PH4442 Advanced Particle Physics 2025/26
Lecture Week 7

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- CKM Matrix
- Neutrino scattering
- Neutral currents
- Gauge invariance

CKM matrix definition

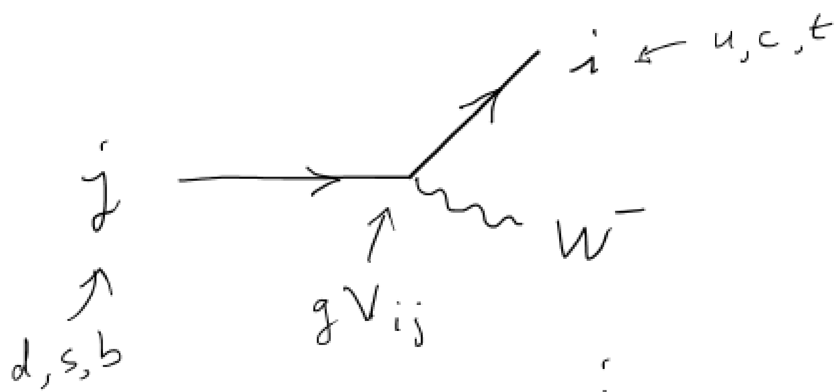
flavour eigenstates

unitary CKM matrix

mass eigenstates

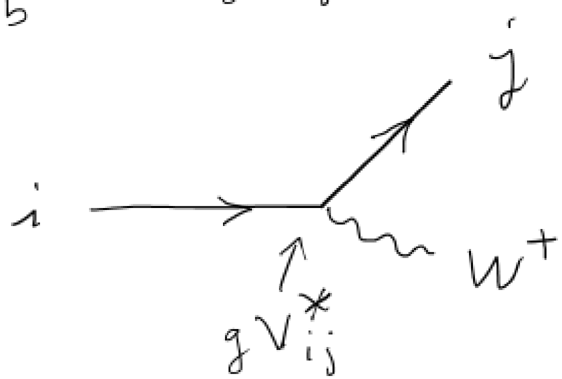
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

row i = u,c,t
column j = d,s,b



$$\frac{-ig}{\sqrt{2}} V_{ij} \gamma^{\mu} \tfrac{1}{2}(1-\gamma^5)$$

V_{ij} if i at start of fermion line



$$\frac{-ig}{\sqrt{2}} V_{ij}^{*} \gamma^{\mu} \tfrac{1}{2}(1-\gamma^5)$$

V_{ij}^{*} if i at end of fermion line

CKM matrix values

Measured magnitudes:

$$V_{\text{CKM}} = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

Wolfenstein parameterisation:

$$VV^\dagger = I + \mathcal{O}(\lambda^4)$$

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\bar{\rho} = \rho(1-\lambda^2/2)$$

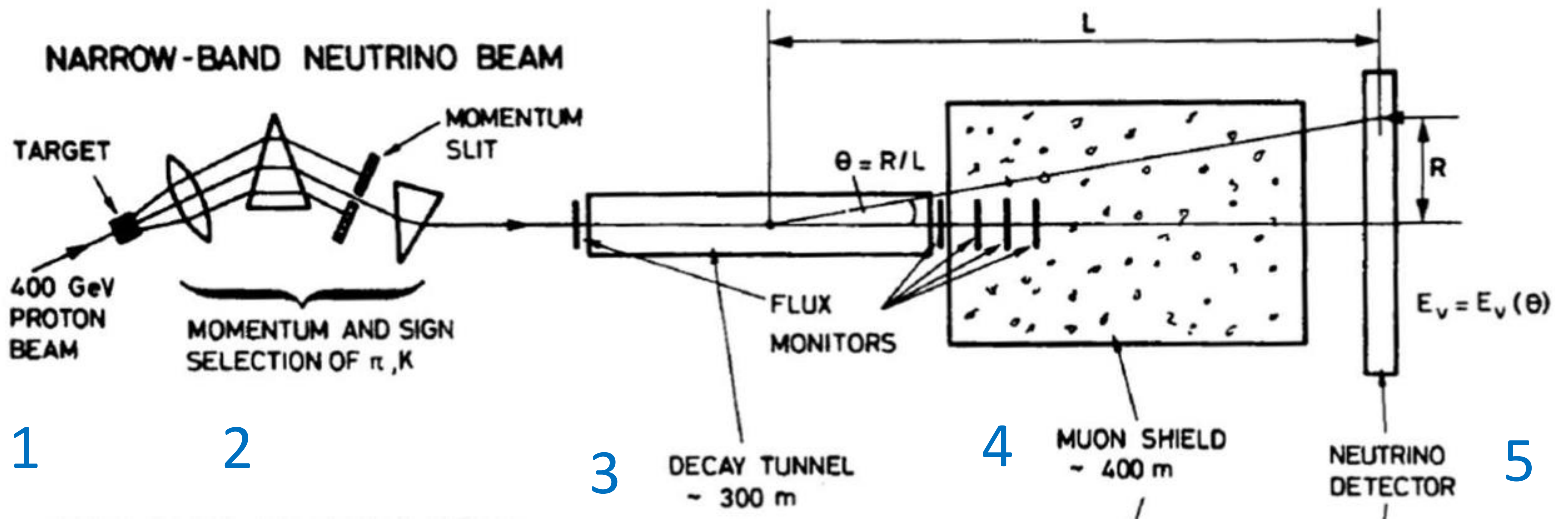
$$\bar{\eta} = \eta(1-\lambda^2)$$

improves unitarity approximation

$$\lambda = 0.22500 \pm 0.00067, \quad A = 0.826^{+0.018}_{-0.015}, \quad \bar{\rho} = 0.159 \pm 0.010, \quad \bar{\eta} = 0.348 \pm 0.01$$

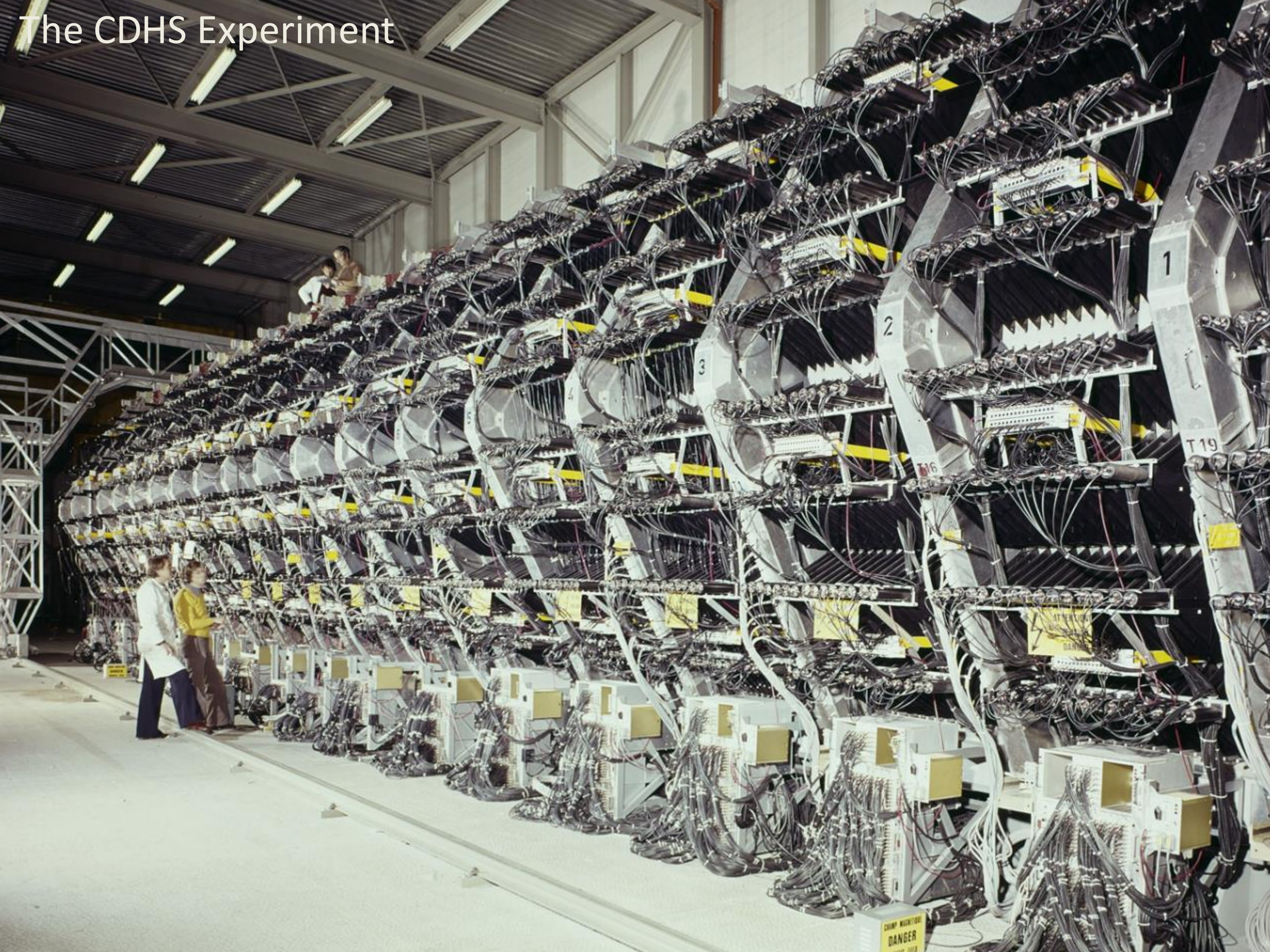
Neutrino scattering experiments

Basic set-up of a neutrino beam:

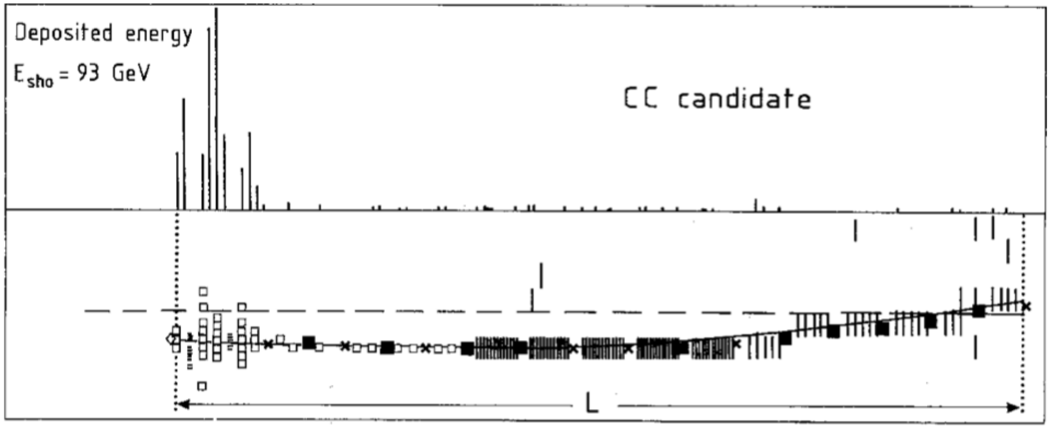


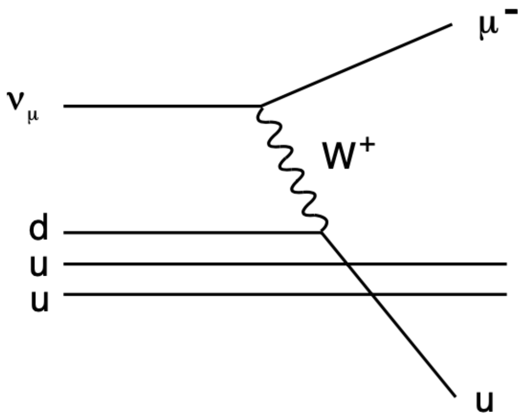
1. High-energy protons collide with target, produce pions, kaons.
2. System of magnets selects momentum and charge of π/K.
3. In decay tunnel, almost all pions decay as $\pi \to \mu\, \nu_\mu$.
4. Muons absorbed in thick shield.
5. Neutrinos proceed to detector, energy from angle to central axis.

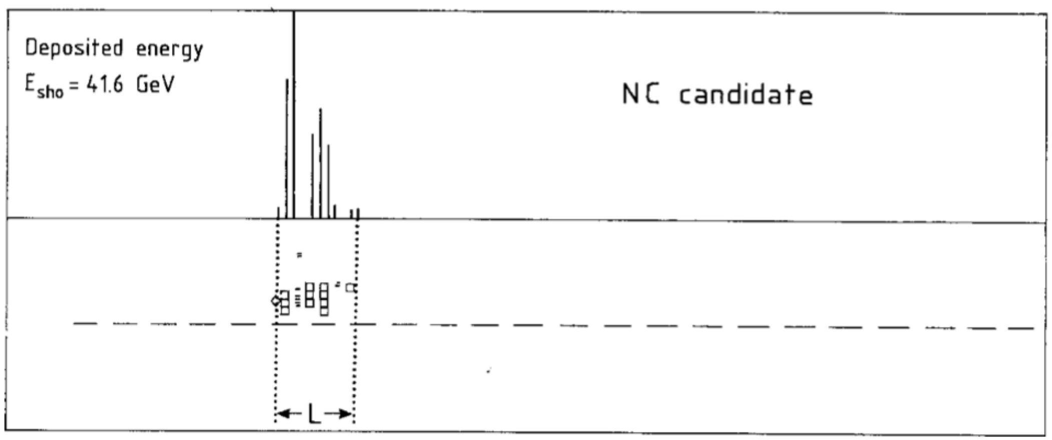
The CDHS Experiment

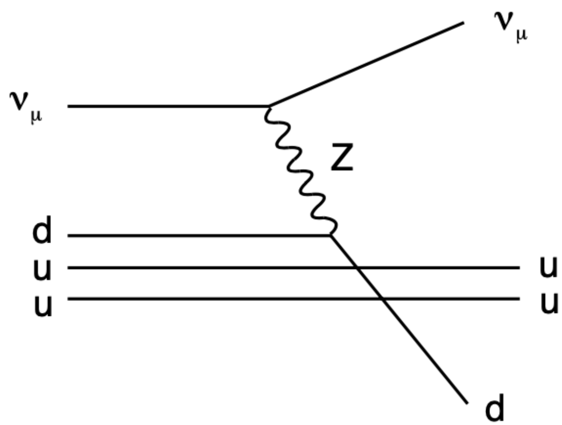


Neutrino scattering events in CDHS







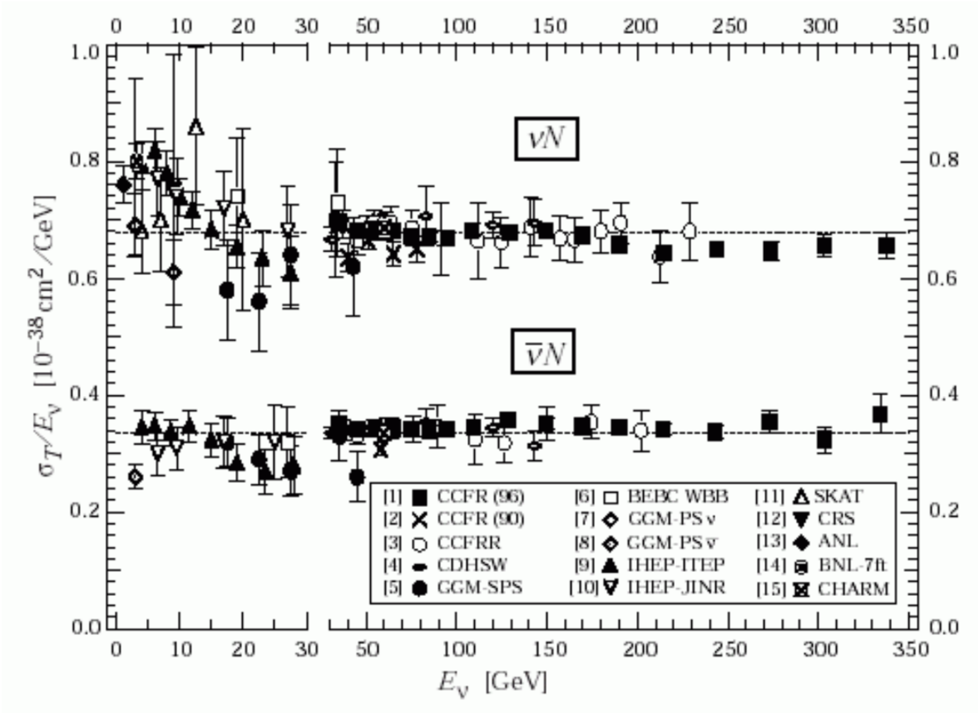


Charged-Current neutrino-nucleon scattering

N = "isoscalar" target (equal numbers of n, p)

Naive prediction where n, p only contain quarks: $$\frac{\sigma(\overline{\nu}N)}{\sigma(\nu N)} = \frac{1}{3}$$

Data show: $\frac{\sigma(\overline{\nu}N)}{\sigma(\nu N)} \approx 0.5$ $\longrightarrow$ evidence of antiquarks in nucleon.



$\sigma \propto E_\nu$ (lab)

$\longrightarrow$ point-like quarks

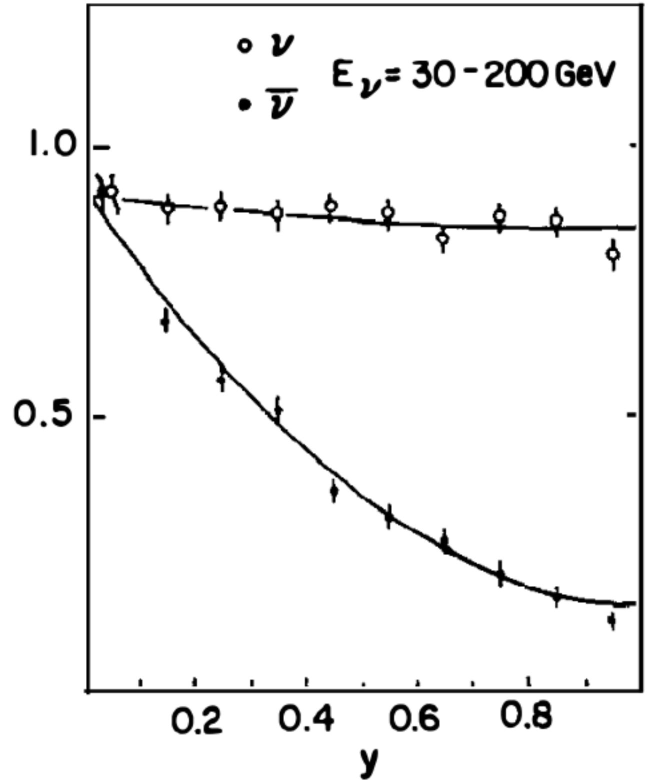
Charged-Current neutrino-nucleon scattering

Inelasticity $y = 1 - \frac{E_\mu}{E_\nu}$ (lab)

For $E_{cm} \gg m_\mu$, $1 - y = \frac{1 + \cos\theta}{2}$ (cm)

Collision	$\frac{d\sigma}{d(\cos\theta)}$	$\frac{d\sigma}{dy}$
$\nu_\mu d$, $\bar{\nu}_\mu \bar{d}$	$\frac{G^2 s}{2\pi}$	$\frac{G^2 s}{\pi}$
$\bar{\nu}_\mu u$, $\nu_\mu \bar{u}$	$\frac{G^2 s}{2\pi}\left(\frac{1+\cos\theta}{2}\right)^2$	$\frac{G^2 s}{\pi}(1-y)^2$

J. G. H. de Groot et al. (CDHS), Z. Phys. C 1, 143 (1979).



Data: $d\sigma(\nu N)/dy$ mostly ~ const.

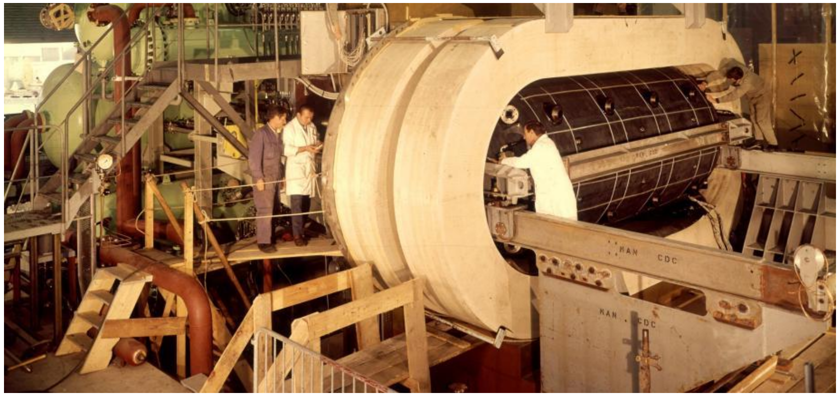
$d\sigma(\bar{\nu}N)/dy$ mostly ~ $(1-y)^2$

Departure from naive prediction because nucleon has some antiquarks.

Gargamelle

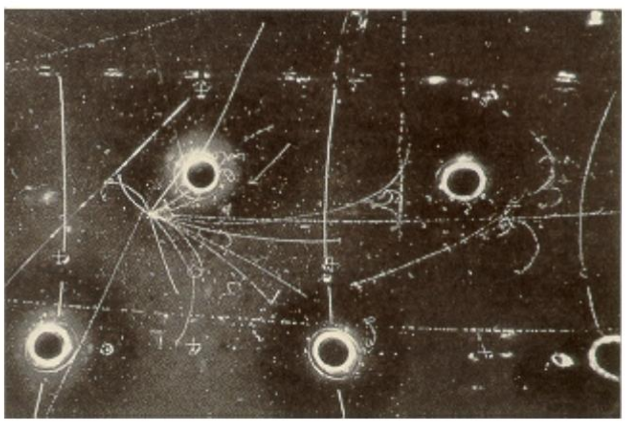
Charged current event in Gargamelle

Photo: CERN

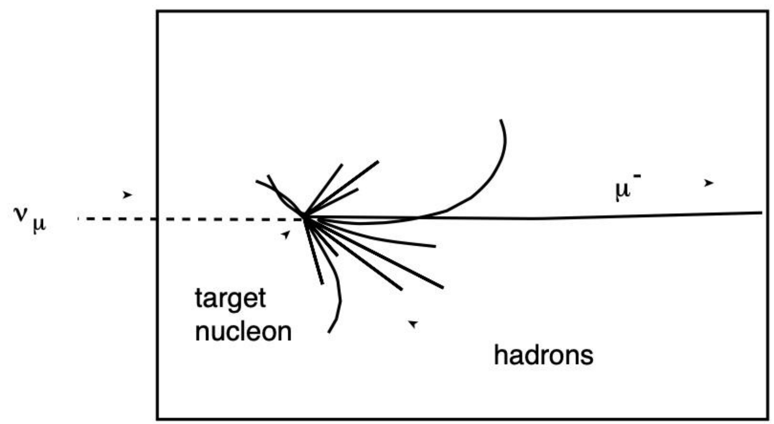


Freon bubble chamber

ν_μ enter from left, hadrons from interaction absorbed, muon exits to the right.



(a)



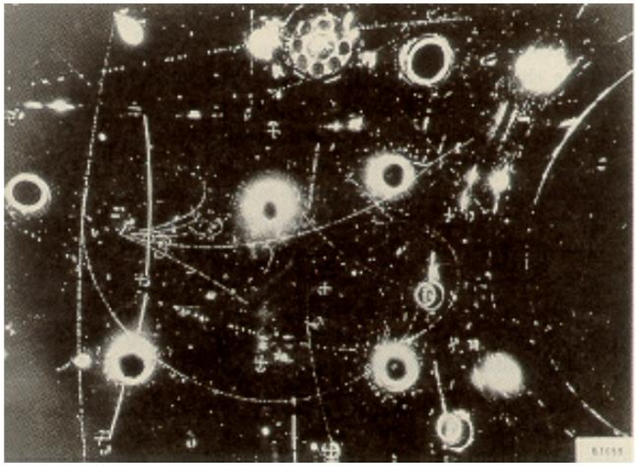
(b)

Figure 9.6: Bubble chamber photograph (left) and its interpretation (right) showing the reaction $\nu_\mu N \rightarrow \mu^- +$ hadrons, where N is a nucleon (from D. Perkins in [22], p. 428). The neutrino enters from the left and the muon exits to the right.

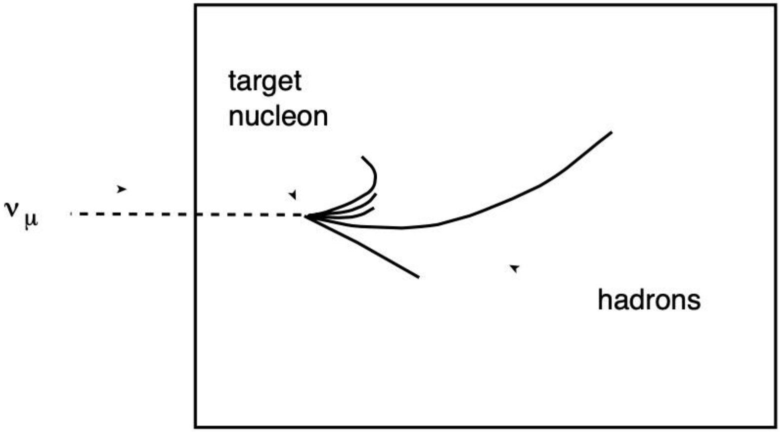
Neutral current event in Gargamelle

No muon seen in final state, only hadrons.

Evidence for Z (CERN, 1973)



(a)

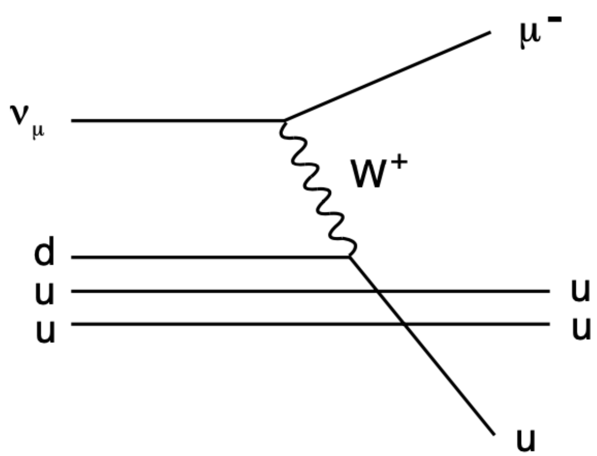


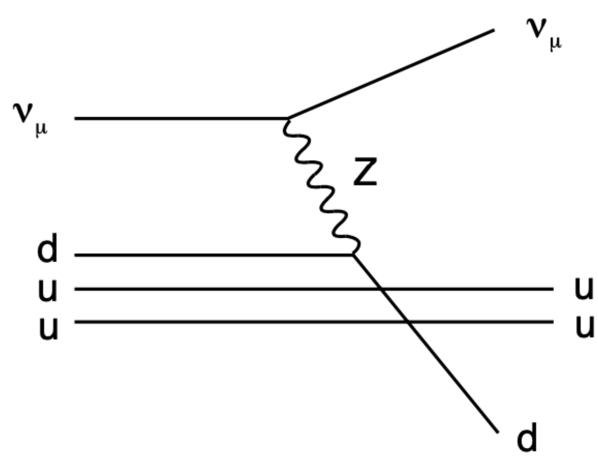
(b)

Figure 9.7: Bubble chamber photograph (left) and its interpretation (right) showing the reaction $\nu_\mu N \to \nu_\mu +$ hadrons (from D. Perkins in [22], p. 428). The neutrino enters from the left. All of the final state particles are identified as hadrons.

The W and Z in the GWS Electroweak Theory

The Glashow-Weinberg-Salam Electroweak Theory predicts Z (1960s), but in 1973 not at all clear that the model was correct.





Discovery of weak neutral currents makes GWS the front runner, convincing confirmation with discovery of W and Z at CERN by UA1 Experiment (1983):

$$M_W = 80.4\,\mathrm{GeV}\,, \qquad M_Z = 91.2\,\mathrm{GeV}\,.$$

Yang-Mills Gauge Theories

The gauge invariance of QED is a special case of a broader class of theory proposed by Yang and Mills in 1954.

Consider the wave equation $(i\gamma^\mu D_\mu - m)\psi = 0$

and the transformation $\psi(x) \to \psi'(x) = U(x)\psi(x)$

where $U(x) = \exp\left[ig\alpha^a(x)T^a\right]$ is an *N* x *N* unitary matrix,

T^a are *N* x *N* Hermitian matrices (generators of the transformation),

$\alpha^a(x)$ are arbitrary functions of x, g is a constant (gauge coupling)

U acts on an *N*-component wave function, where each of the ψ_i is a Dirac spinor.

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \vdots \\ \psi_N(x) \end{pmatrix}$$

Lie groups, Lie algebras

The set of all possible transformations *U* defines the gauge group, which in a Yang-Mills theory is a *Lie* group.

This means that the generators satisfy a Lie algebra:

commutator → structure constants ↙

$$[T^a, T^b] = i f^{abc} T^c$$

as well as the normalisation condition $\mathrm{Tr}(T^a T^b) = \frac{1}{2}\delta^{ab}$

Example: SU(2) = set of unitary 2 x 2 matrices *U* with det *U* = 1

The generators are related to the Pauli matrices: $T^i = \frac{1}{2}\sigma_i,\ i = 1, 2, 3$

The Lie algebra is: $[T^i, T^j] = i\epsilon_{ijk} T^k$ (ε_{ijk} = Levi-Cevita symbol).

Covariant derivative, gauge field transformation

For a gauge group with generators T^a, the covariant derivative is

$$D_\mu = \partial_\mu + igT^a A_\mu^a(x)$$

where we introduce a gauge field $A_\mu{}^a(x)$ for each generator.

The gauge fields transform as

$$A_\mu \to A'_\mu = U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \qquad \text{where} \qquad A_\mu = A_\mu^a T^a$$

For infinitesimal transformations ($|\alpha^a(x)| \ll 1$) we can approximate

$$U(x) \approx 1 + ig\alpha^a(x)T^a, \; U^{-1} \approx 1 - ig\alpha^a(x)T^a$$

and the gauge transformation of the fields becomes

$$A_\mu^a \to A_\mu'^{\,a} = A_\mu^a - \partial_\mu \alpha^a + g f^{abc} \alpha^b A_\mu^c$$

Gauge invariance in Yang-Mills theories

Then as in QED, $D^\mu \psi$ transforms the same as ψ, and as a result the transformed ψ and $A_\mu{}^a$ together are still solutions.

Proof:

$$\begin{aligned} \psi' &= U\psi \\ A'^\mu &= UA^\mu U^{-1} + \frac{i}{g}(\partial^\mu U)U^{-1} \\ D^\mu &= \partial^\mu + igA^\mu \end{aligned}$$

$$\begin{aligned} D'^\mu \psi' &= (\partial^\mu + igA'^\mu)(U\psi) \\ &= (\partial^\mu U)\psi + U\partial^\mu \psi + igA'^\mu U\psi \\ &= (\partial^\mu U)\psi + U\partial^\mu \psi + ig\left[UA^\mu U^{-1} + \frac{i}{g}(\partial^\mu U)U^{-1}\right]U\psi \\ &= U(\partial^\mu + igA^\mu)\psi \end{aligned}$$

Gauge invariance in Yang-Mills theories

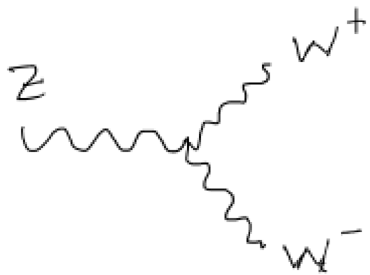
One can then show that (as in QED) $D^\mu\psi$ transforms the same as ψ, as a result the transformed ψ and $A_\mu{}^a$ together are still solutions.

Compare the transformed fields:

$$A'^\mu(x) = A^\mu(x) - \partial^\mu\alpha(x)$$ QED

$$A'^a_\mu = A^a_\mu - \partial_\mu\alpha^a + gf^{abc}\alpha^b A^c_\mu$$ Yang-Mills

A non-abelian Yang-Mills theory (generators do not commute, structure constants nonzero), there is an extra term, which gives rise to gauge boson self coupling.



We will use the gauge groups: $SU(2)_L \times U(1) \rightarrow$ electroweak

$SU(3) \rightarrow$ QCD