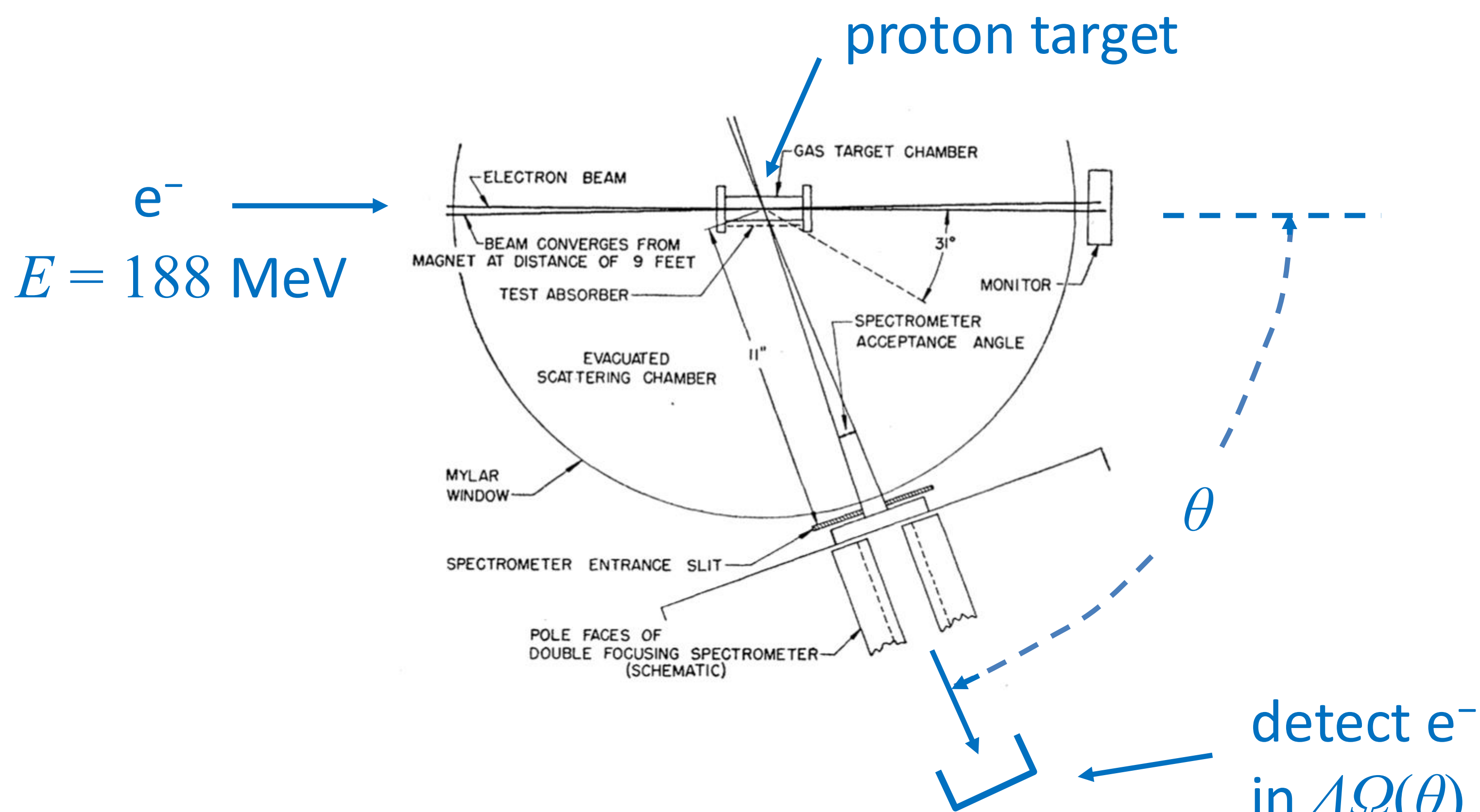
PH4442 Advanced Particle Physics 2025/26
Lecture Week 6

Glen Cowan
Physics Department
Royal Holloway, University of London
g.cowan@rhul.ac.uk
www.pp.rhul.ac.uk/~cowan

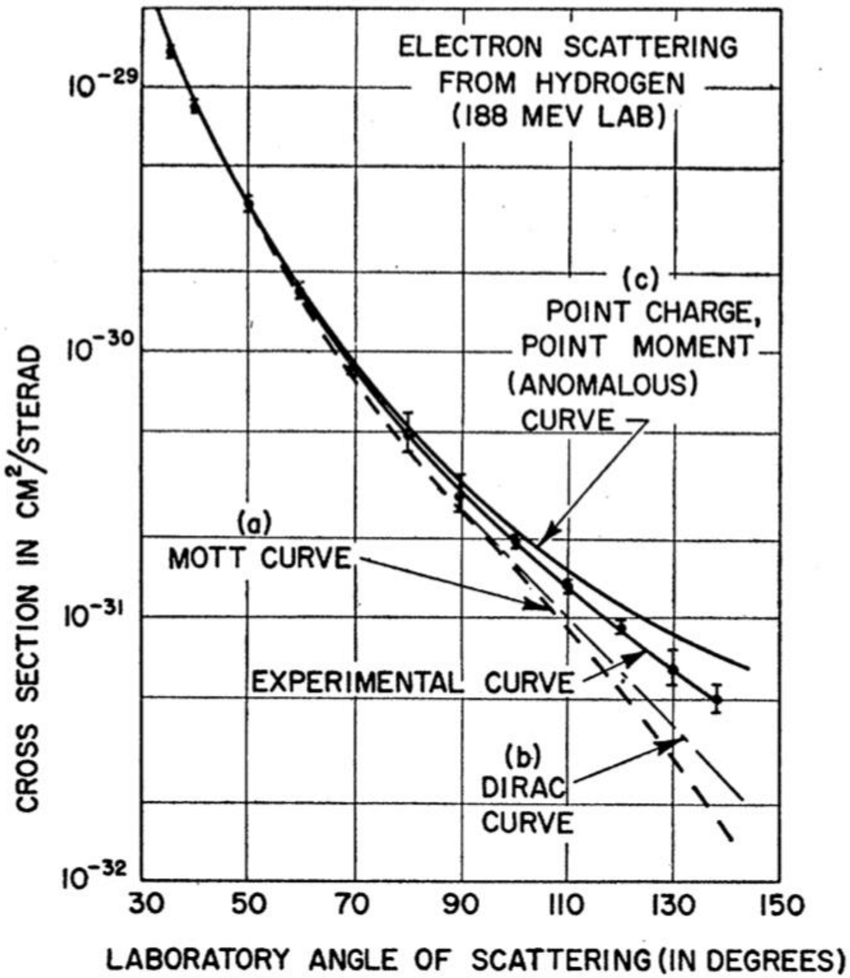
- Comments on problem sheet 3 (ep scattering)
- Comments on problem sheet 2 (W propagator)
- Weak interactions

Comments on elastic electron-proton scattering

Investigated in the 1950s at SLAC by McCallister and Hofstadter.



Size of the proton



The discrepancy between the "Dirac curve" and the data can be explained by assuming the proton is not a point charge, but rather a charge distribution with rms radius

$$r_{\mathrm{p}} = (0.74 \pm 0.24) \times 10^{-15}\ \mathrm{m}$$

This was the first measurement of the proton's size.

Comments on problem sheet 2, Q5

5 a) Wave eq. for massive vector boson

$$\text{(1)} \quad \left[(\partial_\alpha \partial^\alpha + M^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] W_\nu(x) = J^\mu(x)$$

Corresponding eq. for Green function

$$\text{(2)} \quad \left[(\partial_\alpha \partial^\alpha + M^2) g^{\mu\nu} - \partial^\mu \partial^\nu \right] D_{\nu\lambda}(x - x') = \delta^\mu{}_\lambda \, \delta^4(x - x')$$

Claim: sol'n has form

$$\text{(3)} \quad W_\nu(x) = \int d^4x' \, D_{\nu\lambda}(x - x') \, J^\lambda(x')$$

PS2, Q5(a)

Sub ③ into LHS of ①

$$\underbrace{\left[(\partial_\alpha \partial^\alpha + M^2) g^{\mu\nu} - \partial^\mu \partial^\nu\right]}_{\text{derivatives wrt } x, \text{ not } x' \Rightarrow \text{ move in } \int} \int d^4x' \, D_{\nu\lambda}(x - x') J^\lambda(x')$$

$$= \int d^4x' \, J^\lambda(x') \underbrace{\left[(\partial_\alpha \partial^\alpha + M^2) g^{\mu\nu} - \partial^\mu \partial^\nu\right] D_{\nu\lambda}(x - x')}_{\text{use ②}}$$

$$= \int d^4x' \, J^\lambda(x') \, \delta^\mu{}_\lambda \, \delta^4(x - x')$$

$$= J^\mu(x) \quad = \text{RHS of ①} \quad ✓$$

PS2, Q5(b)

5b) Fourier transform of $D_{\nu\lambda}(x-x')$ is

$$\tilde{D}_{\nu\lambda}(q) = \int d^4x \, D_{\nu\lambda}(x-x') \, e^{iq\cdot(x-x')}$$

$$\Rightarrow D_{\nu\lambda}(x-x') = \int \frac{d^4q}{(2\pi)^4} \tilde{D}_{\nu\lambda}(q) \, e^{-iq\cdot(x-x')} \qquad (4)$$

Use (4) in (2) (DE for Green func.)

$$\left[(\partial_\alpha \partial^\alpha + M^2) g^{\mu\nu} - \partial^\mu \partial^\nu\right] \int \frac{d^4q}{(2\pi)^4} \tilde{D}_{\nu\lambda}(q) \, e^{-iq\cdot(x-x')} = \delta^\mu{}_\lambda \, \delta^4(x-x')$$

↳ move into $\int$

$$\Rightarrow \int \frac{d^4q}{(2\pi)^4} \left[(-q^2+M^2) g^{\mu\nu} + q^\mu q^\nu\right] \tilde{D}_{\nu\lambda}(q) \, e^{-iq\cdot(x-x')} = \delta^\mu{}_\lambda \int \frac{d^4q}{(2\pi)^4} e^{-iq\cdot(x-x')}$$

$$\Rightarrow \left[(-q^2 + M^2) g^{\mu\nu} + q^\mu q^\nu\right] \tilde{D}_{\nu\lambda}(q) = \delta^\mu{}_\lambda$$

PS2, Q5(c)

5c) Take $\tilde{D}_{\nu\lambda}(q) = A g_{\nu\lambda} + B q_\nu q_\lambda$, A, B Lorentz scalars

$$\left[(-q^2+M^2)g^{\mu\nu} + q^\mu q^\nu\right](A g_{\nu\lambda} + B q_\nu q_\lambda) = \delta^\mu{}_\lambda$$

Use $g^{\mu\nu} g_{\nu\lambda} = \delta^\mu{}_\lambda$

and $g^{\mu\nu} q_\nu q_\lambda = q^\mu q_\lambda$

$$(-q^2+M^2)(A\delta^\mu{}_\lambda + B q^\mu q_\lambda) + A q^\mu q_\lambda + B q^2 q^\mu q_\lambda = \delta^\mu{}_\lambda$$

From $\mu \neq \lambda$, $\delta^\mu{}_\lambda = 0$ $\Rightarrow$ $B = -\dfrac{A}{M^2}$

For $\mu = \lambda$, $\delta^\mu{}_\lambda = 1$ $\Rightarrow$ $A = -\dfrac{1}{-q^2+M^2}$

$$\Rightarrow \tilde{D}_{\nu\lambda}(q) = \frac{-g_{\nu\lambda} + \dfrac{q_\nu q_\lambda}{M^2}}{q^2 - M^2 + i\varepsilon}$$

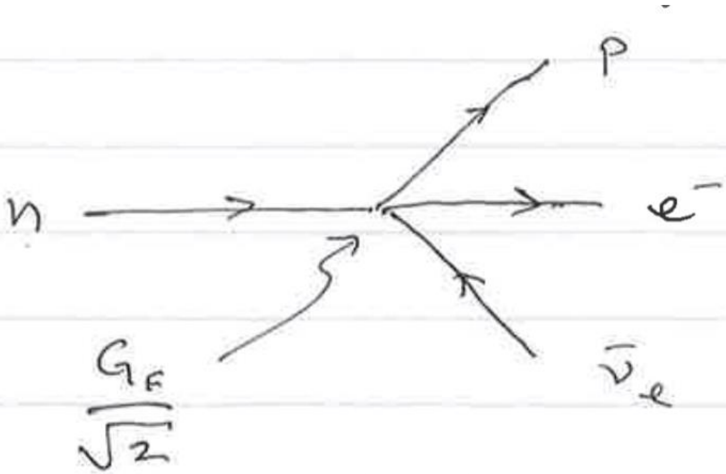
same reasoning as photon propagator

Weak Interactions I: from Fermi's theory to V–A

For a review of main ideas, see PH3520 notes Ch. 9. A brief recap:

Pauli, 1930 proposes neutrino to explain beta decay.

Fermi, 1934, theory of weak interactions that includes neutrino:



four-fermion vertex

$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

Fermi's theory

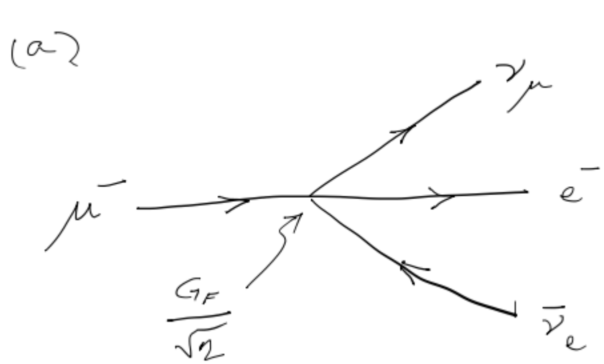
In analogy with QED, interaction amplitude is product of currents:

$$\mathcal{M} = G_{\mathrm{F}}\, J_{\mu}^{\mathrm{had}}\, j_{\mathrm{lep}}^{\mu} = G_{\mathrm{F}}[\overline{u}_p \gamma_\mu u_n][\overline{u}_e \gamma^\mu v_\nu]$$

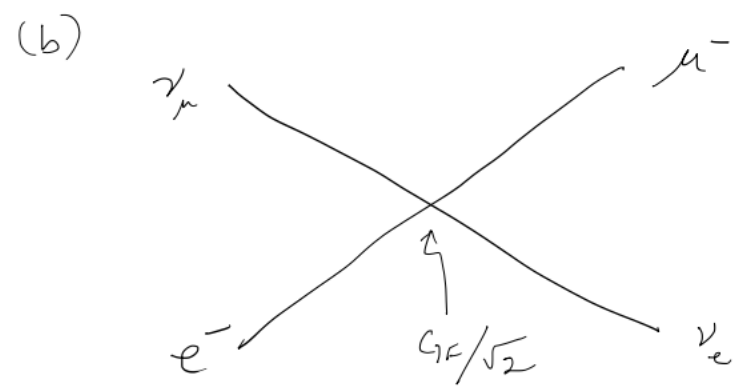
both currents are charge-changing

Initial guess is that both currents are Lorentz vectors, so product is Lorentz scalar (and hence parity conserving).

Other processes in Fermi's theory:



$$\mu^- \to e^- \nu_\mu \overline{\nu}_e$$



$$\nu_\mu e^- \to \mu^- \nu_e$$

Violation of unitarity bound in Fermi's theory

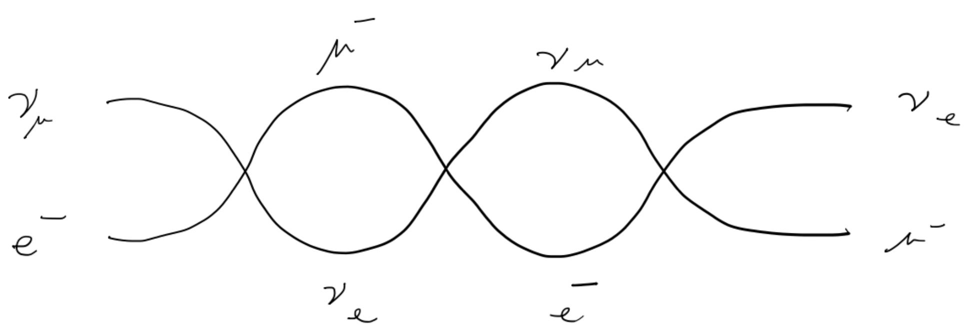
$\mathcal{M}(\nu_\mu e^- \to \mu^- \nu_e) \propto G_\mathrm{F}$ so $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \propto G_\mathrm{F}^2$

G_F and σ both units of E^{-2}, so we need $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \sim G_\mathrm{F}^2 E_\mathrm{cm}^2$

Grows with E_cm, at $E_\mathrm{cm} \sim$ 300 it exceeds the "unitarity bound", i.e., the probability of interaction exceeds unity.

Maybe this is because is based only on first-order diagram?

Include, e.g.,



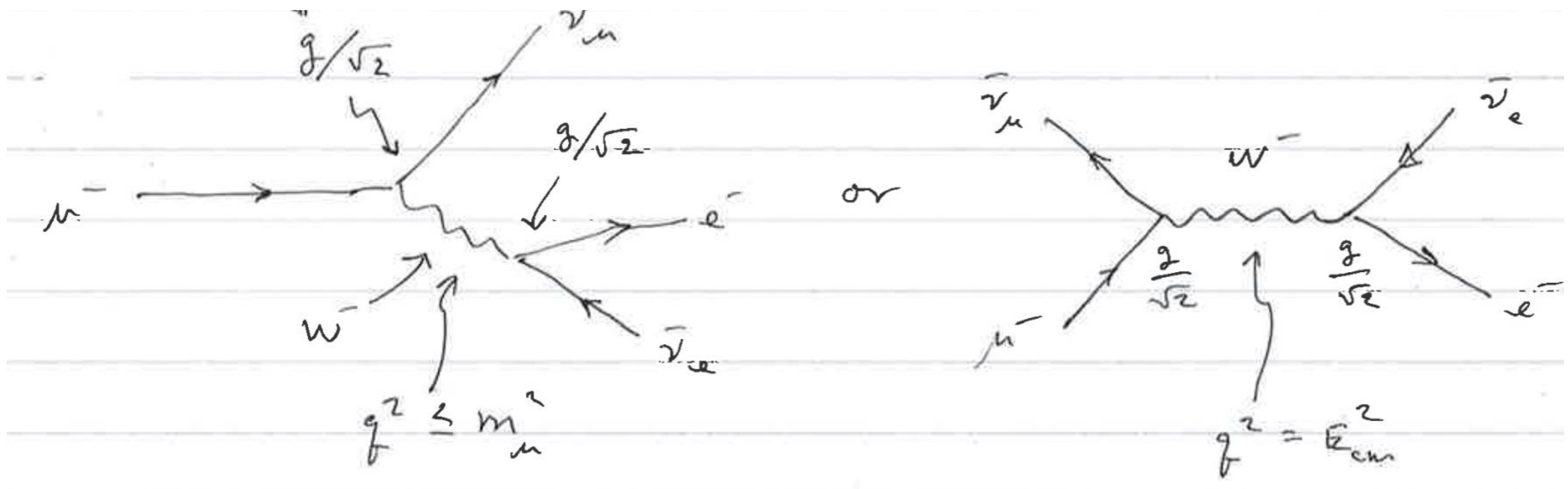
For loops, integrate over all momenta.

Integrals diverge: $\mathcal{M} \to \infty$!

QED also has infinities in higher-order diagrams but finite predictions obtained with "renormalisation"; doesn't work for Fermi's theory.

Violation of unitarity bound → IVB

The problems can be solved if the weak interaction is mediated by exchange of an intermediate vector (spin-1) boson (now the W):



Low ("weak") interaction rate from W-boson's large mass. For a massive boson, wave function obeys the Proca equation:

$$[(\partial_\nu \partial^\nu) + M_W^2] W^\mu - \partial^\mu (\partial_\nu W^\nu) = J^\mu$$

IVB propagator and unitarity bound

The W propagator is found to be:

$$\frac{-ig^{\mu\nu}}{q^2 + i\varepsilon} \quad \to \quad \frac{-i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}\right)}{q^2 - M_W^2 + i\varepsilon}$$

If $q^2 << M_W^2$, then $$\frac{-i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}\right)}{q^2 - M_W^2 + i\varepsilon} \quad \to \quad \frac{ig^{\mu\nu}}{M_W^2}$$

and so the amplitudes ~G_F now go as g^2/ M_W^2

In $\overline{\nu}_\mu \mu^- \to \overline{\nu}_e e^-$ q^2 is E_{cm}^2. For $E_{\rm cm} \gg M_{\rm W}$ we now have

$$\mathcal{M} \sim g^2 \frac{g^{\mu\nu} - \frac{q^\mu q^\nu}{M_W^2}}{q^2 - M_W^2} \to \frac{g^2}{E_{\rm cm}^2}$$

Not obvious but
can neglect $q^\mu q^\nu / M_W^2$

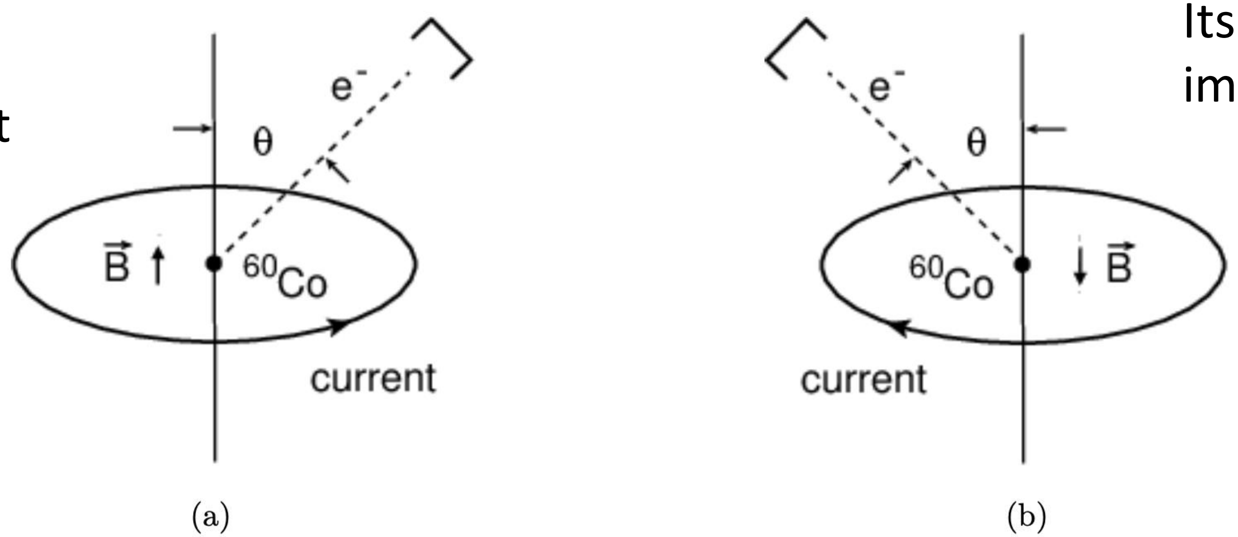
So now $\sigma \sim g^4/E_{\rm cm}^2$ and violation of unitarity bound avoided.

IVB also allows one to construct a renormalisable theory.

Non-conservation of parity

1956 Lee and Yang propose that parity could be violated in weak interactions, e.g., in beta decay:

Beta decay experiment



Its "mirror image"

Figure 9.4: (a) An experiment in which nuclear spins are aligned parallel to a magnetic field created by a current loop, and the number of beta electrons is measured at the angle θ. (b) The mirror image experiment.

In the experiment of C.S. Wu et al., Phys. Rev. 105 (1957) 1413, it was shown that $N(\theta) \neq N(\pi - \theta)$, therefore parity is not conserved.

Spinors for $\boldsymbol{p} = \hat{z} p_z$

$$u_1(p) = \sqrt{E+m} \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix}, \qquad u_2(p) = \sqrt{E+m} \begin{pmatrix} 0 \\ 1 \\ 0 \\ \frac{-p_z}{E+m} \end{pmatrix},$$

$$v_1(p) = \sqrt{E+m} \begin{pmatrix} 0 \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, \qquad v_2(p) = \sqrt{E+m} \begin{pmatrix} \frac{p_z}{E+m} \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Right– and Left-handed projection operators

$$P_R = \tfrac{1}{2}(1+\gamma^5) = \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$P_L = \tfrac{1}{2}(1-\gamma^5) = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}.$$

Helicity suppression

Consider spin up particle, momentum along z (positive helicity) u_1:

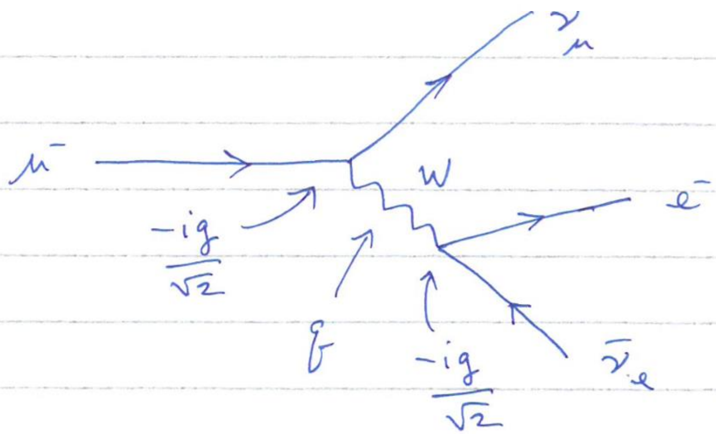
$$P_R u_1 = \tfrac{1}{2}\sqrt{E+m}\left(1+\frac{p_z}{E+m}\right)\begin{pmatrix}1\\0\\1\\0\end{pmatrix} ,$$

$$P_L u_1 = \tfrac{1}{2}\sqrt{E+m}\left(1-\frac{p_z}{E+m}\right)\begin{pmatrix}1\\0\\-1\\0\end{pmatrix} .$$

So left-chiral component of positive-helicity particle goes to zero in relativistic limit.

Similarly, $P_L v_2 \longrightarrow 0$ in relativistic limit.

Muon decay



$$\mathcal{M} = \frac{-ig}{\sqrt{2}}\left[\bar{u}(\nu_\mu)\gamma_\mu \tfrac{1}{2}(1-\gamma^5)u(\mu)\right]\left(\frac{-ig^{\mu\nu} + \frac{iq^\mu q^\nu}{M_W^2}}{q^2 - M_W^2}\right)\left(\frac{-ig}{\sqrt{2}}\right)$$

$$\times\left[\bar{u}(e)\gamma_\nu \tfrac{1}{2}(1-\gamma^5)v(\bar{\nu}_e)\right]$$

$$= \frac{G_F}{\sqrt{2}} J_\mu(\mu) J^\mu(e) \qquad \left(q^2 \ll M_W^2 \Rightarrow \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}\right)$$

Muon decay rate

long calc. because of 3-body final state

unpolarized μ

$$\frac{d\Gamma}{d\Omega\, dx} = \frac{G_F^2 m_\mu^5}{192\pi^4} x^2 \left[3(1-x) + \frac{2}{3}\rho(4x-3) + 6\eta \frac{m_e}{m_\mu}\frac{1-x}{x} \right]$$

$$x = \frac{E_e}{E_{e,max}} = \frac{2E_e}{m_\mu}$$

For V−A theory, $\underbrace{\rho = \frac{3}{4}, \quad \eta = 0}_{\text{Michel parameters}}$

$$\Gamma = \int \frac{d\Gamma}{dx\, d\Omega}\, dx\, d\Omega = \frac{G_F^2 m_\mu^5}{192\pi^3} \approx \Gamma(\mu^- \to e^- \nu \bar{\nu}) \approx \Gamma_{tot}$$

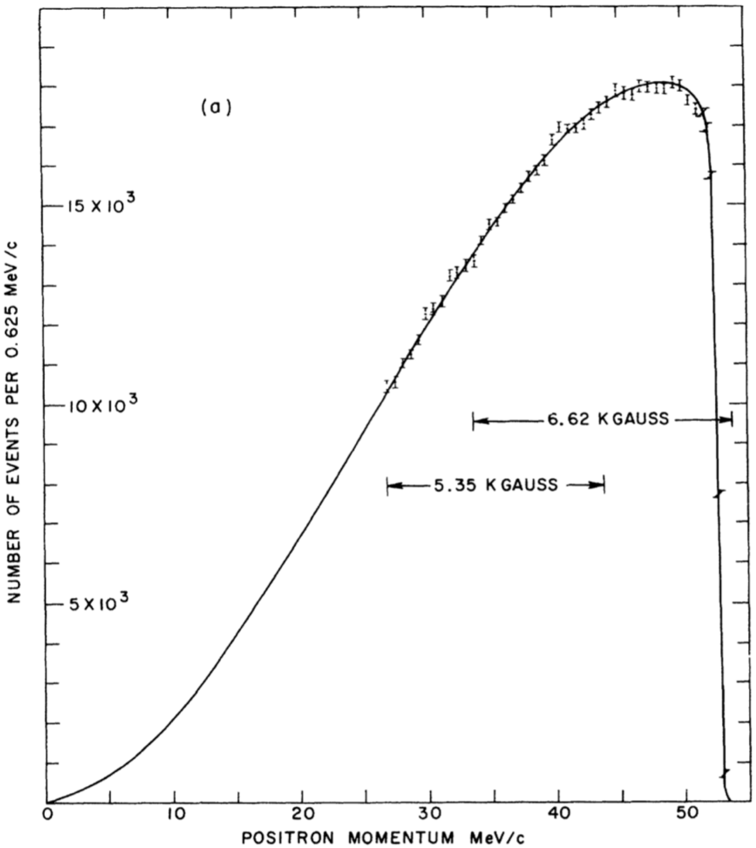
only final state

$$\tau = \frac{1}{\Gamma_{tot}} = 2.2\ \mu s$$

muon lifetime

Energy spectrum of electrons from muon decay

M. Bardon, P. Norton, J. Peoples, A.M. Sachs, and J. Lee-Franzini, *Measurement of the Momentum Spectrum of Positrons from Muon Decay*, Phys. Rev. Lett. 14, 449 (1965)



Excellent agreement with V-A theory of weak interactions (Michel parameter ρ = 0.75).