PH4442 Advanced Particle Physics 2025/26
Lecture Week 10

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- Further properties of the Higgs boson
- Quantum Chromodynamics

Plan for this week

Last week we saw the Higgs mechanism and how it gives masses to the W and Z,

Finish up Higgs couplings (gauge bosons and fermions)

Predict Higgs decay rates, compare to measurements.

Our next topic is the theory of strong interactions, i.e.,

Quantum Chromodynamics:

The need for colour (quick review, see PH3520 notes)
QCD as an SU(3) Yang-Mills gauge theory
Elementary QCD processes, application to $e^+e^- \rightarrow$ hadrons
Confinement, hadronisation, measuring α_s,...

Higgs mechanism recap

Need to include particle masses without breaking gauge symmetry.

Lagrangian gets doublet of complex scalars with Higgs potential:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

$$\mathcal{L}_{\mathrm{H}} = (D_\mu)^\dagger (D^\mu) - V(\phi)$$

$$D_\mu = \partial_\mu + ig\boldsymbol{T} \cdot \boldsymbol{W}_\mu + i\frac{g'}{2} Y B_\mu$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2 \,, \quad \lambda > 0, \; \mu^2 < 0$$

To keep photon massless, only φ^0 gets nonzero VEV:

$$\langle 0|\phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Higgs mechanism recap (2)

Expand about vacuum state: $$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ v + \eta + i\phi_4 \end{pmatrix}$$

Put into Lagrangian and transform to unitary gauge:

$$\begin{aligned} \mathcal{L}_{\mathrm{H}} &= \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{\mu^2}{2}(v+h)^2 - \frac{\lambda}{4}(v+h)^4 \\ &+ \frac{1}{8} g^2 (W_{1\mu} + iW_{2\mu})(W_1^\mu - iW_2^\mu)(v+h)^2 \\ &+ \frac{1}{8}(gW_{3\mu} - g' B_\mu)(gW_3^\mu - g' B^\mu)(v+h)^2 \end{aligned}$$

Rewrite in terms of W^+, W^-, Z, multiply out $(v+h)^2$ and $(v+h)^4$ terms,

φ_1, φ_2, φ_4 “eaten”, W^+, W^-, Z acquire mass $\longrightarrow M_{\mathrm{W}} = \dfrac{gv}{2} = M_{\mathrm{Z}} \cos\theta_{\mathrm{W}}$

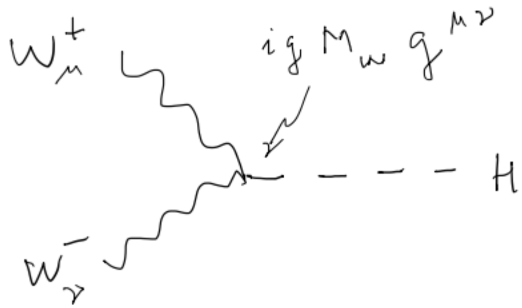
$\eta \longrightarrow h =$ real scalar (Higgs) boson

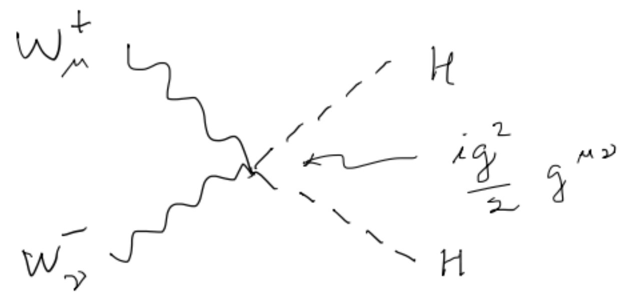
Higgs couplings to gauge bosons

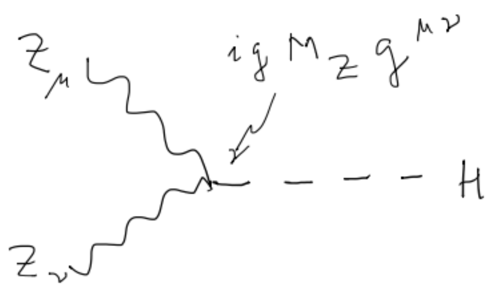
Multiplying out terms with $(v+h)^2$ and $(v+h)^4$ in Lagrangian and expressing W_1, W_2, W_3, B in terms of W^+, W^-, A, Z gives interactions between Higgs and gauge bosons, e.g.,

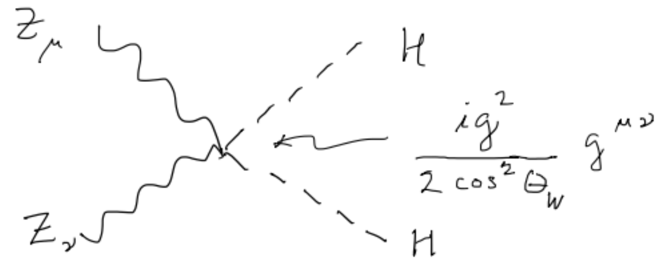
$$\tfrac{1}{2}g^2 v W_\mu^{(-)} W^{(+)\mu} h + \tfrac{1}{4}g^2 W_\mu^{(-)} W^{(+)\mu} h^2$$

and analogous for Z, but no direct coupling to photon.









Fermion mass term breaks SM gauge symmetry

Although we can have massive electrons in (gauge invariant) QED, we cannot just add fermion mass terms to the SM Lagrangian.

Consider $\psi = \psi_L + \psi_R$ where

$$\psi_L = P_L \psi = \frac{1}{2}(1 - \gamma^5)\psi$$

$$\psi_R = P_R \psi = \frac{1}{2}(1 + \gamma^5)\psi$$

Recall $\gamma^0 \gamma^5 = -\gamma^5 \gamma^0$ so $\gamma^0 P_L = P_R \gamma^0$ and $\gamma^0 P_R = P_L \gamma^0$

Also $(\gamma^5)^\dagger = \gamma^5$ so $P_L = P_L^\dagger$ and $P_R = P_R^\dagger$

Therefore $\overline{\psi}_L = (P_L \psi)^\dagger \gamma^0 = \psi^\dagger \gamma^0 P_R = \overline{\psi} P_R$ and $\overline{\psi}_R = \overline{\psi} P_L$

For mass term $-m\overline{\psi}\psi$ with $\psi_L + \psi_R$, $\overline{\psi}_L \psi_L = \overline{\psi} P_R P_L \psi = 0$, $\overline{\psi}_R \psi_R = 0$

The only terms that survive are $-m\overline{\psi}\psi = -m(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L)$

But SU(2) transforms ψ_L and ψ_R differently, so this term breaks the gauge symmetry.

Higgs mechanism for leptons

We can add a gauge invariant term to the Lagrangian involving the complex scalar doublet ϕ that results in masses for leptons.

The SU(2) transformation $U_T = \exp[ig\boldsymbol{\alpha}(x)\cdot\boldsymbol{T}]$ acts on:

the complex scalar doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $\phi' = U_T \phi$,

left-chiral leptons: $L = \begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L$,
$$\begin{aligned} L' &= U_T L \,, \\ \overline{L}' &= (U_T L)^\dagger \gamma^0 = L^\dagger \gamma^0 U_T^\dagger = \overline{L} U_T^\dagger \end{aligned}$$

Right-chiral leptons $R = \psi_{\ell,R}$ have $T_3 = 0$ (do not change under SU(2)).

The transformation is unitary: $U_T^\dagger = U_T^{-1}$ so

$$(\overline{L}\phi)R \rightarrow (\overline{L}'\phi')R' = (\overline{L}U_T^\dagger U_T \phi)R = (\overline{L}\phi)R$$

is invariant under SU(2).

U(1) transformation of $(\overline{L}\phi)R$

Under the U(1)$_Y$ transformation: $U_Y = e^{iY\theta}$, $\theta \equiv g'\beta(x)$,

Hypercharge is $Y = 2(Q - T_3)$ so

$$Y_\phi = 1$$

$$Y_L = -1$$

$$Y_R = -2$$

The components of $(\overline{L}\phi)R$ transform as

$$\overline{L} = L^\dagger\gamma^0 \quad \rightarrow \quad \overline{L}' = (e^{-i\theta}L)^\dagger\gamma^0 = \overline{L}e^{i\theta}$$

$$\phi \quad \rightarrow \quad \phi' = e^{i\theta}\phi$$

$$R \quad \rightarrow \quad R' = e^{-2i\theta}R\,,$$

$\longrightarrow$ $(\overline{L}\phi)R$ is invariant under U(1).

Yukawa Lagrangian

So $(\overline{L}\phi)R$ is invariant under the full gauge group SU(2) x U(1).

We can therefore add to the Lagrangian

$$\mathcal{L}_\ell \;=\; -y_\ell \left[\overline{L}\phi R + (\overline{L}\phi R)^\dagger \right]$$

(y_l = Yukawa coupling, include Hermitian conjugate to ensure *L* real.)

After transform to unitary gauge, $\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$

Using this in the Lagrangian above, the terms involving neutrinos disappear since they are in the upper row of *L* (t_3 = ½).
Substituting the terms gives

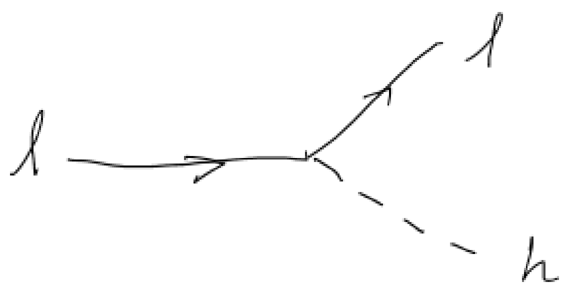
$$\mathcal{L}_\ell = -\frac{y_\ell}{\sqrt{2}} v \underbrace{(\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L)}_{\bar{\ell}\ell} - \frac{y_\ell}{\sqrt{2}} h \underbrace{(\bar{\ell}_L \ell_R + \bar{\ell}_R \ell_L)}_{\bar{\ell}\ell}$$

Yukawa coupling (charged leptons)

We identify the first term as a mass term $-m_\ell \overline{\psi}\psi$ with $m_\ell = \dfrac{y_\ell v}{\sqrt{2}}$

Assign the Yukawa coupling y_l so that it gives the experimentally observed masses. No predicted relation between masses, need an adjustable parameter in the model for each one.

Second term in Yukawa Lagrangian is an interaction between the Higgs boson and a lepton pair with vertex factor



$$-iy_\ell/\sqrt{2} = -im_\ell/v \qquad \propto \underline{m_\ell}$$

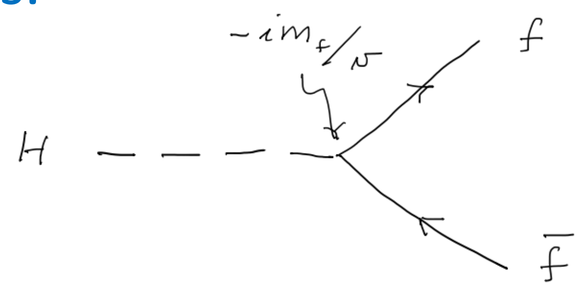
Masses and Higgs couplings for all charged fermions

For down-type quarks, procedure is like charged leptons, but because charged component of ϕ has zero VEV, cannot treat up-type quarks in the same way.

But, by using trick with charge conjugate doublet (see e.g. Thompson Sec. 17.5.5)

$$\phi_c = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}$$

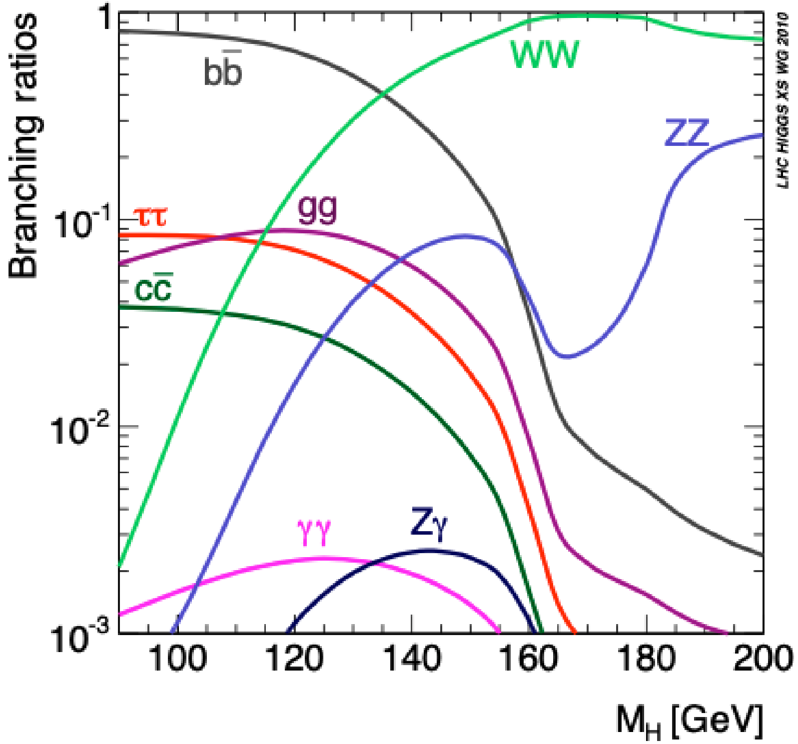
wind up with same recipe for masses and Higgs couplings for all charged fermions:

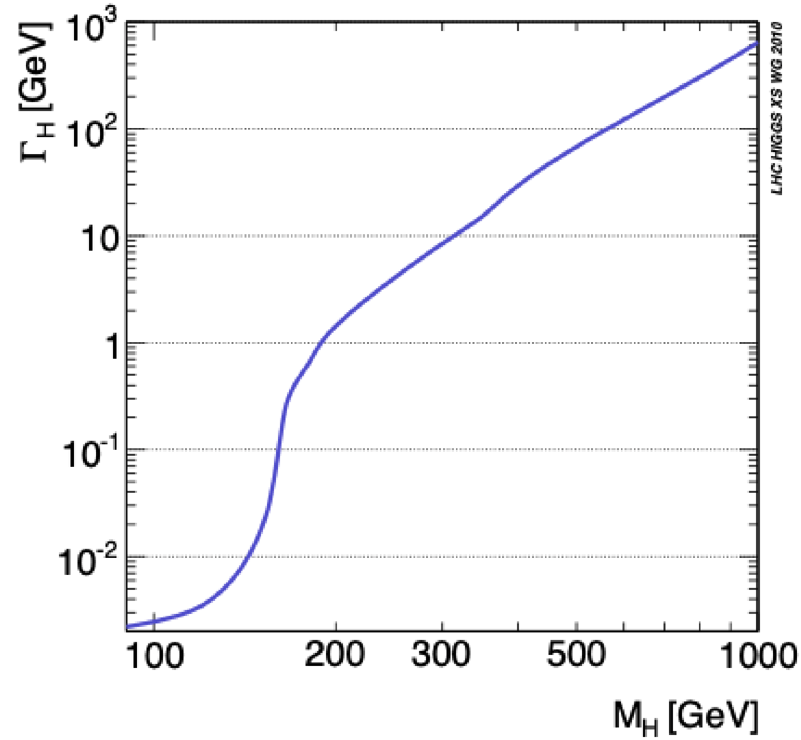


$$y_f = \sqrt{2}\frac{m_f}{v}$$

Higgs branching ratios and decay width

S. Dittmaier et al. (LHC Higgs Cross Section Working Group), *Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables*, CERN-2011-002, arXiv:1101.0593 (2011).





For m_H = 125 GeV, SM predicts: $\Gamma_{\rm H} = 4.1\,{\rm MeV}$

From interference effects in H → ZZ, measure: $\Gamma_{\rm H} = 4.5^{+3.0}_{-2.2}\,{\rm MeV}$

(ATLAS)

Higgs boson couplings versus mass

G. Aad et al. (ATLAS Collaboration), *A detailed map of Higgs boson interactions by the ATLAS experiment ten years after the discovery*, Nature volume 607, pages52–59 (2022).

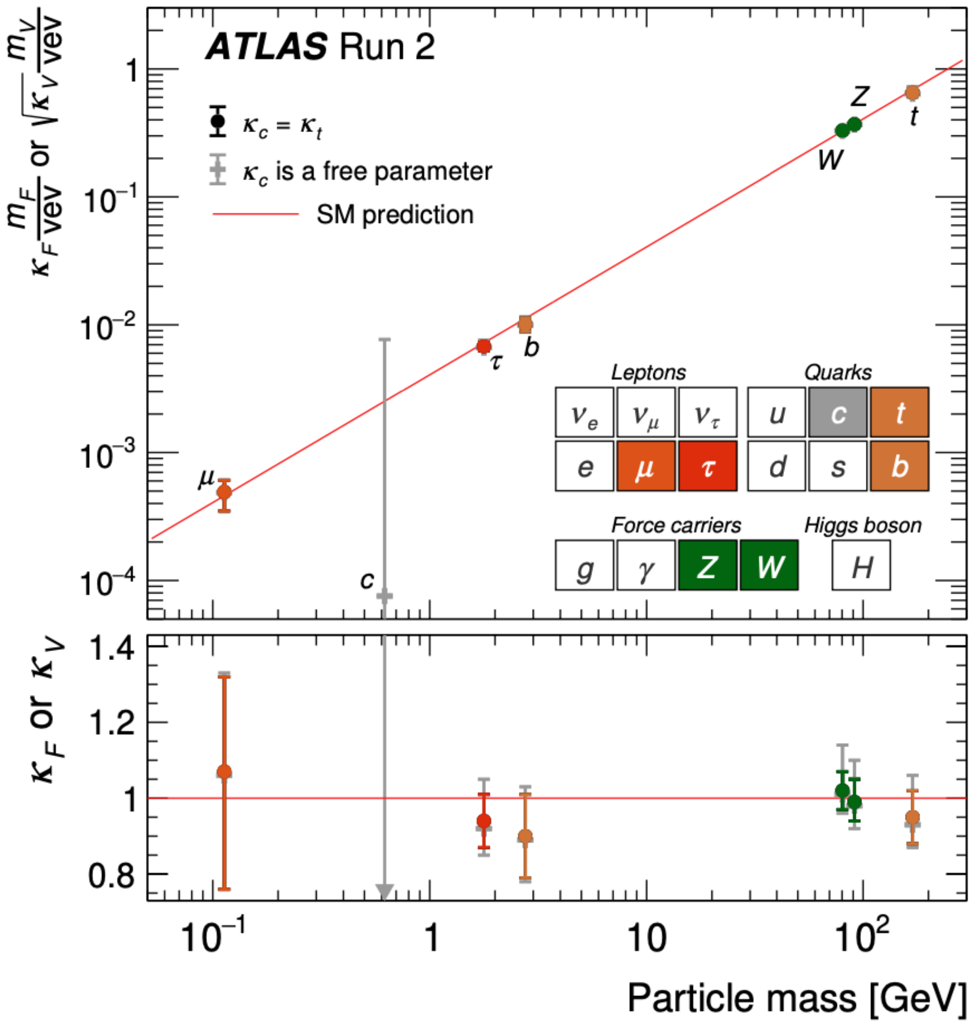


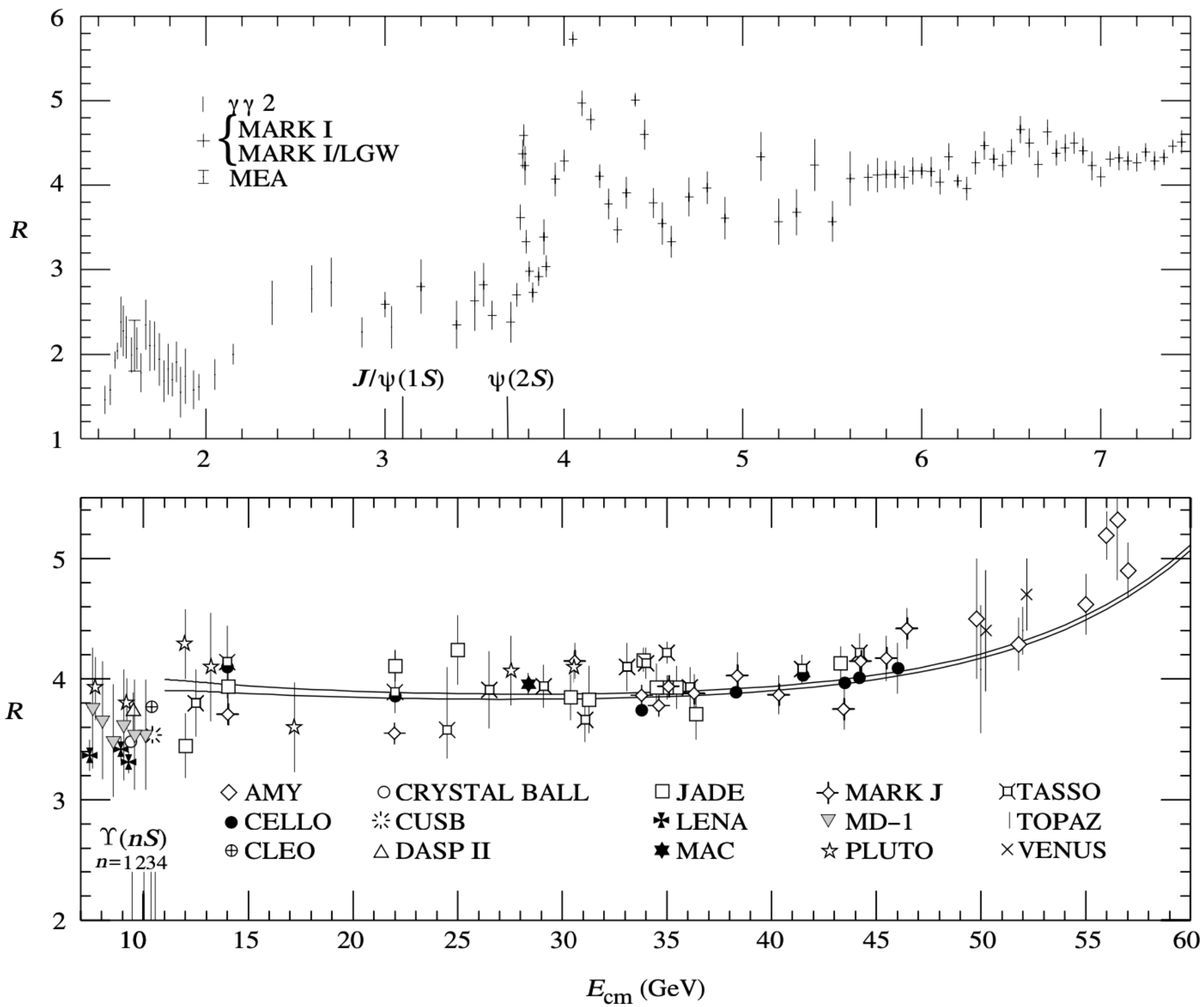
Figure 11.8: Measurements of the coupling strength modifiers (see text) of the Higgs boson to different particles as as function of the particle's mass. (from [50]).

QCD: Evidence for Colour

After the quark model was introduced in 1964 there emerged evidence that quarks have an additional degree of freedom "colour':

- Δ^{++} (uuu), J=3/2 (from angular dist. of πp scattering), L=0 and quark spins all parallel, so spin and spatial wave functions symmetric. But by Pauli exclusion principle, should have antisymmetric total wave function. Works if quarks have antisymmetric "colour" degree of freedom with $N_c \geq 3$.
- $e^+e^- \to$ hadrons interpreted as $e^+e^- \to$ quark-antiquark. Measured cross section $\sim$factor of 3 higher than prediction, suggesting quarks come in $N_c = 3$ colours.
- Decay of neutral pion would involve quark loop, if one for each of N_c colours then decay rate $\propto N_c^2$. Comparison with measured rates indicates $N_c = 3$.

The need for colour: $e^+e^- \to$ hadrons



Ingredients of SU(3)

Generators: $$T^a_{ij} = \frac{\lambda^a_{ij}}{2}\,, \quad a = 1,\ldots,8, \qquad \mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$$

Gell-Mann matrices:

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Structure constants: $$f^{123} = 1,$$

$$[T^a, T^b] = i f^{abc} T^c \qquad f^{147} = f^{156} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2},$$

$$f^{abc} = -f^{bac} = -f^{acb} \qquad f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

(all others zero)

Lagrangian of QCD

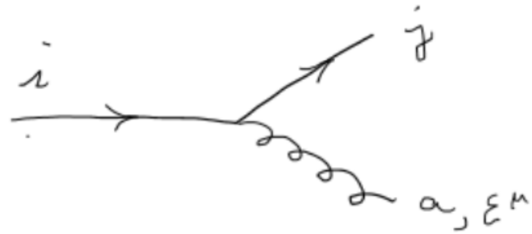
$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \overline{q}(i\not{D} - m_q) q$$

$$= -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \sum_q \sum_{j,k=1}^{3} \overline{q}_j \left[(i\not{\partial} - m_q)\delta_{jk} - g_{\mathrm{s}} \not{G}^a T^a_{jk} \right] q_k$$

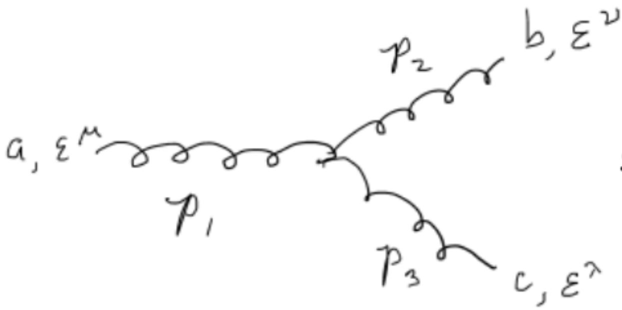
$$D_\mu = \partial_\mu + i g_{\mathrm{s}} T^a G^a_\mu$$

$$F^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_{\mathrm{s}} f^{abc} G^b_\mu G^c_\nu$$

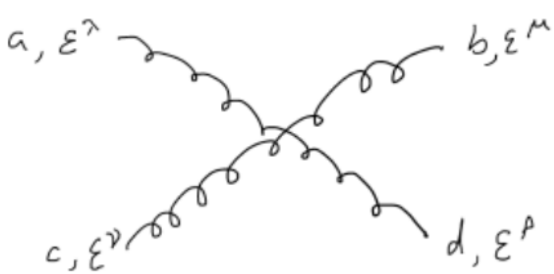
Fundamental vertices of QCD



$$-ig_{\mathrm{s}}\gamma^{\mu}(T^{a})_{ij}$$

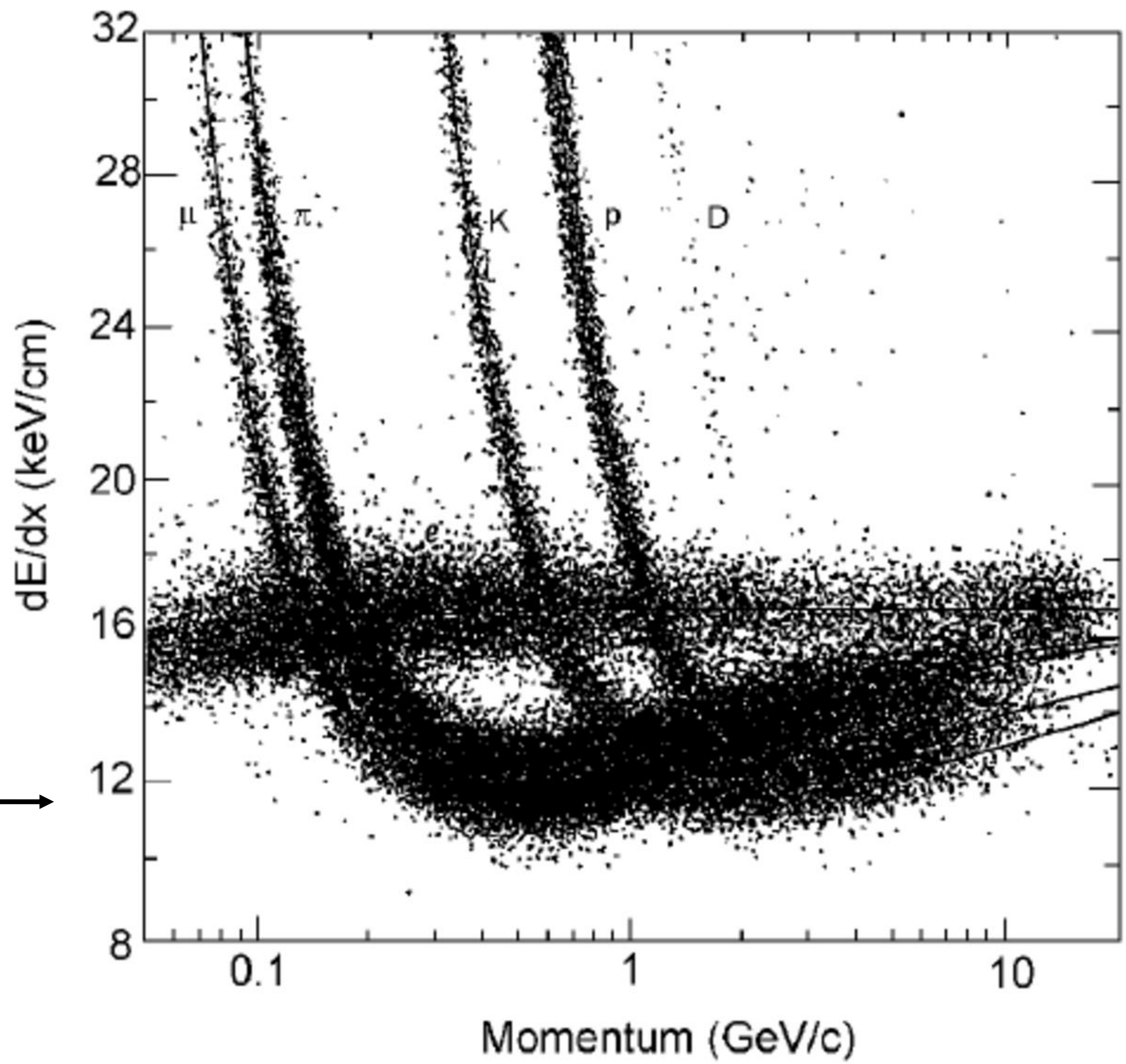


$$g_{\mathrm{s}} f^{abc}\left[g^{\mu\nu}(p_1 - p_2)^{\lambda} + g^{\nu\lambda}(p_2 - p_3)^{\mu} + g^{\lambda\mu}(p_3 - p_1)^{\nu}\right]$$



$$\begin{aligned} -ig_{\mathrm{s}}^{2}\Big[& f^{abf} f^{cdf}(g^{\lambda\nu}g^{\mu\rho} - g^{\lambda\rho}g^{\mu\nu}) \\ & + f^{acf} f^{bdf}(g^{\lambda\mu}g^{\nu\rho} - g^{\lambda\rho}g^{\mu\nu}) \\ & + f^{adf} f^{cbf}(g^{\lambda\nu}g^{\mu\rho} - g^{\lambda\mu}g^{\rho\nu})\Big] \end{aligned}$$

No fractional charges seen

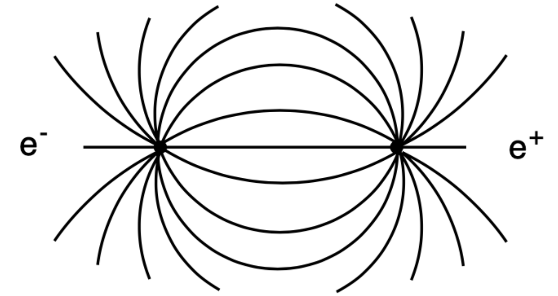


minimum ionising →

~ nothing below

Field configurations between charges

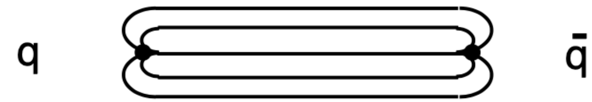
Field configuration between e⁺e⁻:



$$V_{\mathrm{C}}(r) = -\frac{\alpha}{r}$$

$$F_{\mathrm{C}} = -\frac{\partial V_{\mathrm{C}}}{\partial r} = -\frac{\alpha}{r^2}$$

and between quark-antiquark:

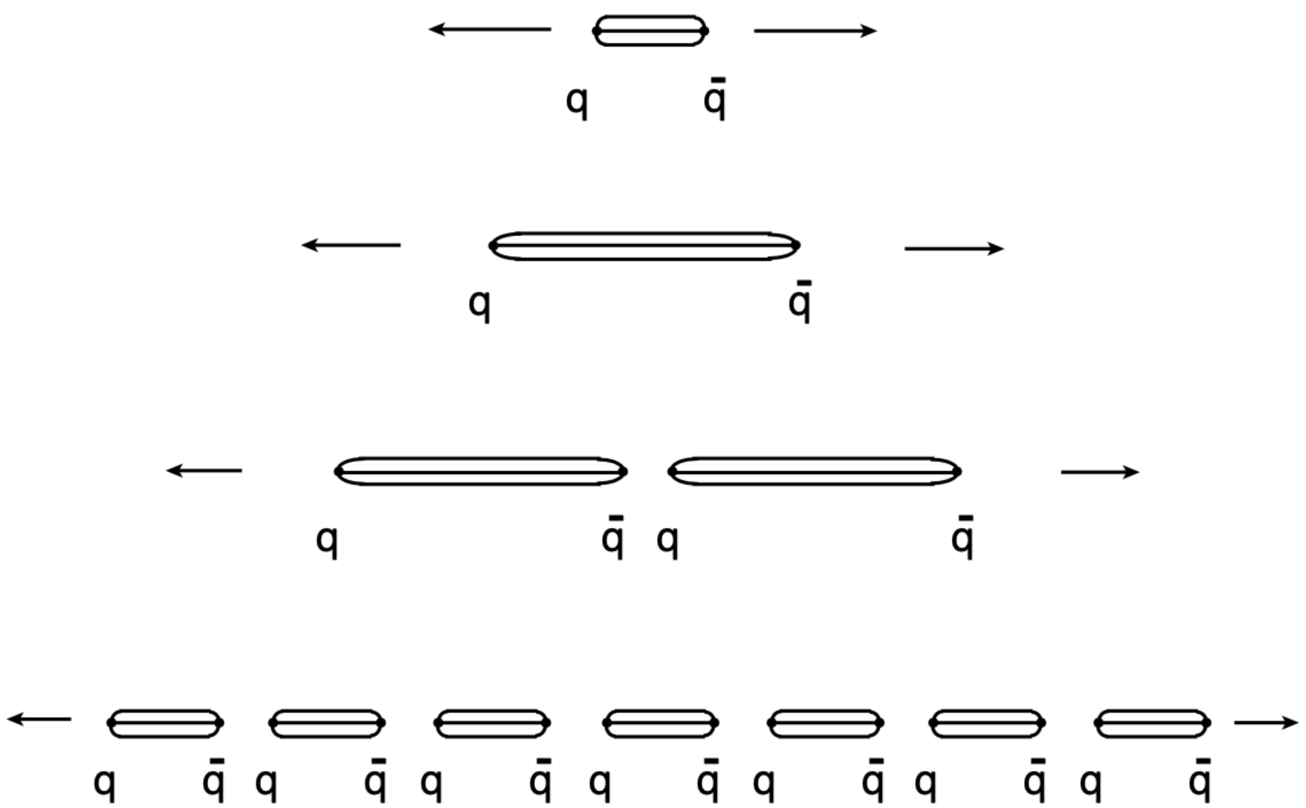


$$V_{\mathrm{QCD}}(r) = -\frac{4}{3}\frac{\alpha_{\mathrm{s}}}{r} + kr$$

$$F = -\frac{\partial V_{\mathrm{QCD}}}{\partial r} = -\frac{4}{3}\frac{\alpha_{\mathrm{s}}}{r^2} - k$$

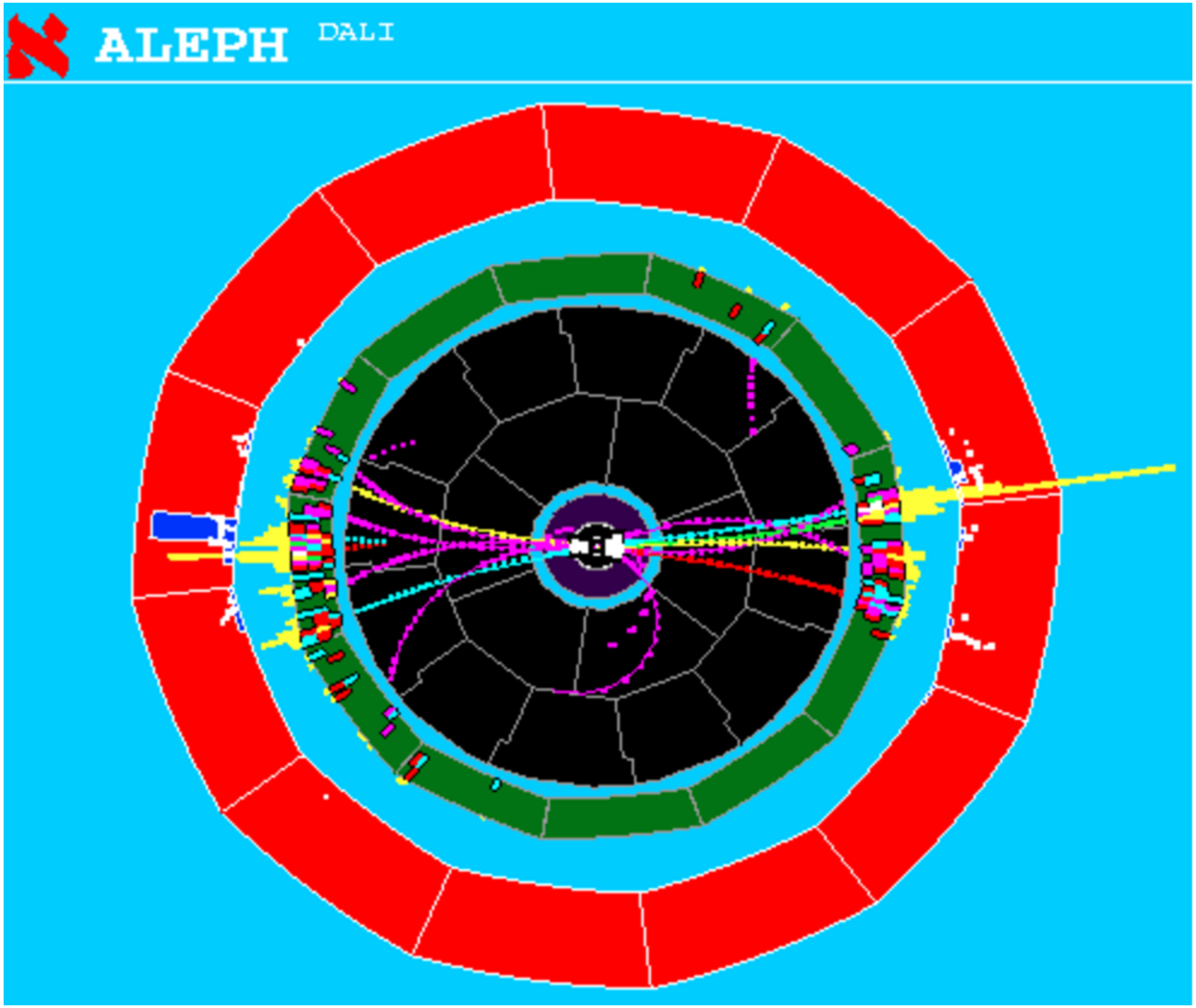
← Force approaches constant.

Hadron production from breaking of flux tube:

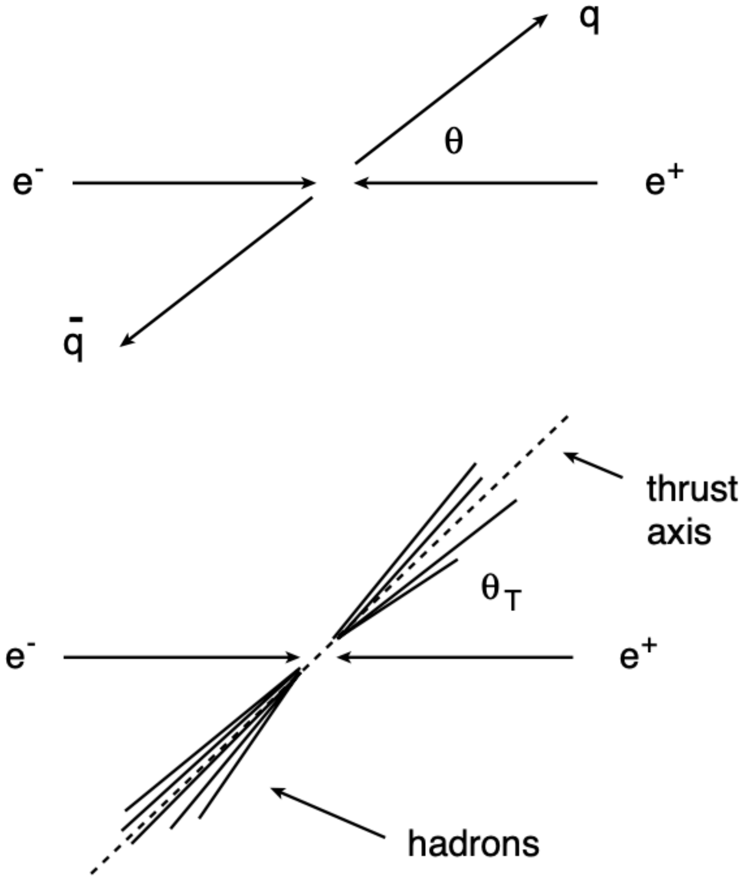


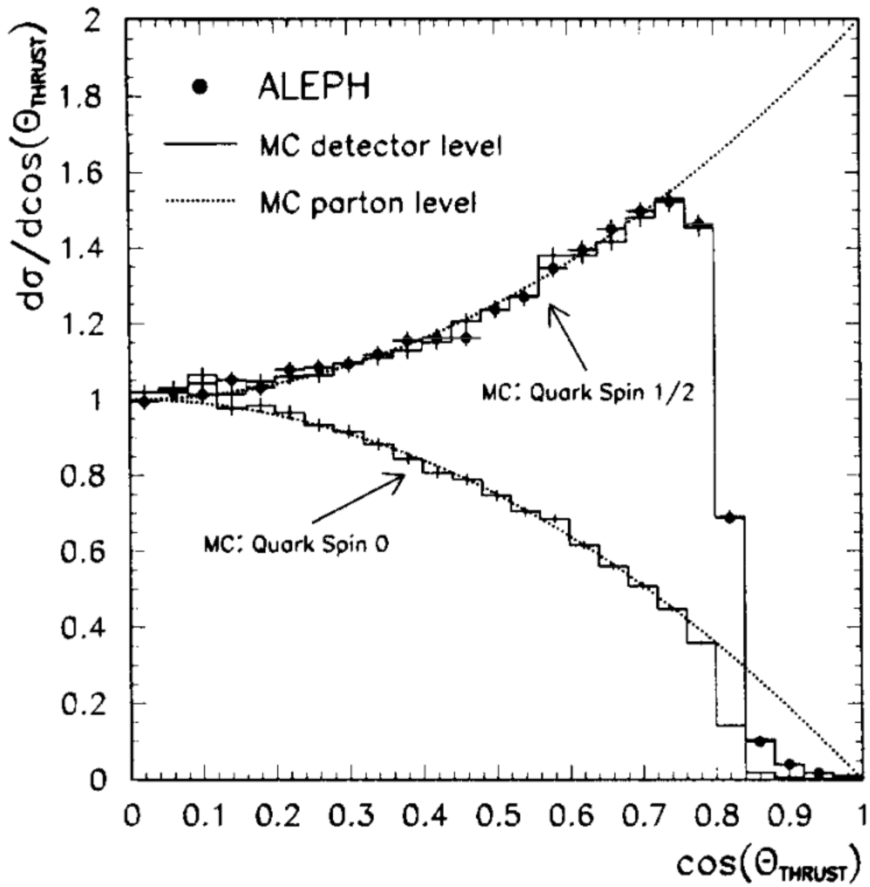
Two-jet event from e^+e^- collision

The ALEPH Collaboration, CERN

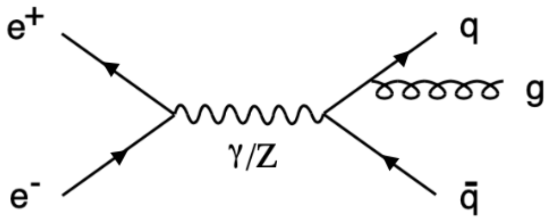


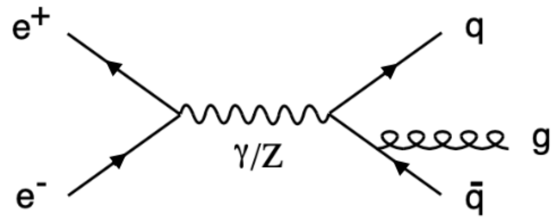
Angular distribution of event axis

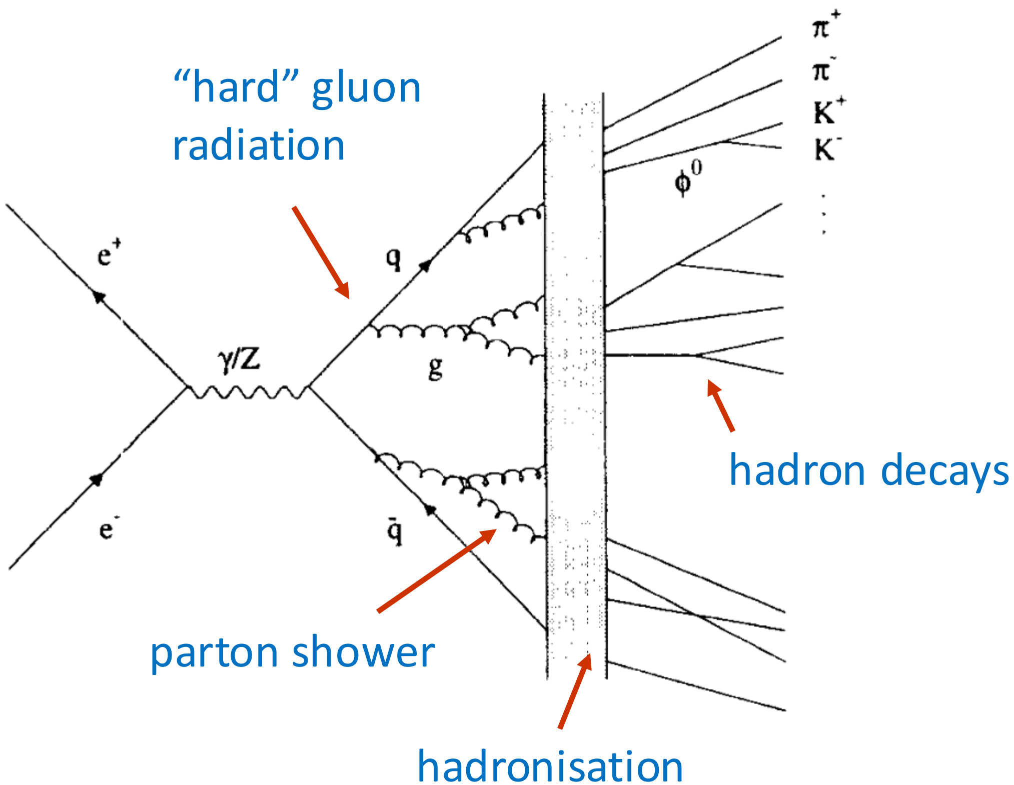




$e^+e^- \rightarrow$ hadrons







Discovery of the gluon at PETRA

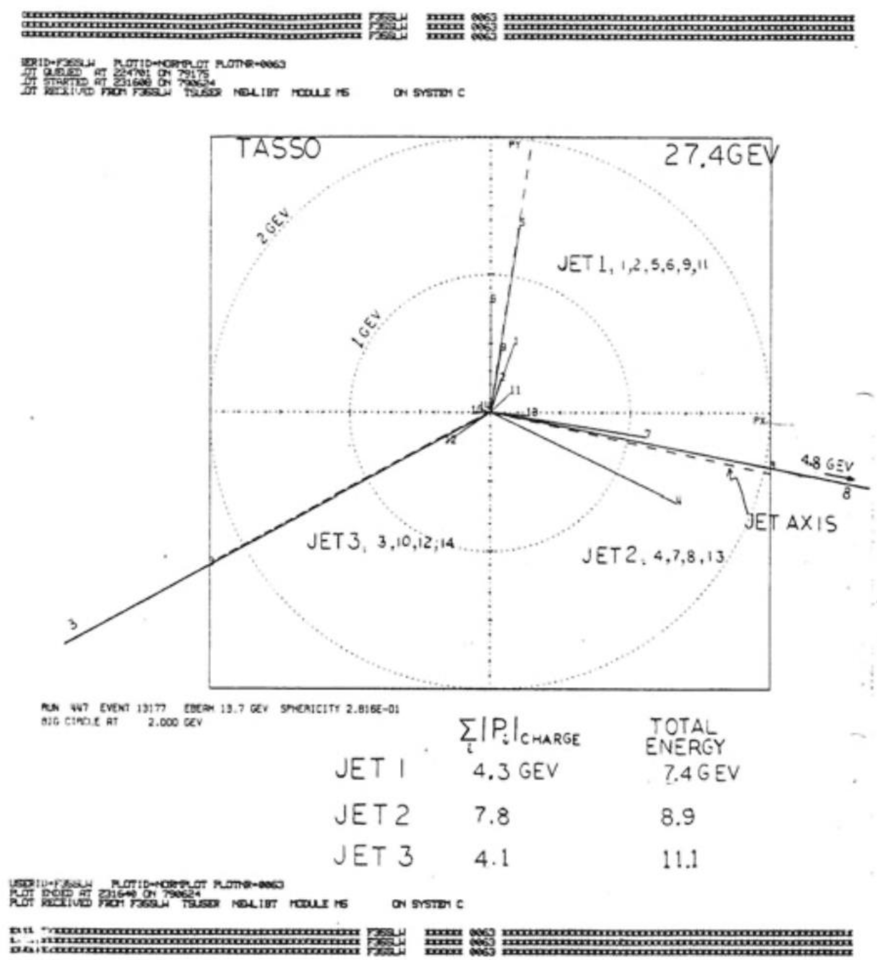
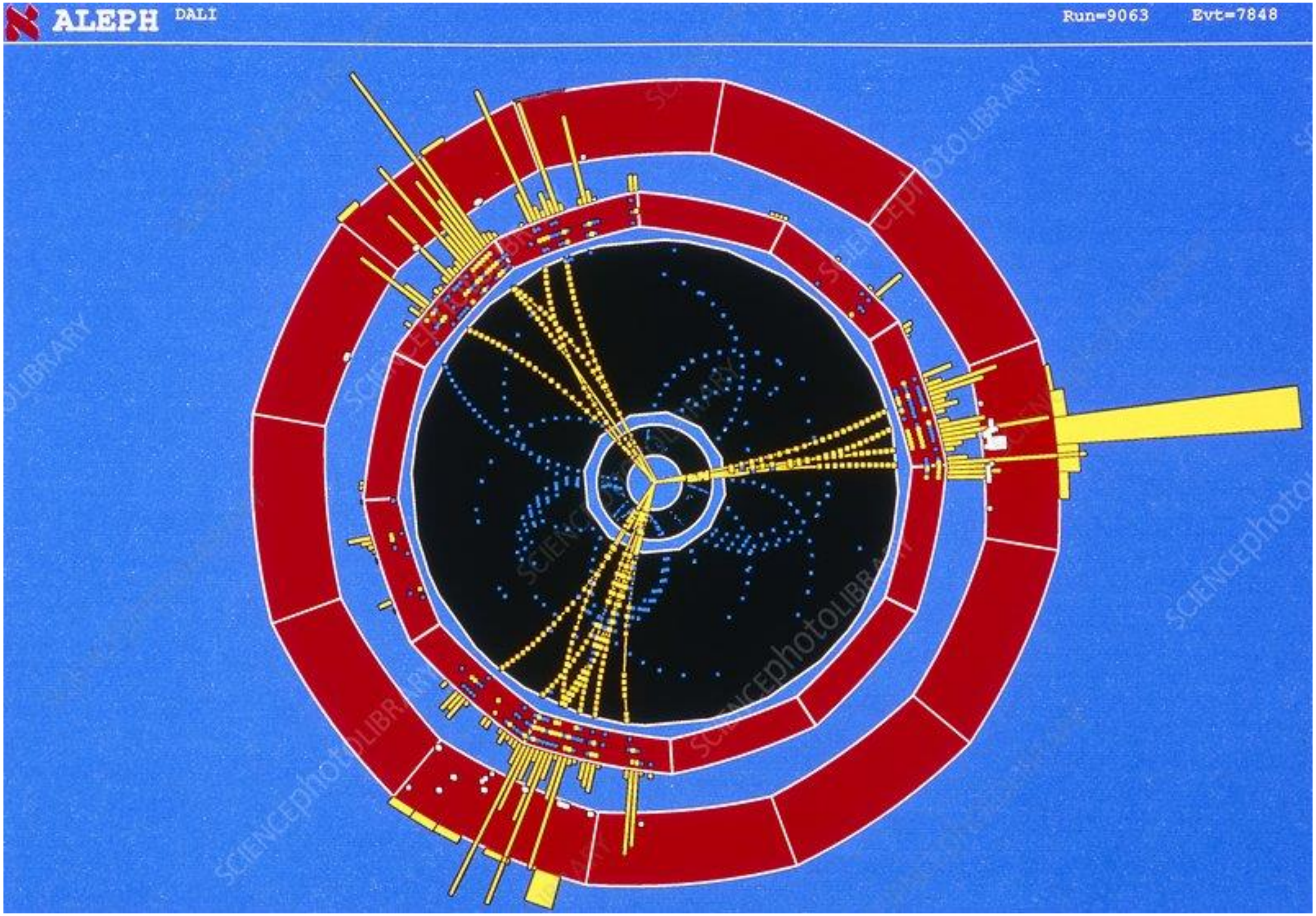


Fig. 6. The computer printout of the first three-jet event found in June 1979 [75, 78]. It shows the momentum vectors of the charged hadrons projected onto the event plane. Note that the event has three separate jets, and does not at all look like a back-to-back two-jet event with one narrow jet and a second, somehow broadened, jet.

Three-jet event from e^+e^- collision

The ALEPH Collaboration, CERN



α_s from thrust distribution

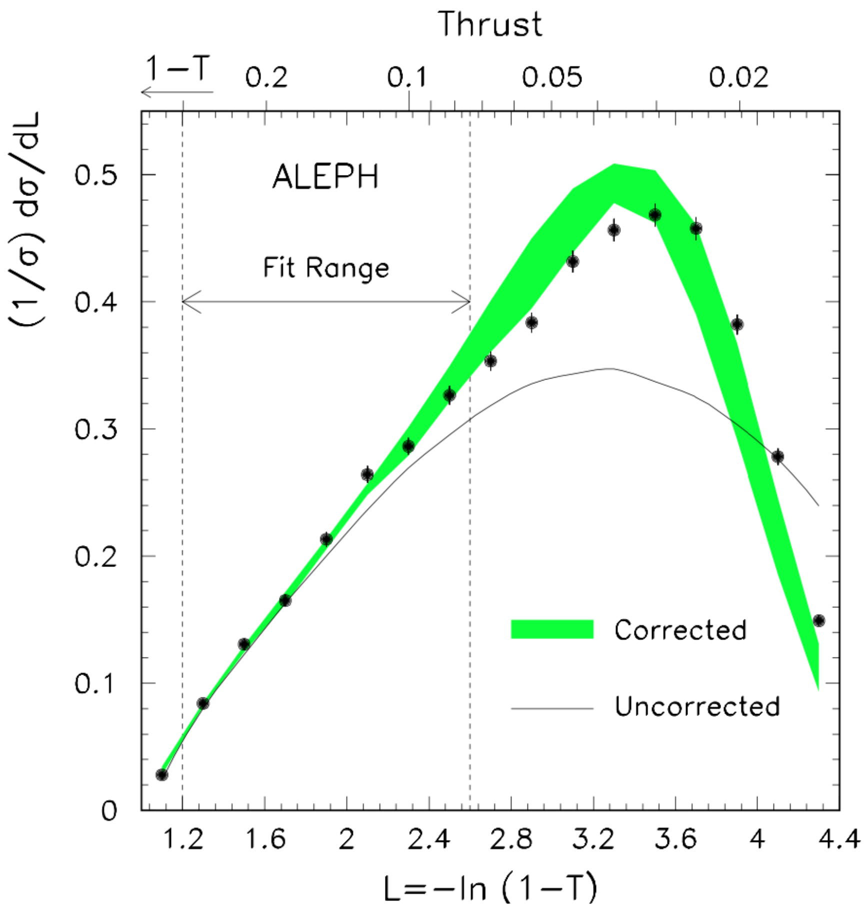


Figure 12.14: The distribution of the event-shape variable $-\ln(1-T)$, where T is the thrust. The QCD prediction (shaded band) has been fitted to the measured distribution (data points) by adjusting the value of α_s [60].

$$\alpha_s = 0.126 \pm 0.007.$$

Dominant uncertainty from QCD prediction.

Test of gluon spin

ALEPH Collaboration / Physics Reports 294 (1998) 1–165

QCD (spin-1 gluon):

$$\frac{d\sigma}{dx_q\,dx_{\bar{q}}} \sim \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

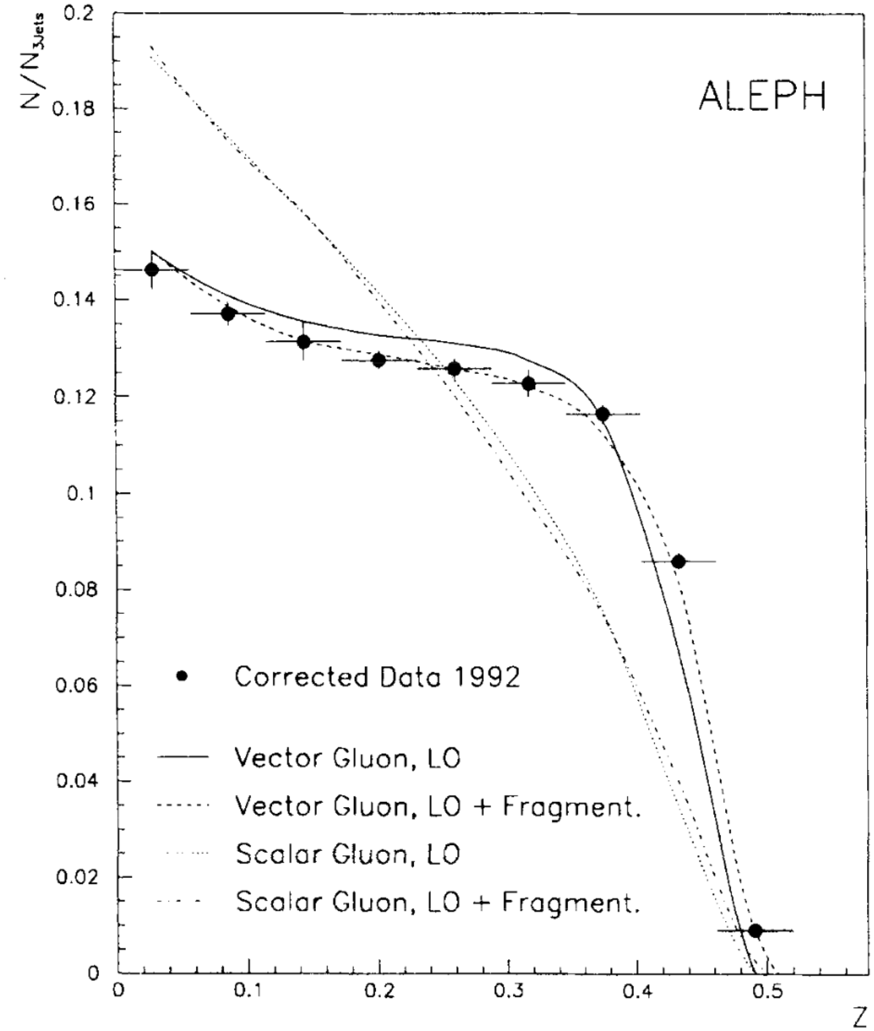
Scalar gluon theory:

$$\frac{d\sigma}{dx_q\,dx_{\bar{q}}} \sim \frac{x_g^2}{(1-x_q)(1-x_{\bar{q}})} + \text{const}$$

Experimentally difficult to distinguish between q, $\bar{q}$, g jets.

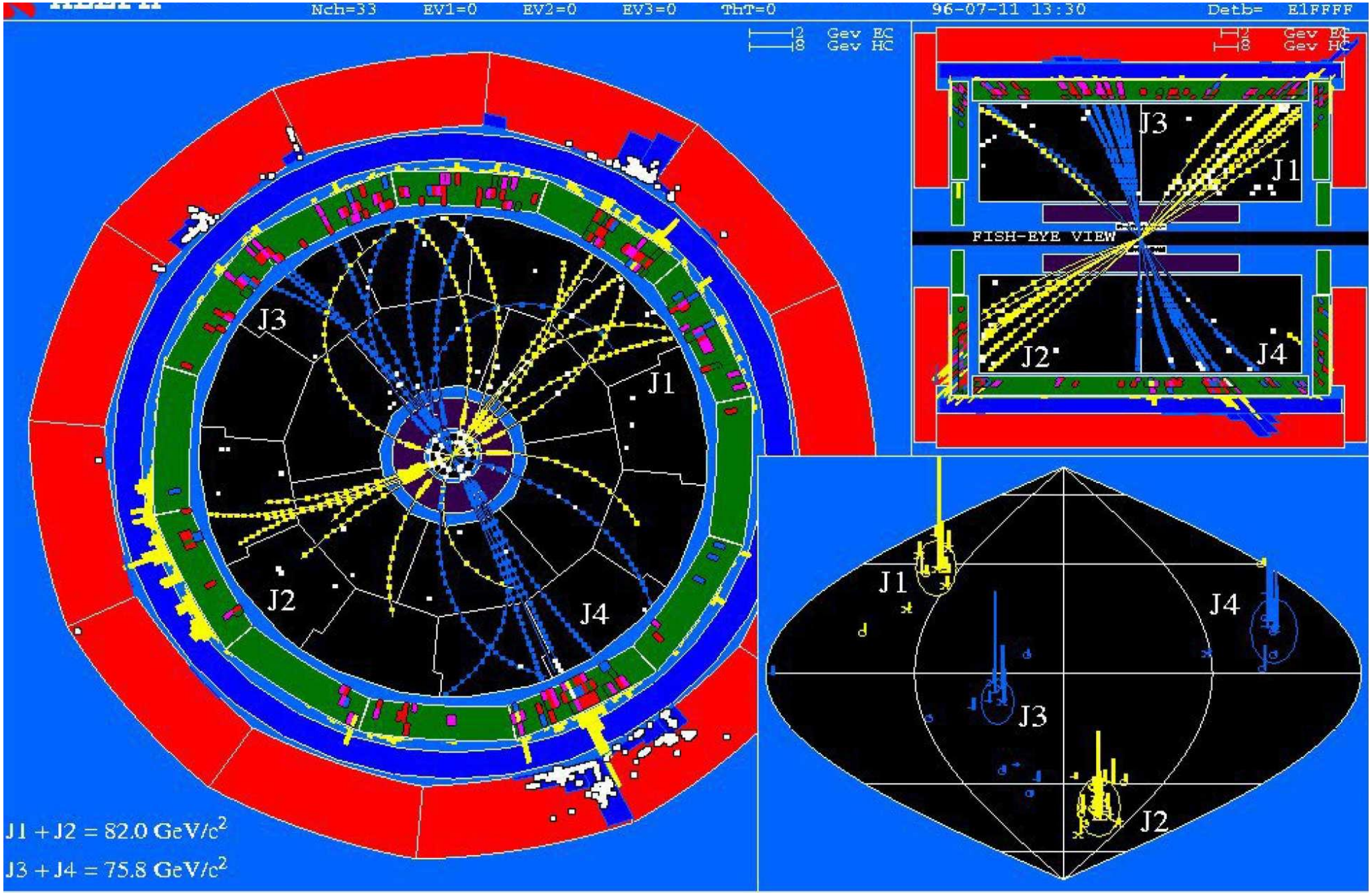
$\rightarrow$ order the x_i by energy: $x_1 > x_2 > x_3$

Define $z = (x_2 - x_3)/\sqrt{3}$



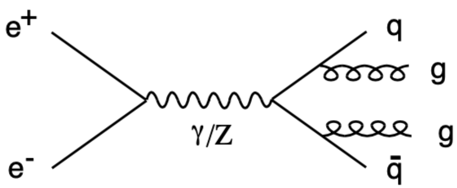
Four-jet event from e^+e^- collision

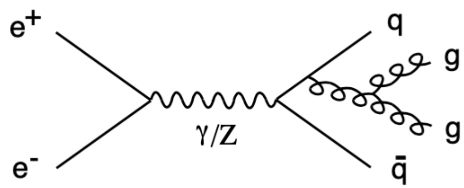
The ALEPH Collaboration, CERN



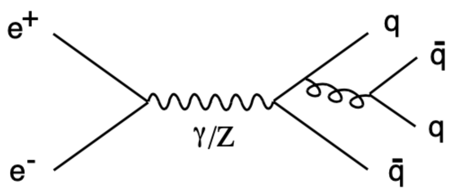
D. Decamp et al. (ALEPH Collaboration), *Evidence for the triple-gluon vertex from measurements of the QCD colour factors in Z decay into four jets*, Physlcs Letters B 284 (1992) 151-162.

Analysis of four-jet events from e^+e^- collisions



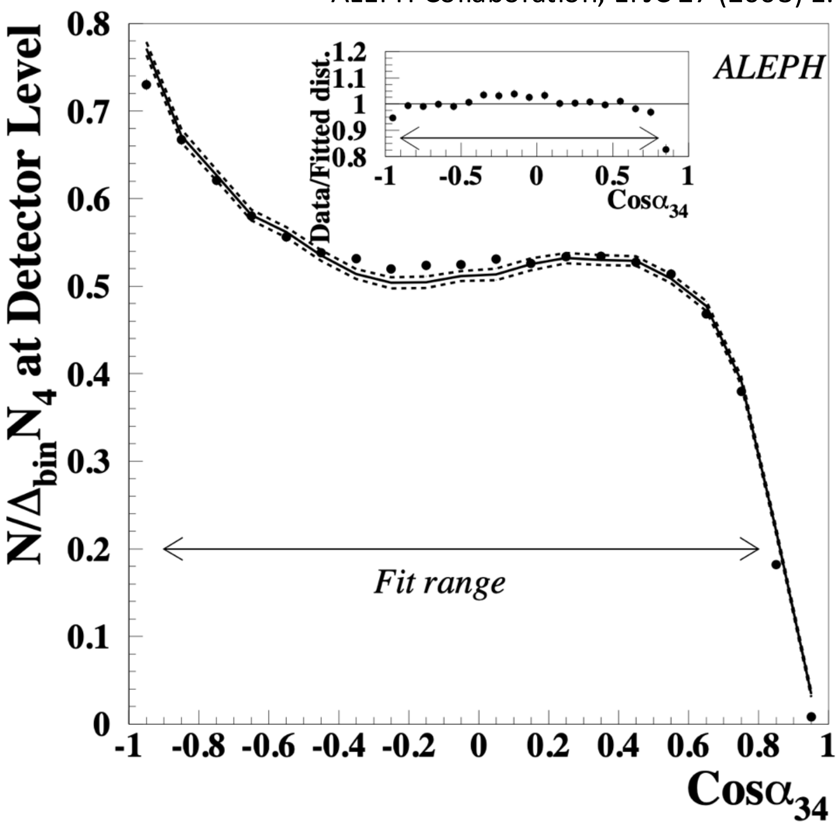


sensitive to
triple-gluon vertex

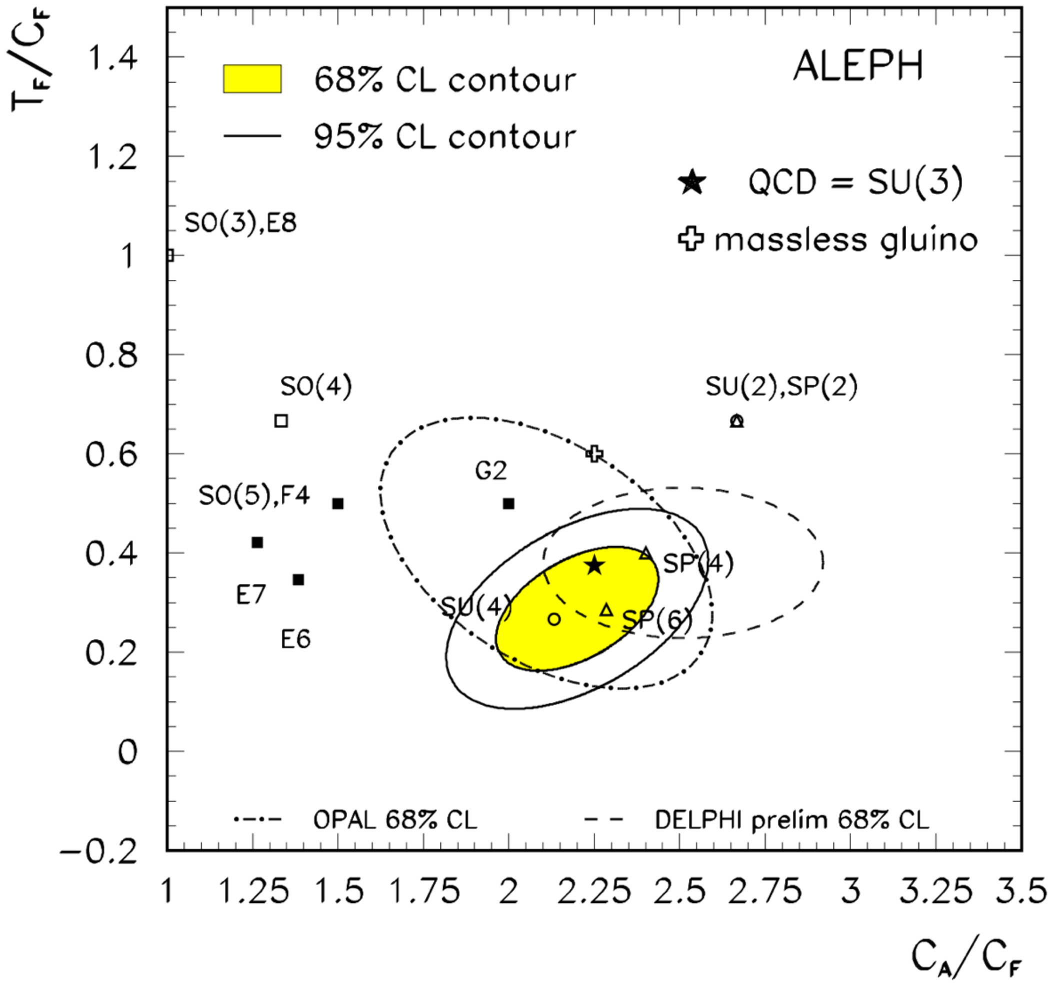


Form variables sensitive to four-jet structure, e.g., α_{34} = angle between least two energetic jets.

ALEPH Collaboration, EPJC 27 (2003) 1.



QCD colour factors from four-jet events



Running α_s