* \gamma^\mu \frac{1}{2}(1-\gamma^5)$$

V_{ij}^* when j (= d,s,b) at head of arrow

ν - nucleon scattering



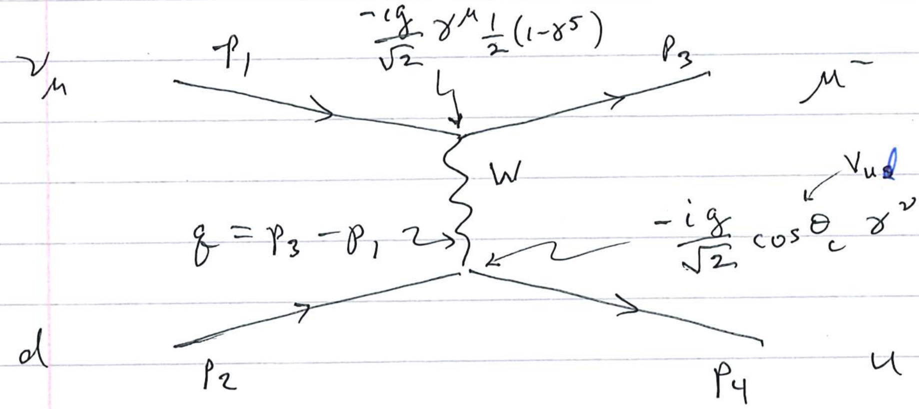
μ^- ~ outgoing μ^- ($\to$ event display)

suppose target has ~ equal #s of p, n ("isoscalar")

Relevant reaction is $\nu_\mu d \to \mu^- u$

No $\nu_\mu u$ since $\mu^- d$ doesn't conserve charge,

$\mu^+ d$ " " lepton number



$\frac{-ig}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5)$

$q = p_3 - p_1$

$\frac{-ig}{\sqrt{2}} \underbrace{\cos\theta_c}_{V_{ud}} \gamma^\nu \underbrace{\tfrac{1}{2}(1-\gamma^5)}_{P_L}$

$$\mathcal{M} = \bar{u}_\mu(p_3)\left[\frac{-ig}{\sqrt{2}} \gamma^\mu \frac{1}{2}(1-\gamma^5)\right] u_{\nu_\mu}(p_1) \left[\frac{-ig_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2}\right]$$

$$\times \bar{u}_u(p_4)\left[\frac{-ig\cos\theta_c}{\sqrt{2}} \gamma^\nu \frac{1}{2}(1-\gamma^5)\right] u_d(p_2)$$

Suppose $|q^2| \ll M_W^2 \Rightarrow [\quad] \approx \frac{ig_{\mu\nu}}{M_W^2}$

Use $\gamma^\mu g_{\mu\nu} \gamma^\nu \longrightarrow \gamma_\mu \gamma^\mu$

and $G_F = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2}$

$$\Rightarrow \mathcal{M} = \frac{-i G_F \cos\theta_c}{\sqrt{2}} \left[\bar{u}(p_3)\gamma_\mu(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_4)\gamma^\mu(1-\gamma^5)u(p_2)\right]$$

From $\mathcal{M} \to |\mathcal{M}|^2$, average over d spin,

ν always left-handed so no factor $\frac{1}{2}$,

(right-handed ν contributes 0 to sum),

sum over final state spins

$$\langle|\mathcal{M}|^2\rangle = \frac{G_F^2 \cos^2\theta_c}{2} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = \sum_{s_1, s_3} \left[\bar{u}(p_3)\gamma_\mu(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_3)\gamma_\nu(1-\gamma^5)u(p_1)\right]^*$$

$$= \sum_{s_1, s_3} \left[\bar{u}(p_3)\gamma_\mu(1-\gamma^5)u(p_1)\right]\left[\bar{u}(p_1)\overline{\gamma_\nu(1-\gamma^5)}u(p_3)\right]$$

where $\overline{\gamma_\nu(1-\gamma^5)} = \gamma^0\left[\gamma_\nu(1-\gamma^5)\right]^\dagger \gamma^0$

can skip to here ↓

$= \gamma^0(1-\gamma^5)^\dagger \gamma_\nu^\dagger \gamma^0$ use $\gamma^{5\dagger} = \gamma^5$

$= \gamma^0(1-\gamma^5)\gamma^0\gamma_\nu \underbrace{\gamma^0\gamma^0}_{=1}$ used $\gamma_\nu^\dagger = \gamma^0\gamma_\nu\gamma^0$

Casimir's trick:

$= \gamma_\nu(1-\gamma^5)$

$$\Rightarrow L_{\mu\nu} = \mathrm{Tr}\left[(\not{p}_3 + m_\mu)\gamma_\mu(1-\gamma^5)\not{p}_1\gamma_\nu(1-\gamma^5)\right]$$

2 sign changes, then $(1-\gamma^5)^2 = 2(1-\gamma^5)$

$$= 2\,\mathrm{Tr}\left[(\not{p}_3 + m_\mu)\gamma_\mu(1-\gamma^5)\not{p}_1\gamma_\nu\right]$$

terms w/ $m_\mu \to 0$ since odd # of gamma mat.

$$L_{\mu\nu} = \underbrace{2\,\mathrm{Tr}(\not{p}_3 \gamma_\mu \not{p}_1 \gamma_\nu)} + 2\,\mathrm{Tr}(\gamma^5 \gamma_\mu \not{p}_1 \gamma_\nu \not{p}_3)$$

$$\hookrightarrow 4[p_{1\mu} p_{3\nu} + p_{1\nu} p_{3\mu} - (p_1 \cdot p_3) g_{\mu\nu}]$$

Use $\mathrm{Tr}(\gamma^5 \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta) = -4i\epsilon_{\alpha\beta\gamma\delta}\, \gamma^\alpha \gamma^\beta \gamma^\gamma \gamma^\delta$

$\uparrow\ \epsilon^{0123} = 1, \quad \epsilon_{0123} = -1$

$$\Rightarrow \mathrm{Tr}[\gamma^5 \gamma_\mu \not{p}_1 \gamma_\nu \not{p}_3] = -4i\epsilon_{\mu\alpha\nu\beta}\, p_1^\alpha p_3^\beta$$

$$\Rightarrow L_{\mu\nu} = 8\left([p_{1\mu} p_{3\nu} + p_{1\nu} p_{3\mu} - (p_1 \cdot p_3) g_{\mu\nu}] - i\epsilon_{\mu\alpha\nu\beta}\, p_1^\alpha p_3^\beta\right)$$

Similarly w/ $p_1 \to p_2$ & $p_3 \to p_4$ find hadron tensor

$$W_{\mu\nu} = \frac{1}{2} 8\left([p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} - (p_2 \cdot p_4) g_{\mu\nu}] - i\epsilon_{\mu\gamma\nu\delta}\, p_2^\gamma p_4^\delta\right)$$

$\uparrow$ average over d spin $\to$ factor of $\frac{1}{2}$.

In computing $L_{\mu\nu} W^{\mu\nu}$,

$$[\ ]_{\mu\nu} [\ ]^{\mu\nu} = 2(p_1 \cdot p_2)(p_3 \cdot p_4) - 2(p_1 \cdot p_4)(p_2 \cdot p_3)$$

$[\ ] \cdot \epsilon \to 0$ since $[\ ]$ symmetric, ϵ antisymmetric

in $\mu \leftrightarrow \nu$ $\qquad$ $\uparrow$ 3 index swaps

$$\epsilon \cdot \epsilon \to 2(p_1 \cdot p_2)(p_3 \cdot p_4) - 2(p_1 \cdot p_4)(p_2 \cdot p_3)$$

(use $\epsilon_{\mu\alpha\nu\beta}\, \epsilon^{\mu\gamma\nu\delta} = \epsilon_{\mu\nu\alpha\beta}\, \epsilon^{\mu\nu\gamma\delta} = -2(\delta_\alpha^\gamma \delta_\beta^\delta - \delta_\alpha^\delta \delta_\beta^\gamma)$)

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = 64\, G_F^2 \cos^2\theta_c\, (p_1 \cdot p_2)(p_3 \cdot p_4)$$

$$\text{Use}\quad \hat{s} \equiv (\underbrace{p_1 + p_2}_{\cancel{m_\nu^2} + \cancel{m_d^2} + 2p_1\cdot p_2})^2 = (\underbrace{p_3 + p_4}_{p_3^2 + p_4^2 + 2p_3\cdot p_4 \,\approx\, m_\mu^2 + 2p_3\cdot p_4})^2$$

$\hat{s}$: E_{cm}^2 of $\nu_\mu d$ system; p_1: ν_μ, p_2: d; p_3: μ, p_4: u

m_ν^2: 0; m_d^2: neglect

$p_3^2 = m_\mu^2$, $p_4^2 = \cancel{m_d^2}$ neglect

$$\Rightarrow \langle |\mathcal{M}|^2 \rangle = 16\, G_F^2 \cos^2\theta_c\, \hat{s}\,(\hat{s} - m_\mu^2)$$

independent of scattering angle θ

$$\text{Use}\quad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 E_{cm}^2}\frac{|\vec{p}_f|}{|\vec{p}_i|}\langle |\mathcal{M}|^2\rangle$$

in c.m.

$$|\vec{p}_i| = \frac{E_{cm}}{2}, \qquad |\vec{p}_f| = \frac{E_{cm}^2 - m_\mu^2}{E_{cm}}$$

show as exercise

$$\Rightarrow \frac{d\sigma}{d\Omega} = \frac{G_F^2\cos^2\theta_c}{4\pi^2}\frac{(\hat{s} - m_\mu^2)^2}{\hat{s}} \approx \frac{G_F^2 \cos^2\theta_c\, \hat{s}}{4\pi^2}, \quad \hat{s} \gg m_\mu^2$$

Total cross section

$$\sigma = \int \frac{d\sigma}{d\Omega}\, d\Omega = 4\pi \frac{d\sigma}{d\Omega}$$

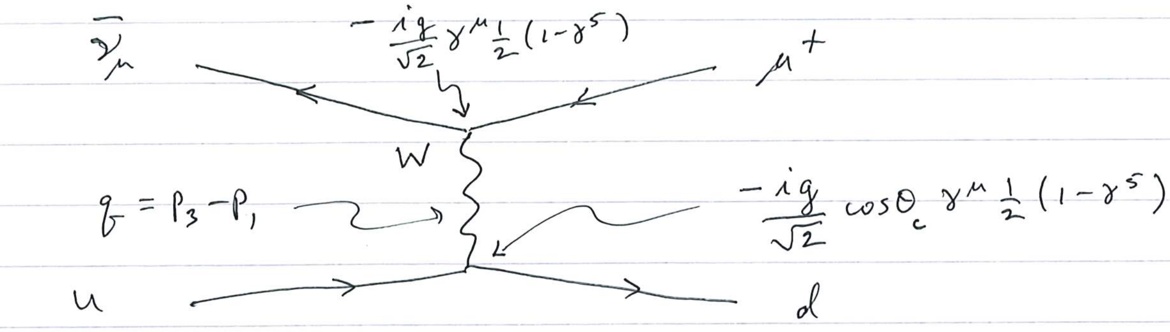
indep θ, ϕ

$$= \frac{G_F^2 \cos^2\theta_c\, \hat{s}}{\pi}, \quad (\hat{s} \gg m_\mu^2)$$

In lab frame, $\hat{s} = 2 m_d E_\nu$ (lab) (assuming d at rest)

$$\sigma_{lab} = \frac{2G_F^2\cos^2\theta_c\, m_d E_\nu}{\pi} \propto E_\nu$$

Now consider antineutrino reaction $\bar{\nu}_\mu u \to \mu^+ d$



$$\Rightarrow \mathcal{M} = \frac{-i\, G_F \cos\theta_c}{\sqrt{2}} \left[\bar{\nu}(p_1)\gamma_\mu(1-\gamma^5)\nu(p_3)\right]\left[\bar{u}(p_4)\gamma^\mu(1-\gamma^5)u(p_2)\right]$$

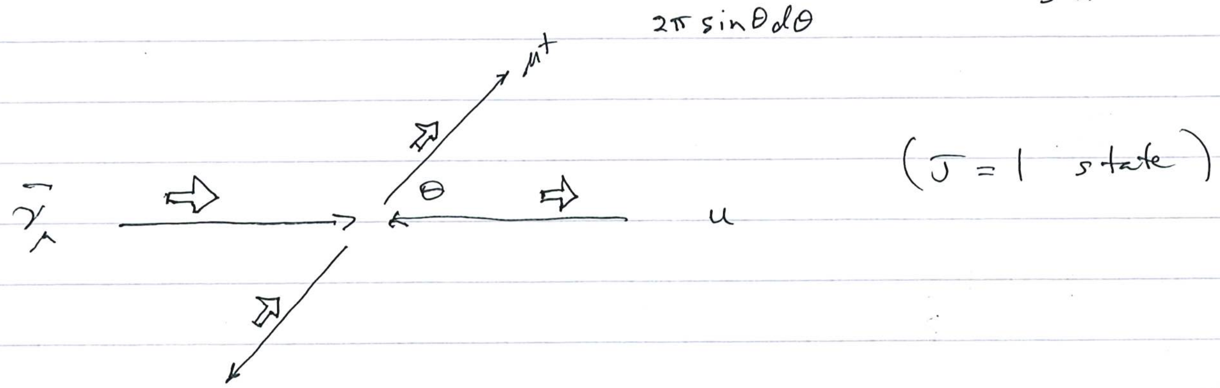
~ same steps as before

All momenta in cm $p \approx \frac{E_{cm}}{2}$

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 \cos^2\theta_c\, \hat{s}}{4\pi^2}\left(\frac{1+\cos\theta}{2}\right)^2 , \qquad \theta = \measuredangle(\bar{\nu}_\mu, \mu^+) \text{ in c.m.}$$

In contrast to $\nu_\mu d \to \mu^- u$, for $\bar{\nu}_\mu$ the μ^+ prefers to follow the direction of incoming $\bar{\nu}_\mu$.

$$\sigma_{tot}(\bar{\nu}_\mu u \to \mu^+ d) = \int \frac{d\sigma}{d\Omega} \underset{2\pi \sin\theta\, d\theta}{\underset{\uparrow}{d\Omega}} = \frac{G_F^2 \cos^2\theta_c\, \hat{s}}{3\pi}$$



$\Rightarrow$ Naively expect

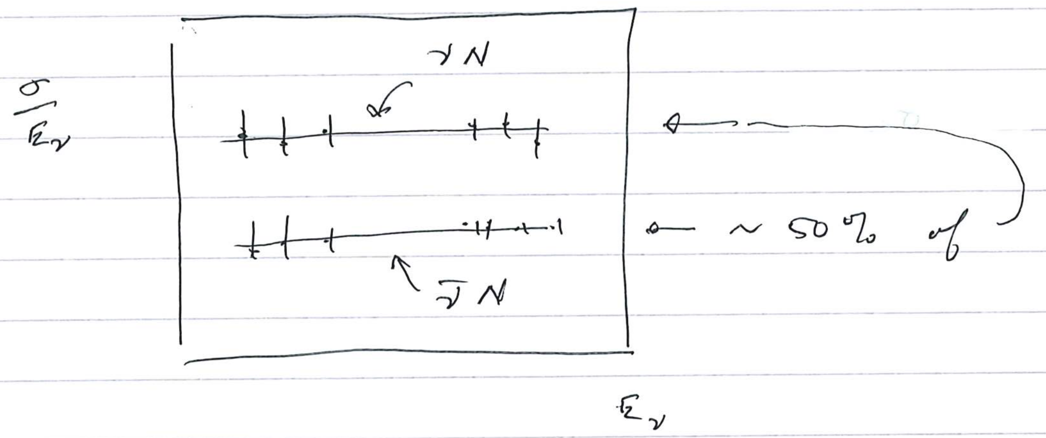
$$\frac{\sigma(\bar{\nu} p)}{\sigma(\nu p)} \approx \frac{\#\ u \text{ quarks in } p}{\#\ d \text{ quarks in } p} \times \frac{1}{3} = \frac{2}{3}$$

or for "isoscalar" target (equal # of n, p $\Rightarrow$ equal #s of u, d)

$$\frac{\sigma(\bar{\nu} N)}{\sigma(\nu N)} = \frac{1}{3}$$

and for both, $\sigma \propto \hat{s} \sim E_\nu^{lab}$

In fact find



$\Rightarrow$ evidence for <u>antiquarks</u> in nucleon

$\longrightarrow$ slides on ν scattering exp.

Kinematics for ν scattering

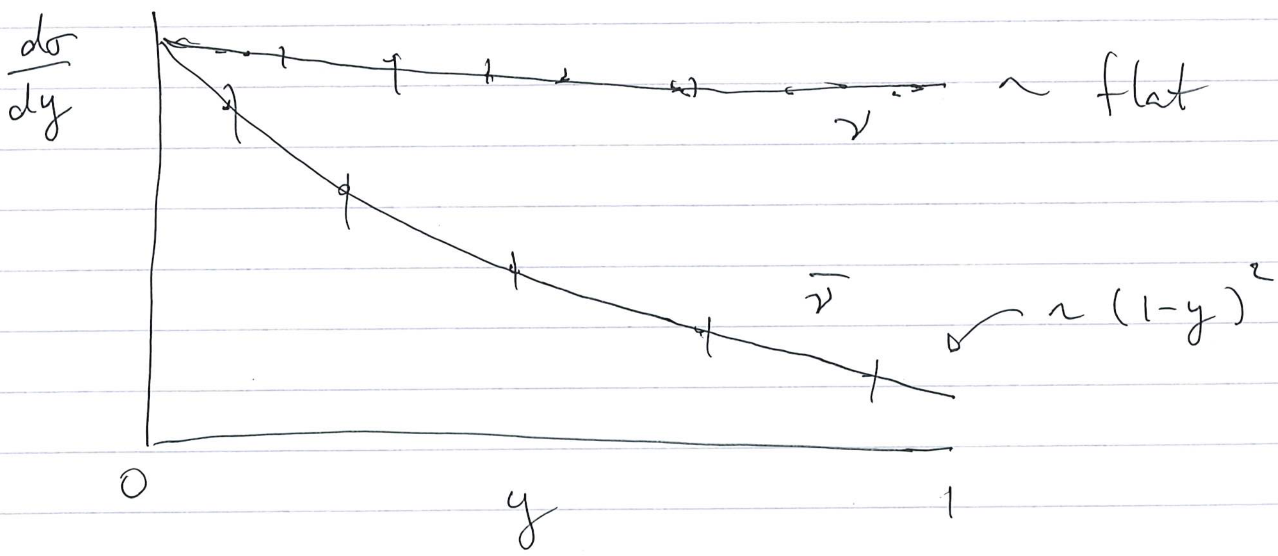
Let $y = 1 - \dfrac{E_\mu}{E_\nu}$ (inelasticity)

In c.m. & ultra-rel. limit,

$$1 - y = \frac{1 + \cos\theta}{2}$$

$\Rightarrow$

Reaction	$\dfrac{d\sigma}{d\cos\theta}$	$\dfrac{d\sigma}{dy}$
$\nu_\mu d$, $\bar{\nu}_\mu \bar{d}$	$\dfrac{G_F^2 s}{2\pi}$	$\dfrac{G_F^2 s}{\pi}$
$\bar{\nu}_\mu u$, $\nu_\mu \bar{u}$	$\dfrac{G_F^2 s}{2\pi}\left(\dfrac{1+\cos\theta}{2}\right)$	$\dfrac{G_F^2 s}{\pi}(1-y)^2$



Departs from naive expectation because nucleon contains some antiquark component

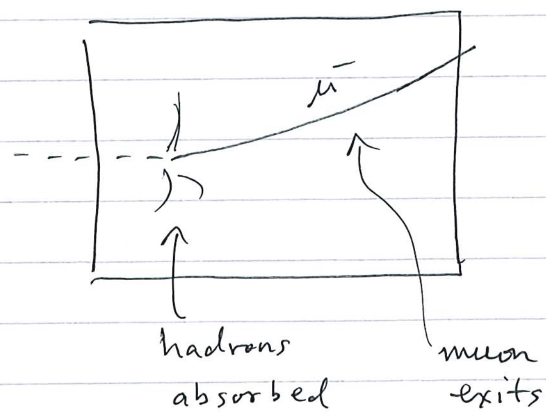
Neutral-current processes

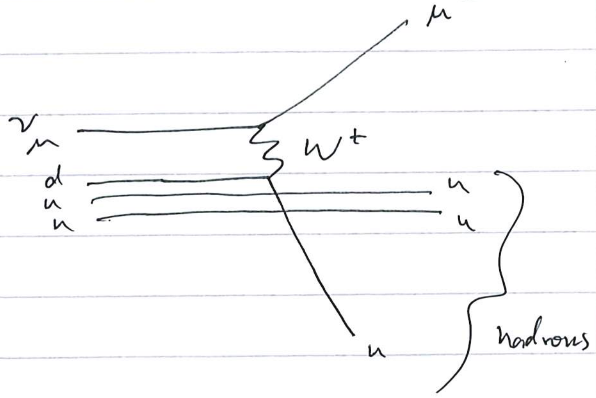
Up to now all weak processes mediated by $W^\pm$ ("charged current processes")

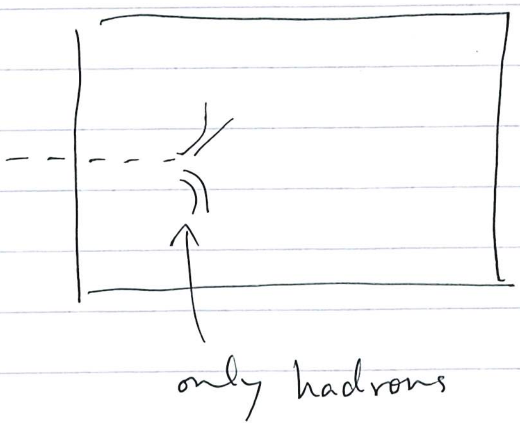
Electroweak SM (next chapter) predicted a neutral intermed. vector boson, the Z.

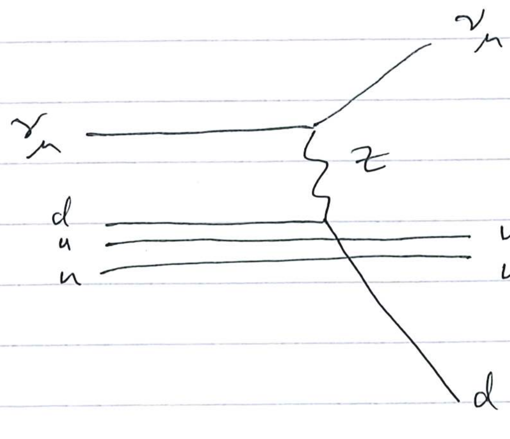
("neutral currents") discovered 1973.

→ slides









W, Z (on shell) discovered in $p\bar{p}$ collisions by UA1 (1983)

$M_W = 80.4$ GeV

$M_Z = 91.2$ GeV

Electroweak Standard Model

Prelim: local gauge sym. for electromag.

Dirac eq. for free ptcl.
$$(i\gamma_\mu \partial^\mu - m)\psi(x) = 0 \qquad ①$$

Suppose sol'n $\psi(x)$ transformed

$$\psi'(x) = e^{i\alpha}\psi(x) \qquad (\alpha = \text{const.})$$

$\psi'(x)$ also a sol'n to ①: Global U(1) symmetry

Now suppose const. $\alpha \rightarrow \alpha(x)$ ~ arb. func. of space-time

"local" transformation

$$\psi'(x) = e^{iq\alpha(x)}\psi(x)$$

a const. (will be charge)

sub. into ①

$$[i\gamma_\mu(\partial^\mu + iq\partial^\mu\alpha) - m]\psi = ?$$

extra term ⇒ ψ' not sol'n

But, recall Dirac eq. in EM potential,

$$(i\gamma_\mu\partial^\mu - m)\psi(x) = q\gamma_\mu A^\mu(x)\psi(x)$$

+ suppose $\psi'(x) = e^{iq\alpha(x)}\psi(x)$

$$A'^\mu(x) = A^\mu(x) - \partial^\mu\alpha(x)$$

Extra term now cancels:

$$[i\gamma_\mu(\partial^\mu + \cancel{iq\partial^\mu\alpha}) - m]\psi = q\gamma_\mu(A^\mu - \cancel{\partial^\mu\alpha})\psi$$

Recall (Sec. 3.1) that gauge trans

$$A^\mu \to A^\mu - \partial^\mu \alpha(x)$$

leaves $\vec{E}$, $\vec{B}$ unchanged.

Equivalent way of formulating trans

Define "covariant derivative" (not same "covariant" as Lorentz indices)

$$D^\mu = \partial^\mu + iqA^\mu$$

$$D^\mu \psi \to (D^\mu \psi)' = \left[\partial^\mu + iq(A^\mu - \partial^\mu \alpha)\right] e^{iq\alpha} \psi$$

$$= e^{iq\alpha}\partial^\mu \psi + iq(\partial^\mu \alpha) e^{iq\alpha}\psi + iqA^\mu e^{iq\alpha}\psi - iq(\partial^\mu \alpha) e^{iq\alpha}\psi$$

$$= e^{iq\alpha}(\partial^\mu + iqA^\mu)\psi$$

i.e. $D^\mu \psi \to (D^\mu \psi)' = e^{iq\alpha} D^\mu \psi$,

same as $\psi \to \psi' = \psi e^{iq\alpha}$

Dirac eq. w/ A^μ becomes

$$(i\gamma_\mu D^\mu - m)\psi = 0 \quad (2)$$

& since since $D^\mu \psi$ and ψ transform in same way, ψ' is a sol'n to (2)

$\Rightarrow$ local U(1) gauge symmetry.

i.e. if we insist theory have local $U(1)$ symmetry, only possible with A^μ.

$\sim$ "predicts" photon

Real history: E.M. seen to have local gauge sym. Guess that weak (+ strong) int. have similar symmetry

$\longrightarrow$ renormalisable theories

Yang-Mills gauge theories (1954)

Consider (again)

$$(i\gamma^\mu D_\mu - m)\psi = 0 \qquad \psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{pmatrix}$$

$\uparrow$ each comp. is Dirac spinor

& suppose more general unitary trans

$$\psi(x) \to \psi'(x) = U(x)\,\psi(x)$$

$$U(x) = \exp\left[i g \underbrace{\alpha^a(x)}_{} T^a \right]$$

$U(x)$: $N \times N$ unitary matrix

g: gauge coupling

$\alpha^a(x)$: arbitrary func. of x

T^a: $N \times N$ Hermitian matrices ("generators")

sum over repeated $a = 1, \dots, n$ $\leftarrow$ depends on N

$\alpha^a = \alpha_a$, $T^a = T_a$ so here don't distinguish upper, lower.

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{pmatrix},$$ each $\psi_i(x)$ is a Dirac spinor

$i =$ gauge index $= 1, \dots, N$

U acts only on gauge indices

Set of all possible transformations $U(x)$

$\equiv$ <u>gauge group</u> $\rightsquigarrow$ see app. A

It is a <u>Lie</u> group (elements labeled by continuous params.)

with Hermitian matrices $T^1, \dots, T^n$ $\rightsquigarrow$ generators

For a Lie group, commutator of generators is linear comb. of the T^a:

$$[T^a, T^b] = i f^{abc} T^c \equiv \text{Lie } \underline{\text{algebra}}$$

sum

structure constants (or f_{abc})

(Conventional) normalisation condition

$$\mathrm{Tr}(T^a T^b) = \underbrace{\tfrac{1}{2}}_{\equiv T_F} \delta^{ab}$$

If generators (+ hence group elements U) do not commute, group is non-Abelian

at least some $f^{abc} \neq 0$.

$\det U = +1$

Example $SU(2)$ = special unitary group in 2-dim.

= set of det = 1 unitary 2×2 matrices

3 generators $T^i = \frac{1}{2}\sigma_i$, $i = 1, 2, 3$

Pauli matrices

Lie algebra: $[T^i, T^j] = i\epsilon_{ijk} T^k$

For $SU(N)$ there are $n = N^2 - 1$ generators

since $\det = 1 \Rightarrow \mathrm{tr}\, T^i = 0$

Now define the covariant derivative as

$$D_\mu = \partial_\mu + i g T^a A^a_\mu(x)$$

one gauge field for each generator

Combined gauge field $A_\mu = A^a_\mu T^a$ ~ $N\times N$ matrix

Under gauge trans. $\psi'(x) = U(x)\psi(x)$,

$$A'_\mu = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

If $\alpha^a(x)$ small, $U \approx 1 + i g \alpha^a(x) T^a$, $U^{-1} \approx 1 - i g \alpha^a(x) T^a$

→ to ☆ p 17

Gauge trans becomes

$$A'^a_\mu = A^a_\mu - \partial_\mu \alpha^a + g f^{abc} \alpha^b A^c_\mu$$

$\Rightarrow D^\mu \psi$ transforms same as ψ $\Rightarrow$ theory invariant under combined trans. of ψ and gauge fields A^a_μ .

Proof: $\psi' = U\psi$

$$A'_\mu = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$$

$$D_\mu = \partial_\mu + i g A_\mu$$

⚠ some books $i \to -i$

$$D'_\mu \psi' = (\partial_\mu + i g A'_\mu)(U\psi)$$

$$= (\partial_\mu U)\psi + U\partial_\mu \psi + i g A'_\mu U \psi$$

$$= \cancel{(\partial_\mu U)\psi} + U\partial_\mu \psi + i g\left[U A_\mu U^{-1} + \cancel{\frac{i}{g}(\partial_\mu U)U^{-1}}\right]U\psi$$

$$= U(\partial_\mu + i g A_\mu)\psi \quad \checkmark$$

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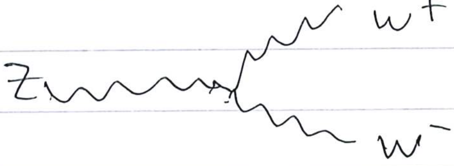
p16 → ☆

QED: $A'_\mu = A_\mu - \partial_\mu \alpha$

Yang-Mills: $A'^a_\mu = A^a_\mu - \partial_\mu \alpha^a + \underbrace{g f^{abc} \alpha^b A^c_\mu}$

↳ present for non-Abelian gauge group, leads to gauge boson self interaction

e.g. 

Will now use Yang-Mills with:

$SU(2)_L \times U(1) \longrightarrow$ electroweak SM

$W^\pm, Z, \gamma$

couplings g, g' $(\to g, e)$

$SU(3) \longrightarrow$ QCD

gluons (8)

coupling g_s