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PH4442 Adv. Ptcl. Physics <u>Week 9</u> 1

<u>Recap</u> (→ slides) : We now have electroweak SM based on $SU(2)_L \times U(1)$, but gauge symmetry only works for massless particles (apparently).

<u>This week</u>: show how to include masses while preserving gauge sym. via <u>Higgs mechanism</u>

For this we first need to introduce some elements of <u>Quantum Field Theory</u>

# Intro to QFT

Lagrangian formalism : system of ptcls

w/ generalised coord. $q_1, \ldots, q_N \equiv \vec{q}$

& " velocities $\dot{q}_1, \ldots, \dot{q}_N \equiv \dot{\vec{q}}$

evolves from $t_1$ to $t_2$ such that the <u>action</u>

$$S = \int_{t_1}^{t_2} L(\vec{q}, \dot{\vec{q}})\, dt \quad \text{is minimum}$$

(or max, saddle)

Lagrangian $L = T - V$

kin. E  pot. E

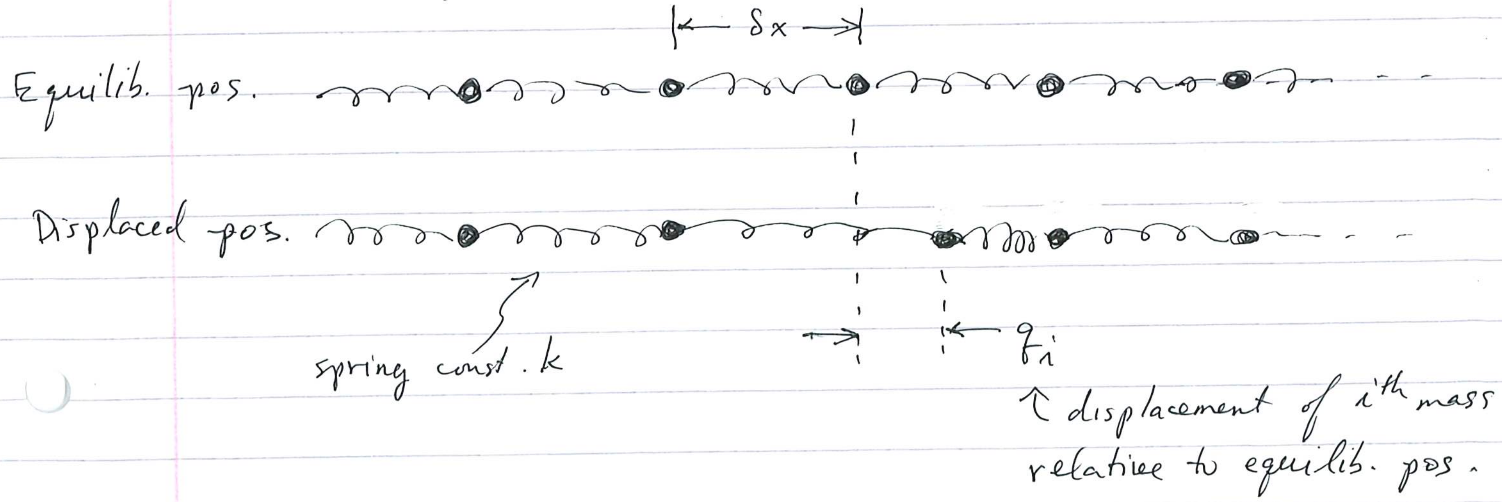
(Hamilton's)  (stationary)

Principle of least action : $\delta S = 0$

$\Rightarrow$ $\vec{q}, \dot{\vec{q}}$ follow Euler-Lagrange eq.

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = 0 , \quad i = 1, \ldots, N$$

Consider system of masses $\delta m$, spacing $\delta x$



$$L = T - V$$

$$= \sum_{i=1}^{N-1} \left[ \frac{1}{2} \delta m \, \dot{q}_i^2 - \frac{1}{2} k (q_{i+1} - q_i)^2 \right]$$

$$= \sum_{i=1}^{N-1} \frac{\delta x}{2} \left[ \left(\frac{\delta m}{\delta x}\right) \dot{q}_i^2 - k \, \delta x \left( \frac{q_{i+1} - q_i}{\delta x} \right)^2 \right]$$

$$= \sum_{i=1}^{N-1} \delta x \, L_i$$

↑ Lagrangian density (per unit length)

In limit $N \to \infty$, $\delta m$, $\delta x \to 0$

$\frac{\delta m}{\delta x} \to$ (linear) density $\rho$

$k \, \delta x \to \tau$ tension

$q_i \to \phi(x)$  $x$ = equilib. pos. of mass $i$

↳ field

$$L \to \int dx \, \underbrace{\frac{1}{2} \left[ \rho \, \dot{\phi}^2(x) - \tau \left( \frac{\partial \phi}{\partial x} \right)^2 \right]}_{\mathcal{L}(x) \,=\, \text{Lagrangian density}}$$

Euler-Lagrange eqs. for field become

$$\frac{\partial \mathcal{L}}{\partial \phi(x)} - \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\phi}(x)} \right) = 0$$

$$\longrightarrow \tau \left( \frac{\partial^2 \phi}{\partial x^2} \right) - \rho \left( \frac{\partial^2 \phi}{\partial t^2} \right) = 0$$

= wave eq. w/ speed $v = \sqrt{\frac{\tau}{\rho}}$

Now consider 3D-space & time

Action becomes

allow $\mathcal{L}$ to depend on all derivs.

$$S = \int_{t_1}^{t_2} dt \int d^3x \; \mathcal{L}(\phi, \partial_\mu \phi)$$

Euler - Lagrange eqs. become

$$\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$$

Classical $\rightarrow$ Quantum FT (brief)

In quantum theory, the masses act like quantum oscillators $E_n = (n + \frac{1}{2}) \underbrace{\sqrt{\frac{k}{m}}}_{\omega}$ $(\hbar = 1)$

Wave-like excitation carries energy / momentum

$\rightarrow$ <u>particle</u> w/ $E, \vec{p}$

( in 3-D ~ lattice of atoms )

Particles of different types = excitations of different fields.

Interactions between fields

$\rightarrow$ ptcls created / destroyed

A <u>state</u> w/ n ptcls w/ momenta

$\vec{p}_1, \ldots, \vec{p}_n$ = vector in Fock space

= direct sum of Hilbert spaces for all n

In QFT, fields (wave func.) $\phi, \psi, A^\mu, \ldots$

become <u>operators</u>

Fourier transform of field operators

→ creation / annihilation operators,

operate on Fock space vector,

creating/destroying ptcls of different momenta

Back to string of masses

position $q_i$

momentum $p_i = \frac{\partial L}{\partial \dot{q}_i}$

QM → operators $\hat{q}_i, \hat{p}_i$

commutator $[\hat{q}_i, \hat{p}_j] = i\,\delta_{ij}$

QFT: $\hat{q}_i \to \phi(\vec{x}, t)$ = field op.

$\hat{p}_i \to \pi(\vec{x}, t) = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$ = momentum density op.

Eq. of motion for scalar field

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m \phi^2 = \mathcal{L}(\phi, \partial_\mu \phi)$$

Euler - Lag. $\quad \frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = 0$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi \Rightarrow \partial_\mu \left( \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \partial_\mu \partial^\mu \phi$$

$\Rightarrow$ Euler - Lag : $\quad -m^2 \phi^2 - \partial_\mu \partial^\mu \phi = 0$

i.e. $\quad (\partial_\mu \partial^\mu + m^2)\phi = 0 \quad$ ( Klein - Gordon eq. )

<u>Complex scalar field</u> $\quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}}$

Take $\phi, \phi^*$ as the 2 indep. fields

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi)^* - m \phi \phi^*$$

Euler - Lagrange give

$$\left.\begin{aligned} (\partial_\mu \partial^\mu + m^2)\phi &= 0 \\ (\partial_\mu \partial^\mu + m^2)\phi^* &= 0 \end{aligned}\right\} \text{ both } \phi, \phi^* \text{ obey K - G.}$$

Field operators & momentum density satisfy equal-time commutation relations

bosons: $\left[ \phi(\vec{x},t), \pi(\vec{x}',t) \right] = i \delta^3(\vec{x}-\vec{x}')$

(Dirac) fermions: $\left\{ \psi_\alpha(\vec{x},t), \psi^\dagger_\beta(\vec{x}',t) \right\} = \delta_{\alpha\beta} \, \delta^3(\vec{x}-\vec{x}')$

spinor components

↳ gives antisymmetric states

→ Pauli exclusion principle

Some important Lagrangians ← these define the theory

Scalar field $\phi(x)$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2$$

↳ does not affect eqs. of motion, chosen to give correct energy density

Euler - Lagrange eqs.

$$\rightarrow \quad (\partial_\mu \partial^\mu + m^2)\,\phi = 0$$

calc. on next page

Klein - Gordon eq. for ptcl. of mass m.

# QED

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma_\mu D^\mu - m)\psi$$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$ field strength tensor

electron $Q=-1$

$D^\mu = \partial^\mu - ie A^\mu$ covariant deriv.

Treat $\psi$, $\bar{\psi}$, $A^\mu$ as indep. fields

Euler-Lagrange $\rightarrow$ $(i\gamma_\mu \partial^\mu - m)\psi = 0$

and $\partial_\nu F^{\nu\mu} = \underbrace{-e\bar{\psi}\gamma^\mu\psi}_{j^\mu}$

## Massive vector field (Proca Lagrangian)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M^2 B_\mu B^\mu$$

$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Euler-Lagrange

$$\rightarrow (\partial_\nu \partial^\nu + M^2) B^\mu - \partial^\mu(\partial_\nu B^\nu) = 0$$

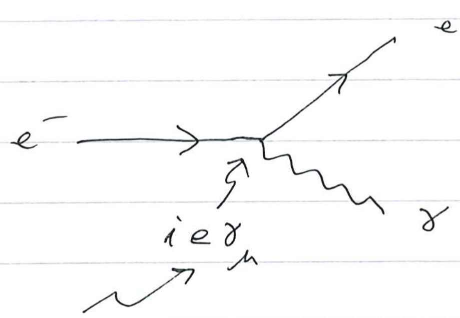
Lagrangian $\rightarrow$ properties of theory

$\mathcal{L}$ not "derived", it defines the theory

If $\mathcal{L}$ contains term w/ product of fields or its derivatives $\rightarrow$ $\exists$ Feynman diag. vertex w/ these fields

e.g. in QED $\mathcal{L}_{int} = e \bar{\psi} \gamma_\mu \psi A^\mu$

$\Rightarrow$




<u>Vertex factor</u> $= i \times$ coeff of fields $\times$ symmetry factor

$N!$

$\#$ of identical ptcls.

<u>Mass terms</u>: quadratic in field.

scalar: $-\frac{1}{2} m^2 \phi^2$

fermion: $-m \bar{\psi} \psi$

vector: $\frac{1}{2} M^2 B_\mu B^\mu$

action

$[S]$ dimensionless: $S = \int d^4x \, \mathcal{L} \Rightarrow [\mathcal{L}] = x^{-4} = E^4$

$\Rightarrow [\phi], [B^\mu] \sim E^1$ ("dimension 1")

$[\psi] \sim E^{3/2}$ ("dimension 3/2")

Symmetry of theory = transformation that leaves action $S$ invariant (+ in most important cases leaves Lagrangian invariant, e.g., Lorentz, gauge sym.)

[cf. eq. of motion covariant: retain same form + sol'ns transform by given rule.]

$\Rightarrow$ easier to study symmetries w/ Lagrangian

E.g. can make $\mathcal{L}$ Lorentz invariant by constructing it out of bilinear covariants that contract to give Lorentz scalar $\mathcal{L}$.

Course on QFT: Noether's theorem

continuous symmetry $\longleftrightarrow$ conserved current

Higgs mechanism for U(1) local gauge sym.

(Toy model that illustrates key points.)

Take: $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$F^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

$\mathcal{L}$ is invariant under local U(1) gauge trans.

$$B^\mu \to B'^\mu = B^\mu - \partial^\mu \alpha(x) \leftarrow \text{easy to check}$$

But, if $\mathcal{L}$ gets a mass term $\frac{1}{2} M^2 B_\mu B^\mu$,

$$\frac{1}{2} M^2 B'_\mu B'^\mu = \frac{1}{2} M^2 B_\mu B^\mu \underbrace{- M^2 \partial_\mu \alpha B^\mu + \frac{1}{2} (\partial_\mu \alpha)(\partial^\mu \alpha)}_{\text{extra terms}}$$

$\rightarrow$ not invariant.

"Clever trick"

Higgs mech: include complex scalar field

w/ nonzero "vacuum expectation value"

→ mass for $B^\mu$

+ massive scalar particle

+ gauge invariance preserved!

[For SM do similar, get masses for $W^\pm$, $Z$, fermions

& Higgs boson (later)]

Introduce to $\mathcal{L}$ a field $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$ complex scalar

In kinetic term $(\partial_\mu \phi)(\partial^\mu \phi)^*$,

① replace $\partial^\mu \to D^\mu = \partial^\mu + i g \alpha(x) B^\mu$ gauge field

② include potential $V$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* (D^\mu \phi) - V(\phi)$$

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2$$

$\lambda > 0$ → $V$ has finite minimum ("vacuum stability")

If $\mu^2 > 0$, $\mu \sim$ mass of $\phi$

$\mathcal{L}$ has $U(1)$ gauge symmetry

$\phi(x) \to \phi'(x) = e^{i g \alpha(x)} \phi(x)$

$B_\mu \to B_\mu' - \partial_\mu \alpha(x)$

Instead, take $\underline{\mu^2 < 0}$ ($\mu^2 \equiv$ the param., not something squared)

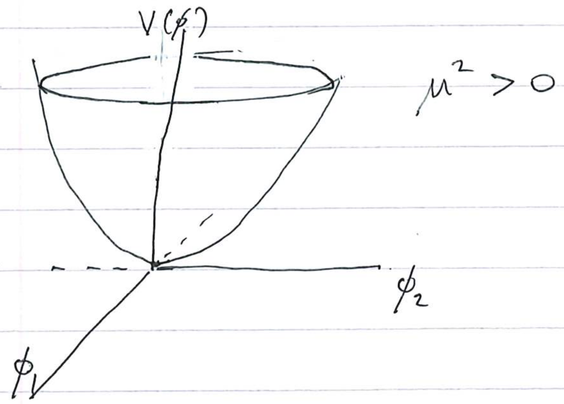
$\rightarrow$ "Mexican hat" potential,

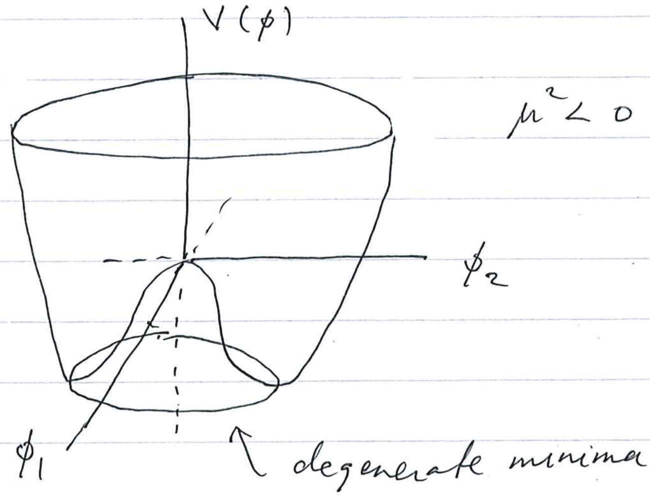
minimum at $\phi_1^2 + \phi_2^2 = \dfrac{-\mu^2}{\lambda} \equiv v^2$

In QFT, VEV $\langle 0 | \phi(x) | 0 \rangle = \dfrac{v}{\sqrt{2}}$  $v$ = vacuum expectation value (VEV)

$\uparrow$ vacuum (lowest E) state

Here can think of $\phi$ as a (c-) number.




$\uparrow$ degenerate minima at $\phi_1^2 + \phi_2^2 = v^2$

For $\mu^2 < 0$,

- $\infty$ # of degenerate minima
- only $\underline{1}$ realised

$\rightarrow$ "spontaneously broken symmetry" (SSB)

i.e. $\mathcal{L}$ invariant under change that leaves $\phi_1^2 + \phi_2^2$ const. but <u>ground state</u> violates the symmetry.

Pick ground state e.g. $(\phi_1, \phi_2) = (v, 0)$

Consider small perturbation of field near vacuum state

$$\phi(x) = \frac{1}{\sqrt{2}}\Big(v + \underbrace{\eta(x) + i\,\xi(x)}_{\text{small}}\Big)$$

i.e. $\phi_1 = v + \eta(x)$ , $\phi_2 = \xi(x)$

$$V(\phi) = \mu^2\overset{-\lambda v^2}{} \phi^*\phi + \lambda(\phi^*\phi)^2$$

$$= \underbrace{-\frac{1}{4}\lambda v^4}_{\text{const.} \rightarrow \text{drop}} + \underbrace{\lambda v^2 \eta^2}_{= -\mu^2\eta^2 \text{ (2 powers of } \eta)} + \lambda v \eta^3 + \frac{1}{4}\lambda\eta^4 + \frac{1}{4}\lambda\xi^4 + \lambda v \eta \xi^2 + \frac{1}{2}\lambda\eta^2\xi^2$$

interaction terms (3,4 powers of $\eta, \xi$)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \mu^2\eta^2 + \text{const.} + \text{int. terms}$$

$$+ \frac{1}{2}\underbrace{\left[(\partial_\mu - igB_\mu)(v+\eta-i\xi)\right]}_{(D_\mu\phi)^*}\underbrace{\left[(\partial^\mu + igB^\mu)(v+\eta+i\xi)\right]}_{D^\mu\phi}$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}g^2v^2 B_\mu B^\mu + gvB_\mu(\partial^\mu\xi)$$

mass term! $m_B = gv$

$$\underbrace{+\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) + \underbrace{\mu^2\eta^2}_{= -\frac{1}{2}m_\eta^2\eta^2}} + \frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + \text{interaction terms}$$

$\hookrightarrow$ $\eta$ is a real scalar, $m_\eta = \sqrt{-2\mu^2}$

Weird stuff:
- $\xi$ = massless scalar "Goldstone boson"
- $gvB_\mu(\partial^\mu\xi)$ ??

$\rightarrow$ $\xi$ not physical field

Trick:

$$\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi) + \frac{1}{2}g^2 v^2 B_\mu B^\mu + g v B_\mu (\partial^\mu \xi)$$

$$= \frac{1}{2}g^2 v^2 \left(B_\mu + \frac{1}{gv}\partial_\mu \xi\right)\left(B^\mu + \frac{1}{gv}\partial^\mu \xi\right) \qquad (1)$$

So carry out gauge transformation

$$\left.\begin{aligned} B^\mu \to B'^\mu &= B^\mu - \partial^\mu \alpha(x) = B^\mu + \partial^\mu\left(\frac{\xi}{gv}\right) \\ \phi \to \phi' &= e^{ig\alpha(x)}\phi = e^{-i\xi/v}\phi \end{aligned}\right\} \quad \text{i.e. } \alpha(x) = \frac{-\xi}{gv}$$

$\equiv$ "unitary gauge" $\to$ same physics

Since $\eta, \xi$ are small perturb. about vacuum state,

$$\phi = \frac{1}{\sqrt{2}}(v + \eta + i\xi) \approx \frac{1}{\sqrt{2}}(v+\eta)\, e^{i\xi/v}$$

$\Rightarrow$ gauge trans is $\quad \phi' = e^{-i\xi/v}\frac{1}{\sqrt{2}}(v+\eta)\, e^{i\xi/v}$

$$= \frac{1}{\sqrt{2}}(v+\eta)$$

$\xi$ disappears! $\quad (1) \to \frac{1}{2}g^2 v^2 B'_\mu B'^\mu$

$\phi$ now a <u>real</u> scalar

Rename <u>$\eta \to h$ = Higgs boson</u> = physical field in unitary gauge

Lagrangian becomes (in unitary gauge, but equivalent to any other gauge)

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_B^2 B_\mu B^\mu \quad \rightsquigarrow \quad m_B = gv$$

$$+ \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 \quad \rightsquigarrow \quad m_h = \sqrt{-2\mu^2} = \sqrt{2\lambda}\, v$$

$$+ \underbrace{g^2 v B_\mu B^\mu h + \frac{1}{2}g^2 B_\mu B^\mu h^2}_{\eta, B \text{ interact.}} \underbrace{- \lambda v h^3 - \frac{1}{4}\lambda h^4}_{\text{Higgs self-interact.}}$$

Did we lose a degree of freedom?

Before including $\phi$, $B^\mu$ massless $\Rightarrow$ 2 d.o.f.

(no longitudinal polarisation)

After including $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

2 d.o.f

$$\phi_1, \phi_2 \rightarrow \overset{\eta}{h} + \xi,$$

$h$ becomes real scalar field

$\xi$ "eaten" by $B^\mu$ to make its long. polar. state.

# Higgs mech. in EW SM

We wrote down SM w/o mass terms to keep from violating $SU(2) \times U(1)$ gauge sym.

$\longrightarrow$ massless $W^\pm, Z, \gamma$

but we <u>know</u> $\underbrace{W^\pm \text{ \& } Z}$ have non zero mass

$\hookrightarrow$ need <u>3</u> long. polar. states

$\Rightarrow$ use $SU(2)$ <u>doublet</u> of two complex scalars (4 fields)

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_H = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \lambda > 0, \quad \mu^2 < 0$$

$$D_\mu = \partial_\mu + i g \vec{T} \cdot \vec{W}_\mu + \frac{i g'}{2} Y B_\mu$$

weak isospin

$\phi$ is $SU(2)$ doublet : $\phi^+$ has $t_3 = 1/2$

$\phi^0$ has $t_3 = -1/2$

Hypercharge of $\phi$, $Y = 2(Q - t_3)$

$\underline{= 1}$ (for both)

We do <u>not</u> want mass for photon, which will interact w/ charged $\phi^+$

$\Rightarrow$ take VEV

$$\langle 0 | \phi | 0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

only $\phi^0$ gets nonzero VEV

Expand about minimum,

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \eta + i\phi_4 \end{pmatrix}$$

$\sim$ same as before w/ $U(1)$

$\phi_1, \phi_2, \phi_4$ are "eaten" by $W^\pm, Z$

$\rightarrow$ longitudinal polarisation states.

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \quad \text{in unitary gauge}$$

$\eta$

Need to work out

$$D_\mu \phi = \left( \partial_\mu + ig \underbrace{\vec{T}}_{\frac{1}{2}\vec{\tau}} \cdot \vec{W} + \frac{ig'}{2} \underbrace{Y}_{1} \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

+ put into $\mathcal{L}_H$

$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{8} g^2 (W_{1\mu} + i W_{2\mu})(W_1^\mu - i W_2^\mu)(v+h)^2 \quad \text{①}$$

$$+ \frac{1}{8}(g W_{\mu 3} - g' B_\mu)(g W_3^\mu - g' B^\mu)(v+h)^2 \quad \text{②}$$

$$- \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 \quad \text{from } V(\phi) \quad \text{③}$$

Recall $W^{(\pm)\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu)$

$$\Rightarrow (W_{1\mu} + i W_{2\mu})(W_1^\mu - i W_2^\mu) = |W_\mu^+|^2 + |M_\mu^-|^2$$

where $|W_\mu^\pm|^2 = (W_\mu^\pm)(W^{\pm,\mu})^*$

$$\Rightarrow \frac{1}{8} g^2 (W_{1,\mu} + i W_{2,\mu})(W_1^\mu - i W_2^\mu) v^2 = \frac{1}{2} M_W^2 \left(|W_\mu^+|^2 + |W_\mu^-|^2\right)$$

↑ mass term for $W^\pm$

$$\Rightarrow M_W = \frac{gv}{2}$$

Also, $A^\mu = \cos\theta_W B^\mu + \sin\theta_W W_3^\mu = (g B^\mu + g' W_3^\mu)/\sqrt{g^2 + g'^2}$

$Z^\mu = -\sin\theta_W B^\mu + \cos\theta_W W_3^\mu = (-g' B^\mu + g W_3^\mu)/\sqrt{g^2+g'^2}$

$$\Rightarrow \frac{1}{8}(g W_{3\mu} - g' B_\mu)(g W_3^\mu - g' B^\mu) v^2 = \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$

with $M_Z = \frac{1}{2} v \sqrt{g^2 + g'^2} = \dfrac{gv}{2\cos\theta_W}$

$$\Rightarrow \boxed{\frac{M_W}{M_Z} = \cos\theta_W} = \sqrt{1 - \frac{e^2}{g^2}}$$

$\Rightarrow$ predicts relation between $M_W$, $M_Z$, $e$, $g$

$\hookrightarrow M_W = M_Z \sqrt{1 - \frac{e^2}{g^2}}$

But $G_F = \frac{\sqrt{2}}{8} \frac{g^2}{M_W^2}$ much more accurately

known than $g$ $\Rightarrow$ express $M_W = M_W(M_Z, \alpha, G_F)$

$\Rightarrow M_{W,pred} = 80.3545 \pm 0.0057$ GeV

agrees* w/ data ($\rightarrow$ plot)      * except CDF

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Multiply out $-\frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4$

$$\Rightarrow -\frac{1}{2}(\underbrace{\mu^2}_{=-\lambda v^2} + 3\lambda v^2)h^2 = -\frac{1}{2}m_H^2 h^2$$

mass term for Higgs

$\Rightarrow m_H = \sqrt{2\lambda}\, v$

Note $v = \frac{2M_W}{g} = 246$ GeV <u>known</u> before discovery of Higgs

But $\lambda$ not known $\rightarrow m_H$ not known

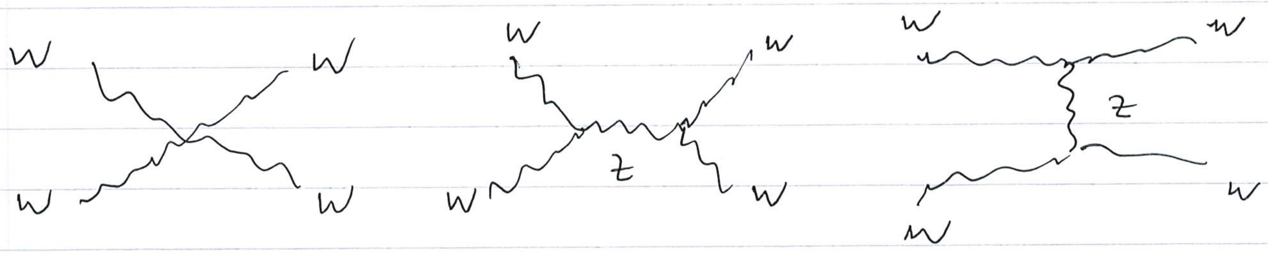
until 2012 Higgs discovery: $m_H = 125$ GeV

$\rightarrow \lambda = 0.129$

But even before "discovery" of Higgs, theory ~accepted because:
- $\rightarrow$ renormalisable theory
- $\rightarrow M_W$
- $\rightarrow$ unitarity limit "solved" (next slide)

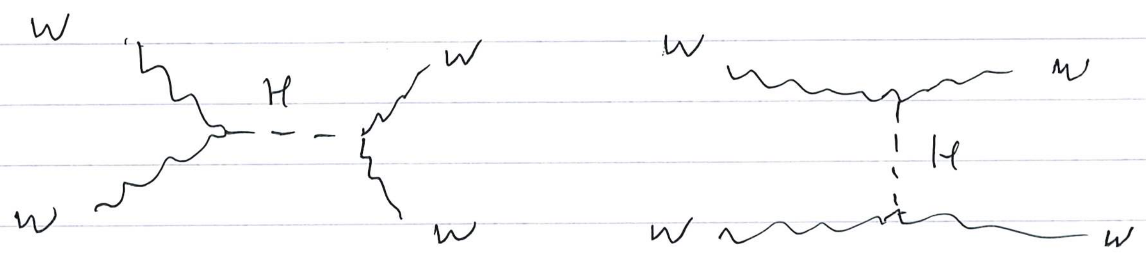
$W_L W_L$ scattering (brief)

w/o Higgs, $\sigma(W_L W_L \to W_L W_L) \propto E_{cm}^2$




$\sigma$ exceeds unitarity bound @ $\sim 1$ TeV

With Higgs, extra diagrams interfere destructively




cancelling $E_{cm}^2$ dependence,

avoids violating unitarity bound!

$\longrightarrow$ slide

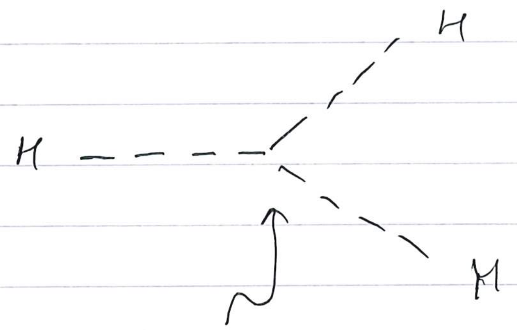
# Higgs self coupling

Multiplying out $-\frac{\lambda}{4}(v+h)^4$ in $\mathcal{L}_H$

$$\rightarrow \quad -\lambda v h^3 - \frac{\lambda}{4} h^4$$

a) triple Higgs $\qquad$ b) quartic Higgs

a)

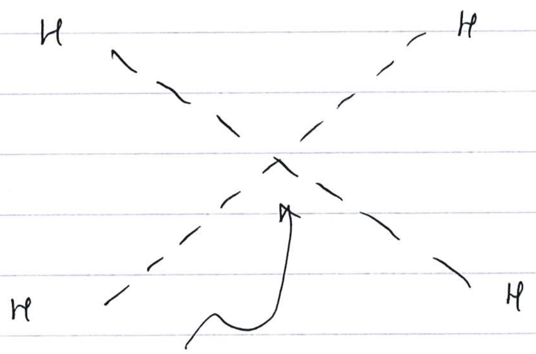



$N = 3$ identical ptcls

$$\Rightarrow \text{vertex factor} = i(-\lambda v)\,3! = -6 i \lambda v$$

$$= -\frac{3 i m_H^2}{v} = -\frac{3 i g m_H^2}{2 M_W}$$

b)




$N = 4$ identical ptcls

$$\Rightarrow \text{vertex factor} = i\left(-\frac{\lambda}{4}\right) 4! = -6 i \lambda$$

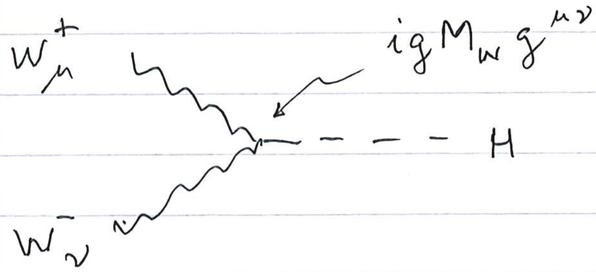
$$= -\frac{3 i m_H^2}{v^2} = -\frac{3 i g^2 m_H^2}{4 M_W^2}$$

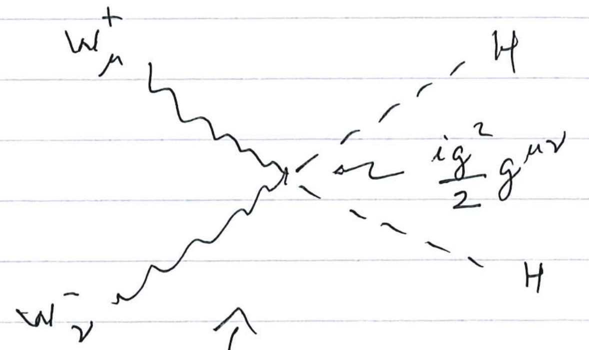
# Higgs couplings

a) to bosons. Multiply out

terms ① + ② from $\mathcal{L}_H$ on p 19

$$\rightarrow \underbrace{\tfrac{1}{2} g^2 v}_{g M_W} W_\mu^{(-)} W^{(+)\mu} h + \tfrac{1}{4} g^2 W_\mu^{(-)} W^{(+)\mu} h^2$$

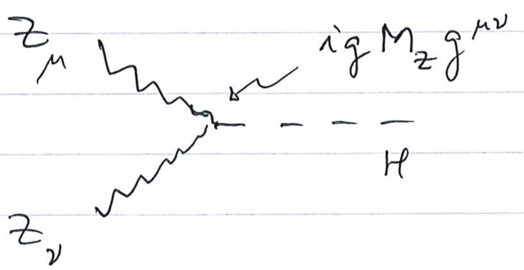


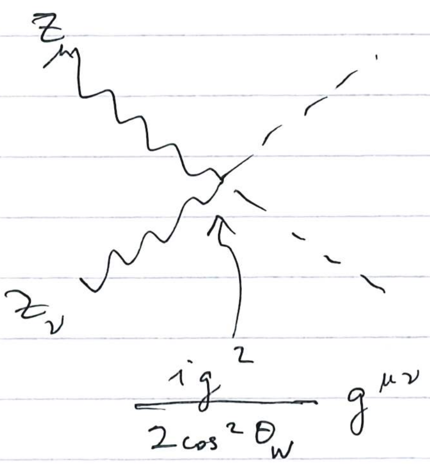




$N=2$ identical ptcls

$\Rightarrow$ sym. factor $N! = 2!$

~ same for Z but $g \rightarrow \frac{g}{\cos\theta_W}$ (or $M_W \rightarrow M_Z$)




$\Rightarrow$ both $WWh$ + $ZZh$, coupling $\propto M_{W/Z}$